

# Randomized Tax Deadlines Can Help Economy

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## Abstract

**Purpose:** While the main purpose of reporting – e.g., reporting for taxes – is to gauge the economic state of a company, the fact that reporting is done at pre-determined dates distorts the reporting results. For example, to create a larger impression of their productivity, companies fire temporary workers before the reporting date and re-hire then right away. The purpose of this study is to decide how to avoid such distortion.

**Design/methodology/approach:** We want to make our solution applicable for all possible reasonable optimality criteria. Thus, we use a general formalism for describing and analyzing all such criteria.

**Findings:** We show that most distortion problems will disappear if we replace the fixed pre-determined reporting dates with individualized random reporting dates. We also show that for all reasonable

optimality criteria, the optimal way to assign reporting dates is to do it uniformly.

**Originality/value:** We propose a new idea of replacing the fixed pre-determining reporting dates with randomized ones. On the informal level, this idea may have been proposed earlier, but what is completely new is our analysis of which randomization of reporting dates is the best for economy: it turns out that under all reasonable optimality criteria, uniform randomization works the best.

**Keywords:** Tax reporting; Disruption caused by fixed reporting dates; Randomized reporting dates; Optimal distribution of reporting dates.

# 1 Formulation of the Problem

## 1.1 *Need for some government regulations and government control*

Until the 20th century, there was not much government intervention in economy, the belief was that the “invisible hand” of the markets – using the famous expression by Adam Smith – would magically bring economic growth and economic prosperity. From this viewpoint, the smaller the government role, the better: medicine, education, etc., are better in private hands. Possibly only the army needs to be controlled by the state – but the supply of the army should be in private hands. The resulting relaxation of government intervention led to economic growth – but also to deep crises.

The last such catastrophic crisis occurred in the late 1920s-1930s. This crisis was the last straw that convinced sceptics all over the world that some government intervention in economy is necessary. Of course, many countries overdid it, and introduced too much government control – which also had a negative effect on economy, but the fact that nowadays, pandemic notwithstanding, the economy is improving all over the world has shown that a correct compromise between too little and too much government intervention has been found.

This intervention occurs on different levels: on the level of the state banks that regulate interest rates and thus, regulate the economy, and on the level of government spending. The government collects taxes and spends them on education, research, and development – with the ultimate goal to help economy – and on the social welfare.

## *1.2 How taxes and government regulations are determined now*

In most countries, taxes are collected on a yearly or quarterly basis: the amount of taxes depends on the financial situation by a certain date. Similarly, the government regulations depend on the state of economy by a certain date – e.g., by the level of economic growth, unemployment, inflation, etc. at the end of each quarter.

## *1.3 Why this is a problem*

One of the main purposes of financial reporting is to provide a clear picture of the state of each company (and of the economy as a whole). In particular, one of the main purposes of tax-related reporting is to make sure that the company pays its fair share of taxes. However, the very fact that this is gauged by the state of the company at a certain date distorts the picture.

For example, the company's productivity – one of the important characteristics determining the company's stock price – is obtained, crudely speaking, by dividing the profit by the number of workers. At first glance, this is exactly what productivity is, but the problem is that, based on the way reporting is set, the profit is the whole profit during the whole reporting period, while the number of workers is the number of workers at the reporting date. So, to create a better impression of the productivity, a company may (and some do) fire temporary workers just before the reporting date and re-hire them once the date has passed.

There are many other similar well-known distortions. It even affects private life. For example, in the US, in many cases, it is better tax-wise to get

married in early January than in December, etc.

The governments are very familiar with these problems, they are always updating the tax rules – but still, some new loopholes are found again and again; see, e.g., (Griffiths, 1992), (Oliveras and Amat, 2003), (Amat and Gowthorpe, 2004), (Muford and Comiskey, 2005), (de la Torre, 2008), (Jones, 2011).

#### *1.4 What we do in this paper*

In this paper, we use the general ideas of decision theory – see, e.g., (Fishburn, 1969), (Fishburn, 1988), (Luce and Raiffa, 1989), (Raiffa, 1997), (Nguyen, Kosheleva, and Kreinovich, 2009), (Nguyen et al., 2012), (Kreinovich, 2014) – to show that to avoid the above problems, there is a straightforward – but somewhat radical – solution: to replace the fixed reporting dates with randomized dates. We also show that the optimal way to arrange these randomized reporting dates is to use the uniform randomization.

## **2 Main Idea**

### *2.1 Doping testing for athletes: situation with similar possible problems*

In professional sports, doping is a big problem, when prohibited chemical substances are used to boost the athletes' performance. To prevent this from happening, athletes are periodically tested for the presence of different possible prohibited substances.

It is well known that in such a situation, tests performed at known dates do not make much sense: the athlete intending to cheat will simply stop using the illegal drug shortly before the test and then resume using it immediately after.

The known solution to this problem is to have tests at *random* times.

## *2.2 Testing at random times is exactly what we propose for economic reporting*

Testing at random times is exactly what we propose to solve the above economic problems. If the company does not know at what day it will be required to report its number of workers, it makes no sense to distort the productivity statistics by firing people only to immediately re-hire them. If tax deadline is randomly determined at an unpredictable time, there is no tax advantage in delaying marriage.

With this change, the reporting will more adequately reflect the current state of the economy.

## *2.3 Additional advantage of the proposed scheme*

At present, when everyone has the same deadline for reporting taxes, accountants who help with this reporting are overworked right before the due date – and under-worked at all other times. Similarly, the tax services are overwhelmed immediately after the tax deadline – which creates delays for taxpayers who over-paid to get money back.

If we make tax dates individually random, then both the accountants who help the taxpayers and the government agency that processes tax returns will have their work spread more equally, thus drastically decreasing delays.

# **3 What Is the Best Way of Implementing This Idea**

## *3.1 Towards a precise formulation of the problem*

The fact that the reporting times are random means that we cannot pre-determine these times, all we can do is determine the probability that the randomized reporting time will happen at different time intervals. One way to describe this is to describe the *density*  $f(t)$  describing reporting times, i.e., the expected number of reporting times per given time interval – so that for

each time interval  $[\underline{t}, \bar{t}]$ , the mean value  $E[N_r([\underline{t}, \bar{t}])]$  of the number  $N_r([\underline{t}, \bar{t}])$  of reporting times within this time interval is equal to

$$E[N_r([\underline{t}, \bar{t}])] = \int_{\underline{t}}^{\bar{t}} f(t) dt.$$

From this viewpoint, the problem is – what is the *optimal* density function  $f(t)$ ?

### 3.2 Clarification

Please note that  $f(t)$  is *different* from the probability density function; e.g.:

- while for a one-year period, the expected number of reporting times is 1:

$$E[N_r([\underline{t}, \bar{t}])] = \int_{\underline{t}}^{\bar{t}} f(t) dt = 1,$$

- for the two-year period, the overall expected number of reporting times is 2:

$$E[N_r([\underline{t}, \bar{t}])] = \int_{\underline{t}}^{\bar{t}} f(t) dt = 2, \text{ etc.}$$

### 3.3 What do we mean by optimal? Problems with the traditional approach to describing optimality

In general, out of several alternatives, we need to select the optimal one. In our case, alternatives are density functions, so our goal is to select the optimal density function  $f(t)$ .

The usual way to describe what is optimal is to select an objective function, and to pick up an alternative for which the value of this objective function is the largest (or, if we are minimizing, the smallest). There are two problems with this usual approach.

The first problem is that often, it is not sufficient to describe a single objective function. Let us give an economy-related example. For a company, a natural objective function is the overall expected profit – taking into account future profits (with appropriate discounts). However, often, there are

several different alternatives with the same expected profit. In this case, a reasonable idea is to use this non-uniqueness to select, among the best-profit alternatives, the one for which, e.g., the risk is the smallest. If there are still several alternatives with the same values of expected profit and the same value of expected risk, we can use the remaining non-uniqueness to select the alternative for which the effect of the environment will be the smallest – or the one that enables the company to preserve most of its workforce. In all these cases, the criterion by which the company selects an alternative is more complicated than using a single objective function.

The second problem with the usual approach is that for different objective functions, we get, in general, different optimal solutions. So, instead of trying to pick a single objective function, it is desirable to come up with a way to find an alternative that is optimal with respect to *all* reasonable objective functions.

Let us see how we can overcome both problems.

### *3.4 Comment: without losing generality, we can consider only maximizing objective functions*

In some cases, we are looking for objective functions that maximize. In other cases, we are looking for alternatives that minimize the given objective function  $F(x)$  – e.g., we want to minimize the risk. This can be reduced to maximization if instead of the original objective function  $F(x)$ , we consider a new objective function  $F_{\text{new}}(x) \stackrel{\text{def}}{=} -F(x)$ . Clearly, maximizing  $F_{\text{new}}(x) = -F(x)$  is equivalent to minimizing  $F(x)$ .

So, every minimization problem can be easily reformulated as a maximization problem. Thus, without losing generality, we can restrict ourselves to the case of maximizing objective functions.

### *3.5 Towards a general description of optimality*

In the usual description of optimality, for maximizing objective functions we select an objective function  $F(x)$ , and we say that an alternative  $a$  is

better than an alternative  $b$  if  $F(a) > F(b)$ . If  $F(a) = F(b)$ , we say that the alternatives  $a$  and  $b$  are of the same value with respect to the given criterion. We say that an alternative  $a$  is optimal if  $F(a) \geq F(b)$  for all alternatives  $b$ .

As we have mentioned earlier, if there are several optimal alternatives, then we can use this non-uniqueness to optimize some other objective function  $G(x)$ . In this case, we have a more complicated criterion for comparing two alternatives:

- we say that an alternative  $a$  is better than an alternative  $b$ , if either  $F(a) > F(b)$ , or we have  $F(a) = F(b)$  and  $G(a) > G(b)$ ;
- we say that an alternative  $a$  is of the same quality as an alternative  $b$  with respect to our optimality criterion if we have  $F(a) = F(b)$  and  $G(a) = G(b)$ .

We say that an alternative  $a$  is optimal if it is either better or of the same quality as all other alternatives.

As we have mentioned, even after this refinement, we can still have several optimal alternatives. In this case, we can use this non-uniqueness to optimize some other objective function  $H(x)$ . Then, we get even more complicated ideas of which alternative is better and what it means for an alternative to be optimal. How can we come up with a general definition that covers all such settings?

From the viewpoint of the decision maker, what we really need is a way to compare the alternatives.

- For some pairs  $(a, b)$  of alternatives, we want to conclude that  $a$  is better than  $b$ ; we will denote this by  $a \succ b$ .
- For some other pairs  $(a, b)$ , we want to conclude that the alternatives  $a$  and  $b$  are of the same quality with respect to the given optimality criterion. We will denote this by  $a \sim b$ .

Of course, these conclusions must be consistent: e.g., if  $a$  is better than  $b$ , and  $b$  is better than  $c$ , then we should be able to conclude that  $a$  is better than  $c$ .



Thus, it makes sense to define a general optimality criterion as a pair of relations  $(\succ, \sim)$ . Once such relations are given, we say that an alternative  $a$  is optimal if for every other alternative  $b$ , we have either  $a \succ b$  or  $a \sim b$ .

If there are several optimal alternatives, this means that the given optimality criterion is not final: we can use this non-uniqueness to optimize some other criterion and thus, in effect, to change the optimality criterion. So, when the criterion is final, there is only one optimal alternative. (Of course, there should be at least one optimal alternative – otherwise, the optimality criterion is useless.)

So, we arrive at the following definition.

### 3.6 Definition 1

Let  $A$  be a set. Elements of this set will be called *alternatives*. By an *optimality criterion*, we mean a pair  $(\succ, \sim)$  of binary relations on the set  $A$  for which the following conditions hold for every three alternatives  $a$ ,  $b$ , and  $c$ :

- if  $a \succ b$  and  $b \succ c$ , then  $a \succ c$ ;
- if  $a \succ b$  and  $b \sim c$ , then  $a \succ c$ ;
- if  $a \sim b$  and  $b \succ c$ , then  $a \succ c$ ;
- if  $a \sim b$  and  $b \sim c$ , then  $a \sim c$ ;
- $a \sim a$  and  $a \not\sim a$ .

We say that an alternative  $a$  is *optimal* if for every other alternative  $b$ , we have either  $a \succ b$  or  $a \sim b$ . We say that the optimality criterion is *final* if there exists exactly one optimal alternative.

### 3.7 From general definition of optimality to our problem

In our case, alternatives are different density functions  $f(t)$ . The main problems with the traditional deterministic setting of reporting dates are caused by the fact that these dates are fixed to some moments of time, while the

reasonable objective functions – prosperity of the country, prosperity of the company, etc. – should not depend on an arbitrarily chosen date. Let us describe this not-depending in precise terms.

Suppose that we change the US tax report date from the current April 15 to some other date, e.g., to April 13. This means, in effect, that what corresponded to day  $t$  now corresponds to day  $t + t_0$ , where, in this case,  $t_0 = 2$  days. So, what was previously the density function  $f(t)$  becomes  $f(t + t_0)$ .

This simple shift should not change the relative quality of two density functions: if we had  $f \succ g$ , then we should have the same relation for the shifted density functions. Thus, we arrive at the following definition.

### 3.8 Definition 2

Let  $(\succ, \sim)$  be an optimality criterion on the set all non-negative functions  $f(t)$ ; we will call such functions *density functions*. For each function  $f(t)$  and for each value  $t_0$ , we can define a shifted function  $S_{t_0}(f)$  for which  $(S_{t_0}(f))(t) = f(t + t_0)$ . We say that the optimality criterion is *shift-invariant* if  $f \succ g$  implies that  $S_{t_0}(f) \succ S_{t_0}(g)$ , and  $f \sim g$  implies that  $S_{t_0}(f) \sim S_{t_0}(g)$ .

### 3.9 Clarification

In this definition,  $f \succ g$  means that the density function  $f$  is better than the density function  $g$  according to some optimality criterion. In our analysis, we do not specify in what sense it is better: it could refer to short-term benefits for the country's economy, it could refer to long-term benefits, it could refer to some other desirable economic objective.

Our main result – described in the following paragraph – is general: no matter what optimality criterion we use, as long as this criterion is shift-invariant, the optimal density function is  $f(t) = \text{const.}$

### 3.10 Main Result

*For every shift-invariant final optimality criterion, the optimal density func-*

tion is a constant.

### 3.11 Discussion

Constant density function means that the probability for a reporting date to be within any time period is proportional to this time period. For example, for each month, the probability that the reporting date will fall in this month is  $1/12$ . For each day of the year, the probability that the reporting date will fall on this particular day, is equal to  $1/365$ . In this sense, the constant density function means that the reporting date should be selected randomly, according to the uniform distribution.

Thus, for all reasonable optimality criteria, the uniform distribution of recording dates works the best.

In particular, with respect to tax reporting, what we propose is that each day, for each individual and for each company, with probability  $1/365$ , we send this person or company a request for tax reporting. The next day, the same procedure repeats again, so other individuals and companies get such requests, etc.

### 3.12 Proof

From the mathematical viewpoint, this result is similar to several results proven in (Nguyen and Kreinovich, 1997).

Let  $f_{\text{opt}}$  be the optimal function. This means that for every other function  $g$ , we have either  $f_{\text{opt}} \succ g$  or  $f_{\text{opt}} \sim g$ . In particular, for every  $g$ , we have  $f_{\text{opt}} \succ S_{-t_0}(g)$  or  $f_{\text{opt}} \sim S_{-t_0}(g)$ . Due to shift-invariance, this implies that either  $S_{t_0}(f_{\text{opt}}) \succ S_{t_0}(S_{-t_0}(g)) = g$  or  $S_{t_0}(f_{\text{opt}}) \sim g$ .

Since this is true for all alternatives  $g$ , this means that the alternative  $S_{t_0}(f_{\text{opt}})$  is also optimal. However, since the optimality criterion is final, there is only one optimal alternative. So, we must have  $S_{t_0}(f_{\text{opt}}) = f_{\text{opt}}$  for all  $t_0$ . This means that  $f_{\text{opt}}(t + t_0) = f_{\text{opt}}(t)$  for all  $t$  and  $t_0$ . For every two values  $t$  and  $t'$ , by taking  $t_0 = t' - t$ , we conclude that  $f_{\text{opt}}(t) = f_{\text{opt}}(t')$ . Thus, the function  $f_{\text{opt}}(t)$  is indeed constant.

The proposition is proven.

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## References

- O. Amat and C. Gowthorpe, *Creative Accounting: Nature, Incidence and Ethical Issues*, Universitat Pompeu Fabra, Barcelona, Department of Economics and Business, Economics Working Paper 749.
- I. de la Torre, *Creative Accounting Exposed*, Palgrave Macmillan, London, UK, 2008.
- P. C. Fishburn, *Utility Theory for Decision Making*, John Wiley & Sons Inc., New York, 1969.
- P. C. Fishburn, *Nonlinear Preference and Utility Theory*, The John Hopkins Press, Baltimore, Maryland, 1988.
- I. Griffiths, *Creative Accounting: How to Make Your Profits What You Want Them to Be*, Routledge, London, UK, 1992.
- M. J. Jones (ed.), *Creative Accounting, Fraud and International Accounting Scandals*, John Wiley and Sons, Chishester, UK, 2011.
- V. Kreinovich, “Decision making under interval uncertainty (and beyond)”, In: P. Guo and W. Pedrycz (eds.), *Human-Centric Decision-Making Models for Social Sciences*, Springer Verlag, 2014, pp. 163–193.
- R. D. Luce and R. Raiffa, *Games and Decisions: Introduction and Critical Survey*, Dover, New York, 1989.
- C. W. Mulford and E. E. Comiskey, *The Financial Numbers Game: Detecting Creative Accounting Practices*, Wiley, New York, 2005.
- H. T. Nguyen, O. Kosheleva, and V. Kreinovich, “Decision making beyond Arrow’s ‘impossibility theorem’, with the analysis of effects of collusion

and mutual attraction”, *International Journal of Intelligent Systems*, 2009, Vol. 24, No. 1, pp. 27–47.

H. T. Nguyen and V. Kreinovich, *Applications of Continuous Mathematics to Computer Science*, Kluwer, Dordrecht, 1997.

H. T. Nguyen, V. Kreinovich, B. Wu, and G. Xiang, *Computing Statistics under Interval and Fuzzy Uncertainty*, Springer Verlag, Berlin, Heidelberg, 2012.

E. Oliveras and O. Amat, *Ethics and Creative Accounting: Some Empirical Evidence on Accounting for Intangibles in Spain*, Universitat Pompeu Fabra, Barcelona, Department of Economics and Business, Economics Working Paper 732.

H. Raiffa, *Decision Analysis*, McGraw-Hill, Columbus, Ohio, 1997.