Everything Is a Matter of Degree: The Main Idea Behind Fuzzy Logic Is Useful in Geosciences and in Authorship

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Abstract This paper presents two applications of the general principle – the everything is a matter of degree – the principle that underlies fuzzy techniques. The first – qualitative – application helps explain the fact that while most earthquakes occur close to faults (borders between tectonic plates or terranes), earthquakes have also been observed in areas which are far away from the known faults. The second – more quantitative – application is to the problem of which of the collaborators should be listed as authors and which should be simply thanked in the paper. We argue that the best answer to this question is to explicitly state the degree of authorship – in contrast to the usual yes-no approach. We also show how to take into account that this degree can be estimated only with some uncertainty – i.e., that we need to deal with interval-valued degrees.

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1 Formulation of the Problem

One of the main ideas behind fuzzy logic is Zadeh's idea that everything is a matter of degree. This idea has been very fruitful in many application areas; see, e.g., [1, 4, 9, 11, 12, 15].

In this paper, we show that there are still many new areas where this idea can be successfully applied. Specifically, we show that it can help to solve puzzling questions in such diverse applications areas as geosciences and authorship.

2 Possible Application to Geosciences

Importance of earthquake studies. Earthquakes are among the most devastating events. Their effect depends on our preparedness. If we know that a certain area is prone to have earthquakes of certain magnitude, then we can strengthen all the buildings and structures, so as the minimize the earthquake's damaging effect. This strengthening is reasonably expensive, so it is only used when we are reasonably confident that earthquakes of such magnitude are possible.

Because of this, predicting the magnitudes and location of possible future earthquakes is one of the main objectives of geosciences.

Seismogenic zones: traditional approach to earthquake study. Up to the 19th century, scientists believed that continents remain the same. Then it turned out that continents – or, to be more precise, plates containing continents or parts of the continents – drift with time. This knowledge formed what is now called *plate tectonics*. It was noticed that most strong earthquakes – as well as most volcanos – occur in the borders between these plates.

Later on, it was found that plates themselves are not unchangeable – they consist of smaller pieces called *terranes* that can also drift with respect to each other. The vast majority of earthquakes occurs close to the *faults* – boundaries of terranes.

This is still the main approach to predicting earthquakes: researchers usually assume that the earthquakes occur only at the faults. So at the faults, they recommend engineering measures to mitigate the effect of possible earthquakes, while in the areas inside the terranes no such measures are recommended.

Recent discovery. A recent statistical analysis of earthquake records has shown that, contrary to the above-described traditional beliefs, earthquakes are not limited to the faults; see, e.g., [2]. Most earthquakes do occur at the faults, but there have been earthquakes in other areas as well: as we move away from the fault, the probability of an earthquake decreases but it never goes to 0.

What does this mean in terms of seismogenic zones? In the traditional approach, there was a clear (crisp) distinction between seismogenic zones where earthquakes are possible and other areas where earthquakes are not possible.

The recent discovery shows that earthquakes are possible literally everywhere. At first glance, this implies that the whole Earth is a seismogenic zone, but this would be a useless conclusion. There should be a difference between zones where earthquakes are frequent – e.g., near the major faults – and zones where earthquakes are so rare that it took several decades to notice them. In other words, some areas are clearly seismogenic zones, while other are barely seismogenic.

In other words, being a seismogenic zone is not a crisp property, it is a matter of degree: some areas are more seismogenic, some are less seismogenic. This is perfectly in line with the main idea that Zadeh placed in the foundation of fuzzy logic (see, e.g., [1, 4, 9, 11, 12, 15]) – that everything is a matter or degree.

What is the physical meaning of this phenomenon. The traditional approach implicitly assumes that a fault is a line. In such a description, we can easily separate regions close to the line from regions which are far away from the line. A similar description was thought to hold when we describe visible cracks: e.g., cracks in rocks, cracks in pavement, etc. A more detailed analysis has shown that visible cracks actually have a fractal structure (see, e.g., [7, 14]):

- there is a main crack line, along which the stress is high,
- at several points in the main line, it branches into then smaller-size crack lines, along which the stress is somewhat smaller;
- at several points in each of these "second-order" lines, the line itself branches into even smaller-size crack lines, with even smaller stress, etc.

Because of this structure, there is, in effect, no area completely without cracks and without stress:

- there are areas around the main fault line, in which the crack is the most visible and the stress is the highest;
- there are areas around the second-order fault lines, where the crack is less visible and the stress is somewhat lower;
- there are areas around the "third-order" fault lines, where the crack is even less visible and the stress is even lower, etc.,
- all the way to areas where cracks are microscopic and the stress is barely measurable.

This is what we directly observe in rock cracks, in the pavement cracks, and this is what we indirectly observe for earthquakes: earthquakes can appear everywhere, just in some areas they are more frequent and stronger, while in other areas they are less frequent and weaker. This means, in effect, that faults are everywhere, just in some areas they are larger and correspond to larger stress, while in other areas, they are weaker and the corresponding stress is smaller.

In other words, for this phenomenon, physics is in perfect agreement with Zadeh's principle – at least on the qualitative level.

3 Possible Application to Authorship

Formulation of the problem. In the past, most researchers worked on their own, and most papers had just one author. Nowadays, research is often performed by big groups of researchers: some of them make significant contributions to the research results, while contributions of others are not as significant. So, a question naturally appears: when the results of this joint research are formulated in a paper, who should be included in the list of the paper's authors? This is a subject of many serious discussions, see, e.g., [13].

Why is this a difficult problem? In our opinion, the problem is difficult because there is no crisp, discrete separation between authors on the one hand and contributors who end up being thanked (but who are not listed as authors) on the other hand. In each group, there is an implicit threshold, so that participants whose contribution level is above this threshold are listed as authors, while those whose level of contribution is below this threshold are not. This threshold level varies between different research communities, between different research groups: e.g., many experimental papers have dozens of authors, while most theoretical papers usually have much fewer ones. And within each group, there is a certain level of subjectivity.

Being an author in several papers is critically important for students to defend their dissertations, important for job search, for promotion. As a result, the degree of subjectivity in deciding who is listed as an author (and who is not) often causes conflicts within the research groups – and these conflicts hinder possible collaboration and thus, in the long run, slow down the progress of science. How can we avoid this subjectivity?

What we propose. We propose to take into account that being an author is, in effect, not a crisp notion. In many cases, it is a matter of degree. So instead of listing some collaborators as authors and others as non-authors, why not list everyone who contributed something intellectual to the result as authors – but with the corresponding degree of authorship?

To some extent, this is already done in some journals – where for each submitted paper, the authors have to agree on percentages of their contributions. But at present, this is only done with respect to participants who have already been declared authors. We propose to extend this idea to all the participants, including those who are usually not included in the authors' list.

Of course, for this idea to work, we need to take into account this degree of authorship when evaluating the quality of a student's dissertation work, or the quality of the researcher. We believe that this – yet another – example of using the above-mentioned Zadeh's principle will help resolve this issue.

How to assign the degree of authorship? For this idea to work, we need to have an acceptable way to assign degrees of authorship. In some cases, the authoring group includes a leader whose opinion everyone respects. In such cases, we can simply ask this trusted leader to provide the degrees of authorship.

However, the very fact that often conflicts appear around this issue means that in many cases, people's opinions differ. In such cases, a natural idea is to ask different participants of the research group to provide such estimates – and then we need to come up with combined estimates that take into account all the opinions.

For every two participants i and j, let us denote the degree assigned to the participant i by the participant j by d_{ij}. In the beginning, we do not know a priori who contributed more – and thus, whose opinion is more informed and more valuable. Thus, in the first approximation, we can simply take the average of all the assigned degrees. In other words, as the first approximation $a_i^{(0)}$ to the authorship degree assigned to each participant i, we can take the value

$$a_i^{(0)} = \frac{1}{N} \cdot \sum_{j \neq i} d_{ij},$$

where N is the normalizing constant – to make sure that the sum of all these degrees is, e.g., 1. If we want full objectivity, we do not ask each participant to estimate his/her degree of authorship, but if participants trust each other to be objective why not? If we add such degree d_{ii}, then the formula takes the simplified form

$$a_i^{(0)} = \frac{1}{N} \cdot \sum_{j} d_{ij}.$$

We can use the same simplified formulas in situations when we do not ask each participant for his/her own degree of authorship: in this case, we can simply take $d_{ii} = 0$, and the previous formula turns into this simplified form.

Now we know – at least approximately – what was each person's contribution to the project, so it makes sense, instead of using the arithmetic average, to take the weighted average – in which the opinion of each participant j is weighted proportionally to this person's degree of authorship. This way, we get the next approximation $a_i^{(1)}$ to the degree of authorship:

$$a_i^{(1)} = \frac{1}{N} \cdot \sum_{j} d_{ij} \cdot a_j^{(0)}.$$

We can use, as weights, these more accurate degrees of authorship, and we can made these updates again and again. In this procedure, onece we know the degree of authorship $a_i^{(k)}$ on the k-th iteration, the degree of authorship on the next iteration can be computed as follows:

$$a_i^{(k+1)} = \frac{1}{N} \cdot \sum_{j} d_{ij} \cdot a_j^{(k)}.$$

Interestingly, this is the same iterative process that leads to PageRank – a numerical criterion that Google search uses to rank possible answers to queries; see, e.g., [6]. In the limit, this process converges to limit values a_i for which

$$a_i = \frac{1}{N} \cdot \sum_j d_{ij} \cdot a_j.$$

From the mathematical viewpoint, the values a_i form an eigenvector of the matrix d_{ij} – and in almost all cases, this is the eigenvector corresponding to the largest eigenvalue.

What if we take into account the uncertainty of the degrees? It is difficult for people to come up with exact numbers d_{ij} describing contributions of others: this is very subjective, and we do not think that it is possible to distinguish between, e.g., 50% and 51%. People are much more comfortable providing a range $[\underline{d}_{ij}, \overline{d}_{ij}]$ of possible values, such as $[0.6, 0.7]$.

In this case, in the first approximation, we come up with interval of possible values of $a_i^{(0)}$:

$$[\underline{a}_i^{(0)}, \overline{a}_i^{(0)}] = \frac{1}{N} \cdot \sum_j [\underline{d}_{ij}, \overline{d}_{ij}],$$

where by sum – or any other operation $\oplus$ – between intervals we mean the range of the values $a \oplus b$ when a and b lie in the corresponding intervals $[\underline{a}, \overline{a}]$ and $[\underline{b}, \overline{b}]$ (see, e.g., [3, 5, 8, 10]):

$$[\underline{a}, \overline{a}] \oplus [\underline{b}, \overline{b}] \stackrel{\text{def}}{=} \{a \oplus b : a \in [\underline{a}, \overline{a}], b \in [\underline{b}, \overline{b}]\}.$$

Once we have an approximation $[\underline{a}_i^{(k)}, \overline{a}_i^{(k)}]$, we can use these interval values as weights to compute the next approximation:

$$[\underline{a}_i^{(k+1)}, \overline{a}_i^{(k+1)}] = \frac{1}{N} \cdot \sum_j [\underline{d}_{ij}, \overline{d}_{ij}] \cdot [\underline{a}_j^{(k)}, \overline{a}_j^{(k)}].$$

The expression in the right-hand side is increasing in terms of $d_{ij}^{(k)}$ and $a_j^{(k)}$, so:

- its smallest possible value is attained when the values $d_{ij}^{(k)}$ and $a_j^{(k)}$ are the smallest possible, and
- its largest possible value is attained when the values $d_{ij}^{(k)}$ and $a_j^{(k)}$ are the largest possible.

In other words, we have

$$\underline{a}_i^{(k+1)} = \frac{1}{N} \cdot \sum_j \underline{d}_{ij} \cdot \underline{a}_j^{(k)},$$

and

$$\overline{a}_i^{(k+1)} = \frac{1}{N} \cdot \sum_j \overline{d}_{ij} \cdot \overline{a}_j^{(k)}.$$

Thus, in the limit, we have an interval $[\underline{a}_i, \overline{a}_i]$, where

$$\underline{a}_i = \frac{1}{N} \cdot \sum_j \underline{d}_{ij} \cdot \underline{a}_j,$$

and

$$\overline{a}_i = \frac{1}{N} \cdot \sum_j \overline{d}_{ij} \cdot \overline{a}_j,$$

i.e., where:

- the values $\underline{a}_i$ form an eigenvector of the matrix $\underline{d}_{ij}$ (corresponding to its largest eigenvalue), and
- the values $\overline{a}_i$ form an eigenvector of the matrix $\overline{d}_{ij}$ (corresponding to its largest eigenvalue).

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