

# How the Pavement Strength Changes With Time: AI Ideas Help to Explain Semi-Empirical Formulas

Edgar Daniel Rodriguez Velasquez<sup>2†</sup> and Vladik Kreinovich<sup>3\*†</sup>

<sup>1</sup>Department of Civil Engineering, Universidad de Piura in Peru (UDEP), Piura, Peru.

<sup>2</sup>Department of Civil Engineering, University of Texas at El Paso, 500 W. University, El Paso, 79968, Texas, USA.

<sup>3</sup>Department of Computer Science, University of Texas at El Paso, 500 W. University, El Paso, 79968, Texas, USA.

\*Corresponding author(s). E-mail(s): [vladik@utep.edu](mailto:vladik@utep.edu);

Contributing authors: [edrodriguezvelasquez@miners.utep.edu](mailto:edrodriguezvelasquez@miners.utep.edu);

<sup>†</sup>These authors contributed equally to this work.

## Abstract

In this paper, we use AI ideas to provide a theoretical explanation for semi-empirical formulas that describe how the pavement strength changes with time, and how we can predict the pavement lifetime.

**Keywords:** AI, Similarity, Invariance, Pavement engineering, Empirical formulas

## 1 Formulation of the Problem

**Formulation of the problem.** In many road segments, one or more pavement layers are stabilized by adding cement, lime, fly ash, and other additives. For example, cement-based concrete is sometimes used as the top layer, cement or lime is often added to the underlying soil, etc. Cement and other additives takes some time to gain strength, so at first, the pavement strength increases with time. Once the desired strength is reached, the strength starts decreasing.

This decrease is caused both by the traffic and by the freeze-thaw and wet-dry cycles. As a result of this decrease, the pavement has a finite lifetime, after which the pavement has to be rehabilitated.

There are semi-empirical formulas that describe both how the pavement strength changes with time and how we can predict the pavement lifetime; these formulas are summarized and tested in [8]. The problem is that these formulas do not have a theoretical explanation, and without such an explanation, practitioners are somewhat reluctant to use them – since there have been many cases when an empirical formula does not work well in situations which are somewhat different from situations from which this formula was derived. To enhance the reliability of these formulas, it is therefore desirable to come up with theoretical explanations.

**What we do in this paper.** In this paper, we describe these formulas, and show that AI ideas can help to provide theoretical explanations for most of these formulas.

## 2 Main Idea Behind Our Explanations

**Need for AI techniques.** In some cases, we have theoretically justified formulas for predicting the future state of a system. A classical example of such a case is celestial mechanics, where Newton’s equations enable us to accurately predict the position of planets and other celestial bodies literally hundreds of years ahead; see, e.g., [4, 15]. However, in many other practical situations, we do not have such formulas, and pavement engineering is one such situation.

In many such cases, however, we have experts who have been successful in solving the corresponding problems: we have expert medical doctors who are very good in diagnosing and curing diseases, we have expert pilots who are very good in controlling complex planes, we have expert engineers who are very good in designing and maintaining durable road pavements, etc. These experts are perfectly willing to share their expertise, but the problem is that they can rarely describe their knowledge and their skills in precise numerical form. It is therefore necessary to be able to translate their non-numerical statements into precise numerical formulas. Such a translation is one of the main tasks of broadly understood Artificial Intelligence.

**AI techniques that we use in this paper.** There are many different types of AI techniques. For example, sometimes, experts can provide rules that use imprecise (“fuzzy”) words from natural language like “small”. To translate such rules into precise numerical dependencies, Lotfi Zadeh invented a special methodology that he called fuzzy; see, e.g., [2, 7, 9, 11, 12, 17].

In some cases, experts can show, on real and/or simulated situations, how they will react. In this case, it is necessary to come up with a general algorithm based on these specific examples. Designing an algorithm based on examples is known as *machine learning*; see, e.g., [3, 5]. At present, the most widely used machine learning techniques is *neural networks*, in particular, *deep neural networks*; see, e.g., [5].

Both fuzzy and neural techniques have been successfully used in many application areas, including pavement engineering. However, in pavement engineering – and in many other application areas – there is an important part of expert knowledge that is not easy to capture by fuzzy or neural techniques: the idea of similarity and symmetry. Indeed, in general, how do we react to a new situation? How does a medical doctor react to a new patient? A usual way to do it is to look for similar situations from our past experiences in which we acted successfully, and to try a similar reaction in this case as well.

Similarities are, in effect, the basis of all the decisions that we make. For example, how do we conclude that Ohm's law  $V = I \cdot R$  relating voltage  $V$ , current  $I$ , and resistance  $R$  will work in a new situation? Because it worked in many similar situations in the past. How do we know that predictions of planet positions based on Newton's equations will turn out to be correct hundred years from now? Because similar predictions worked successfully in the past. Physicists realize that similarities – which they call symmetries or invariances – are very important: for example, in modern physics, many new theories are formulated not in terms of differential equations (as in Newton's time), but by explicitly describing the corresponding symmetries; see, e.g., [4, 15].

In line with this, in this paper, we will use experts' opinion about the corresponding invariances, and we will use techniques similar to the ones used in physics to translate these ideas into numerical dependencies.

### 3 The Newly Built or Newly Rehabilitated Road: How Strength Increases with Time

**Empirical formula.** For the case of a newly built or a newly rehabilitated road, the initial increase in strength  $S$  with time  $t$  is described by the following formula:

$$S = S_0 \cdot p_1^{1 - \frac{1}{1 - \frac{t - t_0}{p_2}}} \quad (1)$$

for some constants  $S_0$ ,  $p_1$ ,  $t_0$ , and  $p_2$ .

If we take the logarithm of both sides, this formula can be equivalently reformulated into a somewhat simpler form:

$$\ln(S) = \frac{a + b \cdot t}{1 + c \cdot t}, \quad (2)$$

for some constants  $a$ ,  $b$ , and  $c$ .

**Our explanation: first approximation.** The above formula describes how the strength  $S$  depends on time  $t$ . The numerical value of time depends on how we select the starting point. A natural idea is to take, as the starting point, the moment when the pavement was open to traffic. However, this starting moment may differ: in some cases, the pavement is released as soon as it is supposed to be ready for traffic, in other cases, the road managers are acting

more conservatively and wait a few extra days to be absolute sure that the road is ready. There is no fixed time for this release.

Depending on the choice of the release time, we get different numerical values of the time  $t$ . Namely, if we release the pavement  $d$  days earlier, then we need to add this number  $d$  to all the numerical values of time:  $t \mapsto t + d$ .

Since there is no preferred release time, it makes sense to assume that the formula for the dependence on  $S$  on  $t$  should not depend on this addition, i.e., that the new dependence  $S(t + d)$  should have the same form as the original dependence  $S(t)$ . Of course, we cannot simply require that  $S(t + d) = S(t)$  for all  $t$  and  $d$ , because that would mean that the function  $S(t)$  is constant, i.e., that the strength does not depend on time at all. However, we can use the fact that such invariance situations are common in physics. For example, the formula  $r = v \cdot t$  describing the distance traveled  $r$  as a function of velocity  $v$  and time  $t$  clearly does not depend on what unit we choose to measure velocity. However, if we use km/h instead of miles per hour, for the formula to remain valid, we need to corresponding re-scale the distance – from kilometers to miles.

Similarly, in our case, the fact that the expression for  $S(t)$  remains the same after changing the starting time means that the formula remain valid if we corresponding re-scale the strength. In contrast to time, for which we can choose different starting points, for strength, there is a natural starting point – 0 strength. However, the numerical value of the strength does depend on the measuring unit. If we choose a unit which is  $\lambda$  times smaller, then all numerical values of strength are multiplied by  $\lambda$ :  $S \mapsto \lambda \cdot S$ .

So, we conclude that for every time shift  $d$ , there exists a value  $\lambda$  – depending on  $d$  – for which  $S(t + d) = \lambda(d) \cdot S(t)$ . It is known – see, e.g., [1, 13] – that all monotonic solutions of this functional equation have the form  $S(t) = A \cdot \exp(b \cdot t)$  for some constants  $A$  and  $b$ . This formula is equivalent to

$$\ln(S) = a + b \cdot t, \quad (3)$$

where we denoted  $a \stackrel{\text{def}}{=} \ln(A)$ .

**From the first approximation to the actual explanation.** For small times  $t$ , the first-approximation formula (3) provides a reasonable description of the actual dependence (2), but for large  $t$ , the formula (3) is not very accurate.

A possible reason for this is that in the formula (2), we used astronomical time, while most processes have their own internal times that better describes their evolution. For example, it is known that the health state of a person is best described not by his/her astronomical time but by so-called biological time which may be different. So, instead of the formula (3), we should have a formula

$$\ln(S) = a + b \cdot \tau, \quad (4)$$

where  $\tau(t)$  is an internal time of the pavement.

For different pavements, we may have different internal times. In other words, we have different scales for measuring time, with possibly nonlinear transformations from one scale to another. What are these transformations?

Of course, we can have different time scales if we use different units for measuring time. Similar to measuring strength  $S$ , if we select a measuring unit which is  $c$  times smaller, then all numerical values are multiplies by  $c$ :  $t \mapsto c \cdot t$ . As we have already mentioned, if we select a new starting point for measuring time which is  $d$  moments earlier than the previous one, then we add  $d$  to all numerical values:  $t \mapsto t + d$ . If we change both the measuring unit and the starting point, then we have a general linear transformation  $t \mapsto c \cdot t + d$ . So, the set  $T$  of all appropriate transformations should include all linear transformations.

It is also reasonable to conclude that if a transformation  $t \mapsto f(t)$  is appropriate, i.e., that it transforms from some scale  $A$  to another scale  $B$  then the inverse transformation  $f^{-1}(t)$  that transforms from scale  $B$  back to scale  $A$  should also be appropriate, i.e., should also belong to the class  $T$ . Similarly, if we have two appropriate transformations, then if we apply them one after another – i.e., if we consider their composition – then we should get an appropriate transformation. Thus, the class  $T$  should be closed under taking the inverse and under taking the composition. In mathematical terms, such classes are known as *transformation groups*.

We want to be able to use these transformations in computer programs that predict how pavement strength changes with time. In a given computer, we can only store finitely many numbers. Thus, we should have a fixed number of numerical parameters that describe all appropriate transformations. In other words, the family of transformations  $T$  should be finite-parametric, or, to use an appropriate mathematical term, finite-dimensional. So, to describe all appropriate transformations, we need to describe all finite-dimensional transformation groups that contain all linear transformations. A description of all such groups was conjectured in [16]. This conjecture was proven in [6, 14]. In our 1-D case, when we consider transformations of one variable, the result is that all appropriate transformations are fractional linear, i.e., that they all have the form

$$\tau = \frac{a + b \cdot t}{1 + c \cdot t}; \quad (5)$$

for the 1-D case, a simpler proof of this conjecture is possible [10].

Substituting the expression (5) for the internal time in the formula (4), we conclude that  $\ln(S)$  is indeed equal to a fractionally linear expression in terms of time, i.e., that we have indeed explained the empirical formulas (1)-(2).

## 4 Exploitation Stage: How Strength Decreases With Use

**Empirical formulas.** Once the additives solidify and the pavement reaches its desired strength, the pavement deteriorates and its strength decreases. As we

have mentioned, the strength decreases both by the traffic and by the freeze-thaw and wet-dry cycles. The empirical formula describing how the strength decreases with the number  $N$  of freeze-thaw and wet-dry cycles is given in [8]:

$$S = a + \frac{b}{1 + \exp(c \cdot N)}. \quad (6)$$

A similar expression – but with different coefficients  $a$ ,  $b$ , and  $c$  – describes how the strength decreases with the number of cars  $N$ . How can we explain these empirical formulas?

**Our explanation.** Our explanation is based on the fact that there is some uncertainty in the number of freeze-thaw and wet-dry cycles: we can start counting them from the very beginning, when the road is started to be built; we can start counting them from the moment when the additives are added to the pavement, or we can start counting from the moment when the road achieved its maximum strength. It is reasonable to require that the dependence  $S(N)$  should be described by a similar formula no matter when we start counting.

If we start counting earlier, then we add some fixed number of cycles  $N_0$  to the previous count, i.e., we replace  $N$  with  $N + N_0$ . It is therefore reasonable to require that the functions  $S(N)$  and  $S(N + N_0)$  corresponding to different starting point should be described by a similar expression. Similarly to the previous case, this means that they may be obtained from each other by some appropriate transformation. In the previous section, we considered only scaling  $S \mapsto \lambda \cdot S$ , but to be as general as possible let us consider the most general – possibly nonlinear – transformations. In the previous section, we have already shown that such transformations are fractionally linear. Thus, we conclude that for each  $N_0$ , the expressions  $S(N + N_0)$  and  $S(N)$  are related to each other by a fractionally linear transformation

$$S(N + N_0) = \frac{a(N_0) + b(N_0) \cdot S(N)}{1 + c(N_0) \cdot S(N)}. \quad (7)$$

It is known – see, e.g., [10] – that every decreasing non-negative function  $S(N)$  that satisfies this functional equation has either the form (6) (or the form  $S(N) = a + b \cdot \exp(-c \cdot N)$ , which is, in effect, a limit case of the formula (6)). This explains the empirical formula (6).

*Comment.* To prove this result, we can differentiate both sides of the formula (7) with respect to  $N_0$  and take  $N_0 = 0$ . Then, we get the following differential equation:

$$\frac{dS}{dN} = A + B \cdot N + C \cdot N^2$$

for some  $A$ ,  $B$ , and  $C$ . We can then conclude that

$$dS = \frac{dN}{A + B \cdot N + C \cdot N^2}$$

and thus, that

$$S(N) = \int \frac{dN}{A + B \cdot N + C \cdot N^2}.$$

There are known formulas for this integral, and selecting decreasing non-negative solutions leads to the above result.

## 5 Exploitation Stage: How Lifetime Depends on Stress Level

**Empirical formulas.** The larger the stress level, the smaller the pavement lifetime  $T$ , i.e., in other words, the smaller the number  $N$  of cycles are which the pavement will need to be repaired. The empirical formula describing this dependence – as given in [8]:

$$T = A \cdot \exp(-b \cdot \sigma). \quad (8)$$

How can we explain this empirical formula?

**Our explanation.** For lifetime, we have a fixed starting point – the moment when the road was built, but, in general, we do not have a preferred measuring unit. With stress, the situation is more complicated: we can measure stress as the pressure caused by the passing traffic, or we can measure the overall pressure – that include the atmospheric pressure. When we take into account atmospheric pressure, we thus add this pressure  $\sigma_0$  to the original stress value  $\sigma$ :  $\sigma \mapsto \sigma + \sigma_0$ . From this viewpoint, the stress is defined modulo an appropriate shift.

It is therefore reasonable to require that the dependence  $T(\sigma)$  should not change if we select a different starting point for measuring stress, i.e., that the expression  $T(\sigma)$  and  $T(\sigma + \sigma_0)$  describe the same dependence – provided, of course, that when we perform the shift, we appropriately re-scale the lifetime, i.e., that

$$T(\sigma + \sigma_0) = C(\sigma_0) \cdot T(\sigma) \quad (9)$$

for some  $C(\sigma_0)$ . As we have already mentioned, it is known – see, e.g., [1, 13] – that all monotonic solutions of this functional equation have the desired form (8).

## 6 Exploitation Stage: How Lifetime Depends on Dry Density of the Soil

**Empirical formulas.** The pavement lifetime  $T$  also depends on the density of the underlying soil: the higher this density, the sturdier the road and the larger the pavement lifetime. The actual density of the soil changes when the soil is dry and when it is wet. We need a value that depends only on the soil itself. A natural idea is to use the dry density  $\rho$ . The empirical formula describing

the dependence of the pavement lifetime  $T$  on the dry density  $\rho$  – as given in [8] – is:

$$\ln(T) = c \cdot \ln\left(\frac{\rho}{\omega}\right). \quad (10)$$

How can we explain this empirical formula?

**Our explanation.** To explain this formula, let us first transform it into a more explicit form, so that it will be describing explicitly the dependence of the lifetime  $T$  on the dry density  $\rho$ . The formula (10) describes  $\ln(T)$ . So, to get a formula for  $T$ , we need to apply  $\exp(x)$  to both sides of the formula (1). Then, the left-hand side of this formula takes the form  $\exp(\ln(T)) = T$ , and the formula itself gets the following equivalent form:

$$T = \exp\left(c \cdot \ln\left(\frac{\rho}{\omega}\right)\right) = \left(\exp\left(\ln\left(\frac{\rho}{\omega}\right)\right)\right)^c = \left(\frac{\rho}{\omega}\right)^c,$$

i.e., the form

$$T = A \cdot \rho^c, \quad (11)$$

where we denoted  $A \stackrel{\text{def}}{=} \omega^{-c}$ .

For lifetime, we have a fixed starting point – the moment when the road was built. For dry density  $\rho$ , we also have a fixed starting point – the density  $\rho = 0$  corresponding to the absence of dry materials. However, in both cases, there does not seem to be any fixed measuring unit. It is therefore reasonable to require that the dependence  $T(\rho)$  should not change if we select a different measuring unit for dry density, i.e., that the expression  $T(\rho)$  and  $T(\lambda \cdot \rho)$  describe the same dependence – provided, of course, that when we re-scale dry density, we appropriately re-scale the lifetime, i.e., that

$$T(\lambda \cdot \rho) = C(\lambda) \cdot T(\rho) \quad (12)$$

for some  $C(\lambda)$ . It is known – see, e.g., [1, 13] – that all monotonic solutions of this functional equation have the desired form (11), i.e., equivalently, the form (10).

## Acknowledgments

This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and HRD-1834620 and HRD-2034030 (CAHSI Includes), and by the AT&T Fellowship in Information Technology.

It was also supported by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478, and by a grant from the Hungarian National Research, Development and Innovation Office (NRDI).



## References

- [1] Aczél, J., Dhombres, J.: *Functional Equations in Several Variables*. Cambridge University Press (2008)
- [2] Belohlavek, R., Dauben, J.W., Klir, G.J.: *Fuzzy Logic and Mathematics: A Historical Perspective*. Oxford University Press, New York (2017)
- [3] Bishop, C.M.: *Pattern Recognition and Machine Learning*, Springer, New York (2006)
- [4] Feynman, R., Leighton, R., Sands, M.: *The Feynman Lectures on Physics*. Addison Wesley, Boston, Massachusetts (2005)
- [5] Goodfellow, I., Bengio, Y., Courville, A.: *Deep Learning*. MIT Press, Cambridge, Massachusetts (2016)
- [6] Guillemin, V.M., Sternberg, S.: An algebraic model of transitive differential geometry. *Bulletin of American Mathematical Society* **70**(1), 16–47 (1964)
- [7] Klir, G., Yuan, B.: *Fuzzy Sets and Fuzzy Logic*. Prentice Hall, Upper Saddle River, New Jersey (1995)
- [8] Li, X., Wang, J., Wen, H., Muhunthan, B.: Field calibration of fatigue models of cementitiously stabilized pavement materials for use in the mechanistic-empirical pavement design guide. *Transportation Research Record* **2673**(2), 427–435 (2019)
- [9] Mendel, J.M.: *Uncertain Rule-Based Fuzzy Systems: Introduction and New Directions*. Springer, Cham, Switzerland (2017)
- [10] Nguyen, H.T., Kreinovich, V.: *Applications of Continuous Mathematics to Computer Science*, Kluwer, Dordrecht (1997)
- [11] Nguyen, H.T., Walker, C.L., Walker, E.A.: *A First Course in Fuzzy Logic*. Chapman and Hall/CRC, Boca Raton, Florida (2019)
- [12] Novák, V., Perfilieva, I. Močkoř, J.: *Mathematical Principles of Fuzzy Logic*. Kluwer, Boston, Dordrecht (1999)
- [13] Rodriguez Velasquez, E.D., Kreinovich, V., Kosheleva, O. Invariance-based approach: general methods and pavement engineering case study. *International Journal of General Systems*. **50**(6), 672–702 (2021)
- [14] Singer, I.M., Sternberg, S.: Infinite groups of Lie and Cartan, Part 1. *Journal d'Analyse Mathématique* **XV**, 1–113 (1965)

- [15] Thorne, K.S., Blandford, R.D.: Modern Classical Physics: Optics, Fluids, Plasmas, Elasticity, Relativity, and Statistical Physics. Princeton University Press, Princeton, New Jersey (2017)
- [16] Wiener, N.: Cybernetics, or Control and Communication in the Animal and the Machine, 3rd edition. MIT Press, Cambridge, Massachusetts (1962)
- [17] Zadeh, L.A.: Fuzzy sets. *Information and Control* **8** 338–353 (1965)