

# Individualized Random Tax Reporting Deadlines Would Be Beneficial for Economy

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## Abstract

**Purpose:** While the main purpose of reporting – e.g., reporting for taxes – is to gauge the economic state of a company, the fact that reporting is done at pre-determined dates distorts the reporting results. For example, to create a larger impression of their productivity, companies fire temporary workers before the reporting date and re-hire them right away. The purpose of this study is to decide how to avoid such distortion.

**Design/methodology/approach:** We want to make our solution applicable for all possible reasonable optimality criteria. Thus, we use a general formalism for describing and analyzing all such criteria.

**Findings:** We show that most distortion problems will disappear if we replace the fixed pre-determined reporting dates with individualized random reporting dates. We also show that for all reasonable optimality criteria, the optimal way to assign reporting dates is to do it uniformly.

**Originality/value:** We propose a new idea of replacing the fixed pre-determining reporting dates with randomized ones. On the informal level, this idea may have been proposed earlier, but what is completely new is our analysis of which probability distribution for reporting dates is the best for economy: it turns out that under all reasonable optimality criteria, uniform distribution works the best.

**Keywords:** Tax reporting; Disruption caused by fixed reporting dates; Randomized reporting dates; Optimal distribution of reporting dates.

# 1 Formulation of the Problem

## 1.1 Need for some government regulations and government control

Until the 20th century, there was not much government intervention in economy, the belief was that the “invisible hand” of the markets – using the famous expression by Adam Smith – would magically bring economic growth and economic prosperity. From this viewpoint, the smaller the government role, the better: medicine, education, etc., are better in private hands. Possibly only the army needs to be controlled by the state – but the supply of the army should be in private hands.

To a large extent, this turned out to be true – capitalism indeed led to unprecedented economic growth and the resulting unprecedented improvement in the level of living. However, this was an improvement *on average*. While on average, the level of living was growing, a significant portion of the population was illiterate – because they could afford education, literally starving, and suffering (and dying) from easily curable diseases because of the absence of affordable health care. Similarly, while there were years of tremendous growth and prosperity, but there were years of deep crises and massive sufferings.

The last such catastrophic crisis occurred in the late 1920s-1930s. This crisis was the last straw that convinced sceptics all over the world that some government intervention in economy is necessary. Of course, many countries overdid it, and introduced too much government control – which also had a negative effect on economy, but the fact that nowadays, pandemic notwithstanding, the economy is improving all over the world has shown that a correct compromise between too little and too much government intervention has been found.

This intervention occurs on different levels: on the level of the state banks that regulate interest rates and thus, regulate the economy, and on the level of government spending. The government collects taxes and spends them on education, research, and development – with the ultimate goal to help economy – and on the social welfare.

## 1.2 How taxes and government regulations are determined now

In most countries, taxes are collected on a yearly or quarterly basis: the amount of taxes depends on the financial situation by a certain date. Similarly, the government regulations depend on the state of economy by a certain data – e.g., by the level of economic growth, unemployment, inflation, etc. at the end of each quarter.

## 1.3 Why this is a problem

The main purpose of tax reporting to provide a clear picture of the state of each company (and of the economy as a whole). However, the very fact that this is

gauged by the state of the company at a certain date distorts the picture.

For example, the company's productivity – one of the important characteristics determining the company's stock price – is obtained, crudely speaking, by dividing the profit by the number of workers. At first glance, this is exactly what productivity is, but the problem is that, based on the way reporting is set, the profit is the whole profit during the whole reporting period, while the number of workers is the number of workers at the reporting date. So, to create a better impression of the productivity, a company may (and some do) fire temporary workers just before the reporting date and re-hire them once the date has passed.

There are many other similar well-known distortions. It even affects private life. For example, in the US, in many cases, it is better tax-wise to get married in early January than in December, etc.

The governments are very familiar with these problems, they are always updating the tax rules – but still, some new loopholes are found again and again.

In the Soviet Union, where one of us (OK) is originally from, there was a similar problem with the planned economy: if an enterprise did not fulfil the yearly plan, it was bad, the leadership could be replaced. However, if due to many efforts, the plan was over-fulfilled, it was also not good – since the next year's plan was based on the current year's achievements, and if all the potential for growth was exhausted this year, next year's plan had a good chance of failing. As a result, enterprises somewhat slowed down before the new year – to avoid over-fulfilment – and then rushed in the last few days. This rush often led to reduced quality of the products, so we all knew to look for the production date and avoid things produced at the end of the year.

Things are definitely better under capitalism, but, as we have mentioned, similar fixed-date reporting problems remain.

## **1.4 What we do in this paper**

In this paper, we show that to avoid the above problems, there is a straightforward – but somewhat radical – solution: to replace the fixed reporting dates with randomized dates. We also show that the optimal way to arrange these randomized reporting dates is to use the uniform distribution.

## **2 Main Idea**

### **2.1 Doping testing for athletes: situation with similar possible problems**

In professional sports, doping is a big problem, when prohibited chemical substances are used to boost the athletes' performance. To prevent this from happening, athletes are periodically tested for the presence of different possible prohibited substances.

It is well known that in such a situation, tests performed at known dates do not make much sense: the athlete intending to cheat will simply stop using the illegal drug shortly before the test and then resume using it immediately after.

The known solution to this problem is to have tests at *random* times.

## 2.2 Testing at random times is exactly what we propose for economic reporting

Testing at random times is exactly what we propose to solve the above economic problems. If the company does not know at what day it will be required to report its number of workers, it makes no sense to distort the productivity statistics by firing people only to immediately re-hire them. If tax deadline is randomly determined at an unpredictable time, there is no tax advantage in delaying marriage.

With this change, the reporting will more adequately reflect the current state of the economy.

## 2.3 Additional advantage of the proposed scheme

At present, when everyone has the same deadline for reporting taxes, accountants who help with this reporting are overworked right before the due date – and under-worked at all other times. Similarly, the tax services are overwhelmed immediately after the tax deadline – which creates delays for taxpayers who over-paid to get money back.

If we make tax dates individually random, then both the accountants who help the taxpayers and the government agency that processes tax returns will have their work spread more equally, thus drastically decreasing delays.

# 3 What Is the Best Way of Implementing This Idea

## 3.1 Towards a precise formulation of the problem

The fact that the reporting times are random means that we cannot pre-determine these times, all we can do is determine the probability that the randomized reporting time will happen at different time intervals. One way to describe this is to describe the *density*  $f(t)$  of reporting times, i.e., the expected number of reporting times per given time interval – so that for each time interval  $[t, \bar{t}]$ , the expected number of reporting times within this interval is equal to  $\int_t^{\bar{t}} f(t) dt$ .

From this viewpoint, the problem is – what is the *optimal* density  $f(t)$ ?

### 3.2 What do we mean by optimal? Problems with the traditional approach

The usual way to describe what is optimal is to select an objective function, and to pick up an alternative for which the value of this objective function is the largest (or, if we are minimizing, the smallest). There are two problems with this usual approach.

The first problem is that often, it is not sufficient to describe a single objective function. Let us give an economy-related example. For a company, a natural objective function is the overall expected profit – taking into account future profits (with appropriate discounts). However, often, there are several different alternative with the same expected profit. In this case, a reasonable idea is to use this non-uniqueness to select, among the best-profit alternatives, the one for which, e.g., the risk is the smallest. If there are still several alternatives with the same values of expected profit and the same value of expected risk, we can use the remaining non-uniqueness to select the alternative for which the effect of the environment will be the smallest – or the one that enables the company to preserve most of its workforce. In all these cases, the criterion by which the company selects an alternative is more complicated than using a single objective function.

The second problem with the usual approach is that for different objective functions, we get, in general, different optimal solutions. So, instead of trying to pick a single objective function, it is desirable to come up with a way to find an alternative that is optimal with respect to *all* reasonable objective functions.

Let us see how we can overcome both problems.

### 3.3 Comment: without losing generality, we can consider only maximizing objective functions

In some cases, we are looking for objective functions that maximize. In other cases, we are looking for alternatives that minimize the given objective function  $F(x)$  – e.g., we want to minimize the risk. This can be reduced to maximization if instead of the original objective function  $F(x)$ , we consider a new objective function  $F_{\text{new}}(x) \stackrel{\text{def}}{=} -F(x)$ . Clearly, maximizing  $F_{\text{new}}(x) = -F(x)$  is equivalent to minimizing  $F(x)$ .

So, every minimization problem can be easily reformulated as a maximization problem. Thus, without losing generality, we can restrict ourselves to the case of maximizing objective functions.

### 3.4 Towards a general description of optimality

In the usual description of optimality, for maximizing objective functions we select an objective function  $F(x)$ , and we say that an alternative  $a$  is better than an alternative  $b$  if  $F(a) > F(b)$ . If  $F(a) = F(b)$ , we say that the alternatives  $a$  and  $b$  are of the same value with respect to the given criterion. We say that an alternative  $a$  is optimal if  $F(a) \geq F(b)$  for all alternatives  $b$ .

As we have mentioned earlier, if there are several optimal alternatives, then we can use this non-uniqueness to optimize some other objective function  $G(x)$ . In this case, we have a more complicated criterion for comparing two alternatives:

- we say that an alternative  $a$  is better than an alternative  $b$ , if either  $F(a) > F(b)$ , or we have  $F(a) = F(b)$  and  $G(a) > G(b)$ ;
- we say that an alternative  $a$  is of the same quality as an alternative  $b$  with respect to our optimality criterion if we have  $F(a) = F(b)$  and  $G(a) = G(b)$ .

We say that an alternative  $a$  is optimal if it is either better or of the same quality as all other alternatives.

As we have mentioned, even after this refinement, we can still have several optimal alternatives. In this case, we can use this non-uniqueness to optimize some other objective function  $H(x)$ . Then, we get even more complicated ideas of which alternative is better and what it means for an alternative to be optimal. How can we come up with a general definition that covers all such settings?

From the viewpoint of the decision maker, what we really need is a way to compare the alternatives.

- For some pairs  $(a, b)$  of alternatives, we want to conclude that  $a$  is better than  $b$ ; we will denote this by  $a > b$ .
- For some other pairs  $(a, b)$ , we want to conclude that the alternatives  $a$  and  $b$  are of the same quality with respect to the given optimality criterion. We will denote this by  $a \sim b$ .

Of course, these conclusions must be consistent: e.g., if  $a$  is better than  $b$ , and  $b$  is better than  $c$ , then we should be able to conclude that  $a$  is better than  $c$ .

Thus, it makes sense to define a general optimality criterion as a pair of relations  $(>, \sim)$ . Once such relations are given, we say that an alternative  $a$  is optimal if for every other alternative  $b$ , we have either  $a > b$  or  $a \sim b$ .

If there are several optimal alternatives, this means that the given optimality criterion is not final: we can use this non-uniqueness to optimize some other criterion and thus, in effect, to change the optimality criterion. So, when the criterion is final, there is only one optimal alternative. (Of course, there should be at least one optimal alternative – otherwise, the optimality criterion is useless.)

So, we arrive at the following definition.

### 3.5 Definition 1

Let  $A$  be a set. Elements of this set will be called *alternatives*. By an *optimality criterion*, we mean a pair  $(>, \sim)$  of binary relations on the set  $A$  for which the following conditions hold for every three alternatives  $a$ ,  $b$ , and  $c$ :

- if  $a > b$  and  $b > c$ , then  $a > c$ ;

- if  $a > b$  and  $b \sim c$ , then  $a > c$ ;
- if  $a \sim b$  and  $b > c$ , then  $a > c$ ;
- if  $a \sim b$  and  $b \sim c$ , then  $a \sim c$ ;
- $a \sim a$  and  $a \not\sim a$ .

We say that an alternative  $a$  is *optimal* if for every other alternative  $b$ , we have either  $a > b$  or  $a \sim b$ . We say that the optimality criterion is *final* if there exists exactly one optimal alternative.

### 3.6 From general definition of optimality to our problem

In our case, alternatives are different density functions  $f(t)$ . The main problems with the traditional deterministic setting of reporting dates are caused by the fact that these dates are fixed to some moments of time, while the reasonable objective functions – prosperity of the country, prosperity of the company, etc. – should not depend on an arbitrarily chosen date. Let us describe this not-depending in precise terms.

Suppose that we change the US tax report date from the current April 15 to some other date, e.g., to April 13. This means, in effect, that what corresponded to day  $t$  now corresponds to day  $t + t_0$ , where, in this case,  $t_0 = 2$  days. So, what was previously the density  $f(t)$  becomes  $f(t + t_0)$ .

This simple shift should not change the relative quality of two densities: if we had  $f > g$ , then we should have the same relation for the shifted densities. Thus, we arrive at the following definition.

### 3.7 Definition 2

Let  $(>, \sim)$  be an optimality criterion on the set all non-negative functions  $f(t)$ ; we will call such functions *density functions*. For each function  $f(t)$  and for each value  $t_0$ , we can define a shifted function  $S_{t_0}(f)$  for which  $(S_{t_0}(f))(t) = f(t + t_0)$ . We say that the optimality criterion is *shift-invariant* if  $f > g$  implies that  $S_{t_0}(f) > S_{t_0}(g)$ , and  $f \sim g$  implies that  $S_{t_0}(f) \sim S_{t_0}(g)$ .

### 3.8 Main Result

*For every shift-invariant final optimality criterion, the optimal density function is a constant.*

### 3.9 Discussion

Thus, for all reasonable optimality criteria, the uniform distribution of recording dates works the best.

This result is similar to several results proven in [1].

### 3.10 Proof

Let  $f_{\text{opt}}$  be the optimal function. This means that for every other function  $g$ , we have either  $f_{\text{opt}} > g$  or  $f_{\text{opt}} \sim g$ . In particular, for every  $g$ , we have  $f_{\text{opt}} > S_{-t_0}(g)$  or  $f_{\text{opt}} \sim S_{-t_0}(g)$ . Due to shift-invariance, this implies that either  $S_{t_0}(f_{\text{opt}}) > S_{t_0}(S_{-t_0}(g)) = g$  or  $S_{t_0}(f_{\text{opt}}) \sim g$ .

Since this is true for all alternatives  $g$ , this means that the alternative  $S_{t_0}(f_{\text{opt}})$  is also optimal. However, since the optimality criterion is final, there is only one optimal alternative. So, we must have  $S_{t_0}(f_{\text{opt}}) = f_{\text{opt}}$  for all  $t_0$ . This means that  $f_{\text{opt}}(t + t_0) = f_{\text{opt}}(t)$  for all  $t$  and  $t_0$ . For every two values  $t$  and  $t'$ , by taking  $t_0 = t' - t$ , we conclude that  $f_{\text{opt}}(t) = f_{\text{opt}}(t')$ . Thus, the function  $f_{\text{opt}}(t)$  is indeed constant.

The proposition is proven.

## References

- [1] H. T. Nguyen and V. Kreinovich, *Applications of continuous mathematics to computer science*, Kluwer, Dordrecht, 1997.