Why Ideas First Appear in Informal Form? Why It Is Very Difficult to Know Yourself? Fuzzy-Based Explanation

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Abstract To a lay person reading about history of physics, it may sound as if the progress of physics comes from geniuses whose inspiration leads them to precise equations that – almost magically – explain all the data: this is what Newton did with mechanics, this is what Schroedinger did with quantum physics, this is what Einstein did with gravitation. However, a deeper study of history of physics shows that in all these cases, these geniuses did not start from scratch – they formalized ideas that first appeared in imprecise ("fuzzy") form. In this paper, we explain – on the qualitative level – *why* ideas usually first appear in informal, imprecise form. This explanations enables us to understand other seemingly counterintuitive facts – e.g., that it is much more difficult for a person to know him/herself than to know others. We also provide some general recommendations based on this explanation.

1 Formulation of the Problem

Ideas first appear in informal form. From a purely mathematical viewpoint, it may seem that progress in physics and in other disciplines appears as a stroke of genius: suddenly, someone comes up with equations, these equations fit the experiments perfectly, Nobel prize and fame follows – and the cycle continues with the next genius equation.

- Newton came up with equations of mechanics,
- Maxwell came up with equations of electrodynamics,

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- Schroedinger came up with equations of quantum mechanics,
- Einstein came up with equations of General Relativity that describe gravity, etc.

However, physicists know better. They know that equations did not come out of nowhere. In most cases, first, there is an informal vague idea, then this idea is getting more and more precise – until finally someone comes up with absolutely precise equations. Newton based his mechanics equations on ideas of Galileo and others that have been analyzing mechanical motion for centuries. Schroedinger formalized ideas of de Broglie and others on the wave-particle duality. Einstein had vague ideas – that gravity disappears in a falling elevator and is, thus, largely equivalent to changing a coordinate system – that took him 10 years (and many unsuccessful attempts) to translate into precise equations that would fit with experimental data; see, e.g., [2, 8].

But why? At first glance, modern physics was formed explicitly as precise science, where everything is described by precise equations – as opposed to vague ancient and medieval thinking about the physical world. Physicists strive to be as precise as possible, their objective is never to form vague ideas – so why do vague ideas form such an important role in physics – that they are practically a ubiquitous step in all scientific discoveries?

What we do in this paper. In this paper, we provide a possible explanation for this phenomenon, an explanation using the ideas of fuzzy techniques. The main ideas behind this explanation enable us to explain other somewhat mysterious phenomena as well – e.g., the well-known fact that while a person knows much more about him/herself than about others, analyzing oneself is much more difficult than analyzing others.

2 Analysis of the Problem Explains the Need for Informal Ideas

What physicists do. Let us describe what physicists do in general terms. We have a set of observations – in many cases, a very large set of observations – and we want to find equations that describe this set and that therefore will enable us to predict similar future phenomena.

Usually, this is not easy. Sometimes, these equations are reasonably easy to find. For example, when Ohm simultaneously measured how changing the voltage V changes the current I flowing through a fixed conductor, he plotted his results and saw that this dependence is linear $I = \text{const} \cdot V$: the famous Ohm's law. However, such cases are rare. In most real-life situations, there is no easy way to directly deduce the equations from the measurement results. In other words, there is a big distance between the experimental results and the equations.

So how do physicists solve their problems: a general description. If you read some popular books on history of physics, you may get a false impression that evolution of physics is a straight line: each new idea bring us closer and closer to the correct equations.

But this is a false impression. Einstein formulated many wrong theories until he came up with the right equations – and many other physicists formulated even more wrong theories.

From this viewpoint, if one takes all these failed attempts into account, the progress of physics looks not so much like a straight line but rather as a random walk on each stage – a random walk that after many attempts, reaches the desired objective (of fitting all the experimental data).

So how do physicists solve their problems: a more detailed description. The problem with the random walk idea is that it means, in effect, that if we want to go from the initial state (experimental data) to the desired state (equations explaining this data) separated by some distance from the initial state, then we have to consider almost all possible models within this distance from the desired state. As we have mentioned, in most real-life situations, the distance R between the initial state and the desired state is large. In such situations, there are many possible models in the corresponding vicinity of the initial state. The number of these models is proportional to the volume $\sim R^d$ of this vicinity (where d is the dimension of this space), and is, therefore, large. Thus, it is not realistic to try all possible models in this vicinity.

And this is not how models actually appeared. It is not that Schroedinger tried all possible differential equations of a given complexity – that would have taken forever. So how do physicists operate?

Random walk works well when the distance R between the starting state and the desired state is small. In this case, we have not that many possible models within a vicinity of radius R, and it is indeed possible to try all of them.

So, a natural idea is to somehow come up with intermediate objectives. Instead of looking for a precise model that explains all the data, it is desirable to come up with several intermediate objectives, so that:

- the first desired state is close to the data,
- the second desired state is close to the first desired state, etc., and
- at the end, the last intermediate state is close to the desired final state.

This is similar to crossing a mountain stream: if this stream is wide, it is very difficult to cross, but if we have rocks showing out of water, we can often easuly go from one rock to another until we reach the other bank.

So how can we form such intermediate states?

How to form intermediate states: a natural idea. The ultimate objective is to have a model that fits all the data exactly – i.e., that is consistent with all the available data (of course, if we take into account that all available data comes from measurement uncertainty; see, e.g., [7]).

Coming up with a model that exactly describes all the data often requires that we determine the values of a large number of parameters p. In general, the number of possible models – i.e., the number of possible combinations of these parameter values – grows with p as N^p, where N is the number of possible values of each parameter. This number grows exponentially with p, so for large p, it becomes not feasible to try all of them.

Thus, as an intermediate step, we need to select class of models that is characterized by fewer parameters. Of course, if we need a large number of parameters to fit the data *exactly*, there is no way to fit the data exactly with fewer parameters. So, on the intermediate stages, we need to use to use models that fit the data *approximately*.

This naturally leads to all kind of ways to describe what the word "approximate" means. This word has some formalizations, but in general, it is an informal word – and thus, a natural general way to describe this imprecise word in precise terms is to use techniques for translating imprecise words from natural language into precise language, and this is exactly what fuzzy techniques (see, e.g., [1, 3, 4, 5, 6, 9]) are about.

How ideas evolve – from this viewpoint. Once we found a model that provides an approximate description of the corresponding phenomena, a natural next objective is to try to modify this approximate model so that the resulting modification provides a more accurate description, etc.

The whole idea of imprecision (fuzziness) is related to the fact that we are dealing with an approximation. The more accurate the model, the smaller the need for approximation, and thus, the smaller the need to take fuzziness into account. Thus, ideas that initially appear in a very fuzzy form become more and more precise, less and less fuzzy – until someone makes the last step and comes up with precise equations.

3 From This Viewpoint, Measurement Errors Are the Blessing in Disguise

Unexpected conclusion. The above analysis leads to a somewhat unexpected conclusion: that measurement errors are good for cognition. At first glance, this sounds counter-intuitive: the more accurate the measurement, the more information we gain about the world, so the easier it is to cognise it. However, this idea starts making sense if we take into account that at first, we want to come up with an approximate model.

Historical examples. For example:

- while Ohm's law is, in many cases, very accurate, and in many situations, the voltage V is indeed proportional to the current I,
- in reality, the dependence of V on I is not linear.

If Ohm was able to perform very accurate measurements, he would never be able to find a complex formula that exactly describes the empirical dependence. However, since his measurements were not very accurate (there was a significant measurement error), linear dependence provided – within this accuracy – a very good fit for all the measurement results.

Similarly, if Newton had access to all modern super-precise astronomical measurements, he would never be able to come up with his theory of gravitation: the exact description of this data requires much more complex equations of General Relativity. However, since in Newton's days, measurements were not that accurate, he was able to come up with equations that – within his-time accuracy – fit all the observations perfectly.

We can avoid the curse of too accurate measurements, but this requires creativity. Of course, this does not mean that if we have very accurate measurements, we cannot come up with theories – it just means that in this case, we need to come up with a reasonable approximate description, we need to know what needs to be taken into account and what details can be ignored. This is not easy – and this is an important part of the art of being a physicist; see, e.g., [2, 8].

For example, it is a known fact that the great 20 century mathematician David Hilbert came up with the same equations of General Relativity as Einstein – his paper was submitted only two weeks later than Einstein's. However, the big difference was that:

- while Hilbert simply presented the same complex system of non-linear differential equations,
- Einstein also provided approximate solutions and thus, the possibility to experimentally test this theory.

It is not that Einstein was a better mathematician knowing some trick of solving non-linear differential equations that Hilbert did not – far from it. The reason why Einstein was more successful is that he knew which terms in these equations can be safely ignored, which made computations much easier.

4 Why Is It Difficult to Know Oneself

Another consequence of the above general idea is that it explains a seemingly counterintuitive fact that even for people who are capable of providing great insights into the souls of others – who can predict the others' behavior, who can explain the other's behavior and who can help others change their behavior if needed – it is much more difficult to understand one's own soul, one's motivations and behaviors. For example, it is known that psychoanalysts – even the most advanced ones who can penetrate deeply into the patients' subconscious activity, need help from others to understand their own behavior and their own problems.

At first glance, it should be the other way around: while we may know a lot about others, we definitely know much more about ourselves. So, based on all this abundant information, we should be able to get a better description of ourselves than of others. However, the above argument shows that the reality is the opposite. When we know a few facts about another person, it is easier to fit these facts into a pattern. In contrast, when since we have am abundance of facts about ourselves, it is very difficult to come up with a description that fits all these facts.

5 How Can All This Be Used

So, we have provided a qualitative explanation of the empirical phenomena. This may be intellectually satisfying, but from the pragmatic practical viewpoint, is this explanation useful? We cannot yet claim that we have used this explanation in practice, but we can envision how it can be used.

In the past, data was scarce. Nowadays, in many application areas, we have an abundance of information, an abundance of data – to the extent that engineers and scientists are actively working on the problem on how to store and process all this data. Often, there is so much data that it its difficult to come up with models describing all this data. This is a problem, e.g., with climate change: none of the existing models provides a perfect fit for all the observations. Our argument shows that this is not a reason to conclude – as some philosophers have been doing for centuries – that many processes in the world are not cognizable. If we cannot find a perfect fit, we should go for an approximate fit – e.g., fuzzify the data and try to fit it with fuzzy rules. (This, by the way, is exactly what has made fuzzy control so successful.)

We should not desperately look for models that fit with $p < 0.05$ (or whatever criteria we use): if we get such models, great, but if not, we should start with approximate models and improve them until we finally (hopefully) get the accurate equations. This is what climatologists are doing, ignoring all criticisms from statisticians, this is what we should recommend to all the sciences which are trying to find an explanation for all the available new data.

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