Zadeh's Legacy – Successes and Challenges: Preface

Vladik Kreinovich

Zadeh's legacy. In 2021, we celebrated 100th birthday of Lotfi Zadeh, and this volume is a part of this celebration. Most papers in this volume are related to Zadeh's legacy, emphasizing and exploring its successes and providing new steps towards solving remaining challenges.

Professor Zadeh lived a long and fruitful life, his results and ideas cover many areas, and it is not possible to cover them all in a short preface, so let us try to cover the main ones.

Main successes: fuzzy techniques. Undoubtedly, the main successes of Zadeh's legacy are related to his invention and development of fuzzy techniques. These techniques enable researchers and practitioners to take into account an important part of expert knowledge – namely, the part that is expressed by using imprecise ("fuzzy") natural-language words like "small" and "large".

Need to take this knowledge into account was the main motivation for Zadeh's 1965 invention of fuzzy. By then, Lotfi Zadeh, one of the world's leading specialists in optimal control and one of the two authors of the most popular control textbook, was puzzled by the fact that in many situations, human operators control the corresponding systems more efficiently than the best automatic controllers.

He realized that the main reason for this efficiency is that human operators use additional knowledge – e.g., imprecise if-then rules, and came up with way to incorporate this knowledge into automatic controllers. One of the main ideas behind this incorporation is that, in contrast to precise statements like "x is positive" – statements that can be either true or false – an imprecise statement like "x is small" is only true to a certain degree.

Partial success and remaining challenges: everything is a matter of degree. The success of degree-based fuzzy techniques led Lotfi Zadeh to formulate a general principle: that everything is a matter of degree. This simple but deep idea helped to

Vladik Kreinovich
Department of Computer Science, University of Texas at El Paso
500 W. University, El Paso, TX 79968, USA, e-mail: vladik@utep.edu

resolve many seemingly unsolvable problems – and it still has a lot of potential. For example, many researchers and practitioners have realized that it is not possible to provide absolute security, absolute privacy, absolute fairness, that all this is possible only to a degree – and the remaining challenges are how to gauge these degrees, and how to find the best way of maximizing these degrees within the given limited amount of resources.

Current controversy in statistical hypothesis testing – where repeated experiments show that several seemingly experimentally confirmed empirical biomedical hypothesis are actually wrong – is largely explained by the fact that empirical testing never leads to an absolute confirmation of a hypothesis. No matter how many experiments we perform, the hypothesis is only confirmed to some degree. How to process these degrees, how to take them into account in decision making are still important challenges.

In modern physics, where symmetry is one of the most important basic notions, it is well understood that in real life, all symmetries are approximate, that they are only satisfied to a degree – and again, how to describe these degrees is an important challenge. Similar need to consider degrees helped to understand what is random and what is not random – via Algorithmic Complexity theory – and will, hopefully, help to better understand the important notion of causality.

Remaining open problems: from t-norms to world's interdependence. Yet another aspect of Zadeh's legacy – an aspect that is currently one of the least developed – is description of dependency. In the traditional statistical approach to data processing, if we have no information about the dependence between the two events, a usual recommendation is to assume that these events are independent. Under this assumption, if we know the probabilities $a = p(A)$ and $b = p(B)$ of these events, then the probability $p(A\,\&\,B)$ that both events will occur is equal to the product $a \cdot b$.

Independence is a good first approximation, but, of course, we know that, in reality, many world events are interdependent. How can we take this interdependence into account? One possible approach to this challenge comes from Zadeh's legacy, according to which, once we know our degrees of confidence $a = d(A)$ and $b = d(B)$ in two statements, we estimate our degree of confidence in $A\,\&\,B$ as $f_{\&}(a,b)$, for some "and"-operation ("t-norm") $f_{\&}(a,b)$ that may be different in different situations. Since subjective probabilities can be naturally viewed as particular cases of fuzzy degrees, we can try (and some did) to use a similar formula $f_{\&}(a,b)$ to estimate the probability $p(A\,\&\,B)$ as well – and, in general, to use Zadeh's ideas to describe interdependence.

Looking forward. Of course, what we described is just a small selection of previous successes and remaining challenges. We hope that papers collected in this book – that provide new solutions and new ideas – will inspired others to move even further.

Enjoy the book!