Why Inverse Layers in Pavement? Why Zipper Fracking? Why Interleaving in Education? A General Explanation*

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Abstract. In many practical situations, if we split our efforts into two disconnected chunks, we get better results: a pavement is stronger if instead of a single strengthening layer, we place two parts of this layer separated by no-so-strong layers; teaching is more effective if instead of concentrating a topic in a single time interval, we split it into two parts separated in time, etc. In this paper, we provide a general explanation for all these phenomena.

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1 Formulation of the Problem

General idea. This research was motivated by the fact that in several application areas, there appears a similar empirical phenomenon, a phenomenon that,

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in each of these areas, is difficult to explain. In this paper, we provide a general explanation for this phenomenon. Let us list the examples of this phenomenon.

Pavement engineering. Road pavement must be strong enough to sustain the traffic loads. To strengthen the pavement, usually, the pavement is formed by the following layers (see, e.g., [1]):

- First, on top of the soil, we place compacted granular material; this is called the *sub-base*.
- On top of the sub-base, we place granular material strengthened with cement; this layer is called the *base*.
- Finally, the top layer is the granular material strengthened by adding the liquid asphalt; this layer is called the *asphalt concrete layer*.

In this arrangement, the strength of the pavement comes largely from the two top layers: the asphalt concrete layer and the base.

Empirical evidence shows that in many cases, the inverse layer structure, where the base and sub-base are switched – so that the two strong layers are separated by a weaker sub-base layer – leads to better pavement performance; see, e.g., [3, 6–8, 10, 11, 16–18, 20, 22, 25].

Fracking. Traditional methods of extracting oil and gas leave a significant portion of them behind. They were also unable to extract oil and gas that were concentrated in small amounts around the area. To extract this oil and gas, practitioners use the process called *fracking*, when high-pressure liquid is injected into the underground location, cracking the rocks and thus, providing the path for low-density oil and gas to move to the surface. Usually, several pipes are used to pump the liquid. Empirically, it turned out that the best performance happens when not all the pipes are active at the same time, but when there is always a significant distance between the active pipes. One way to maintain this distance – known as *zipper fracking* – is to activate, e.g., every other pipe, interchanging activations of pipes 1, 3, 5, etc., with activating the intermediate pipes 2, 4, 6, etc. (This particular technique is known as *Texas two-step*.) For more information, see, e.g., [19, 26] and references therein.

Education. In education, best learning results are achieved when there is a pause between two (or more) periods when some topic is studied; this pedagogical practice is known as *interleaving*. Several studies show that interleaving enhances different types of learning, from learning to play basketball [9, 13] to learning art [12] to learning mathematics [14, 23, 24], to training and re-training medical doctors [2, 21]; see also [4, 5, 15].

2 Towards an Explanation

General idea. What is the ideal situation?

- The ideal pavement would mean that all layers are strong.
- The ideal fracking would mean that all the pipes are active all the time.

– The ideal study process would mean that we study all the time.

So, a natural way to compare the quality of different strategies is to see which ones are closer to this ideal case.

A general mathematical description of the problem. Let us formulate this general setting in precise terms.

In general, we have a certain range: this can be the range that describes:

– strength as a function of depth,
– study intensity as a function of time, etc.

From the mathematical viewpoint, we can always change the starting point to be 0. For example, for studying, we can measure time starting with the moment when we started the whole study process. In this case, the range will take the form $[0, T]$ for some $T > 0$. So, for simplicity, let us assume that this range has the form $[0, T]$.

Ideally, we should have full intensity at all points from this range:

– we should have full strength at all depth,
– we should have full study intensity at all moments of time, etc.

Again, from the mathematical viewpoint, we can re-scale intensity by taking this level as a new unit for measuring intensity. After this re-scaling, the value of the high level of intensity will be 1. So, the ideal case (I) is described by a function that takes the value 1 on the whole interval $[0, T]$; see Fig. 1.

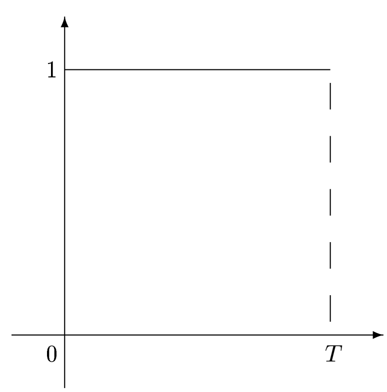

Fig. 1. Ideal case (I)

The problem is that in all the above applications, the ideal case is not realistic. In practice, we can have full strength only over a small portion of this range, a portion of overall size ε.

- We can have this strength portion concentrated on a connected (C) subrange (see Fig. 2) – as is the case, e.g., of the traditional pavement.
- Alternatively, we can divide this portion into two (or more) disconnected (D) subranges, as in Fig. 3.

In both cases, the value of intensity:

- is equal to 1 on a small part of the range, and
- is equal to 0 for all other values from the range.

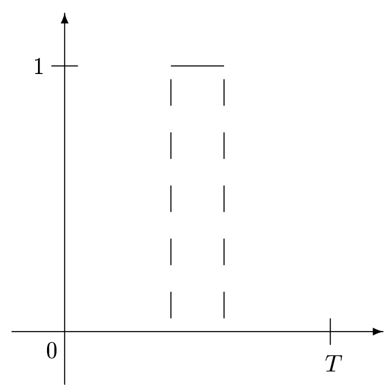

Fig. 2. Connected portion (C)

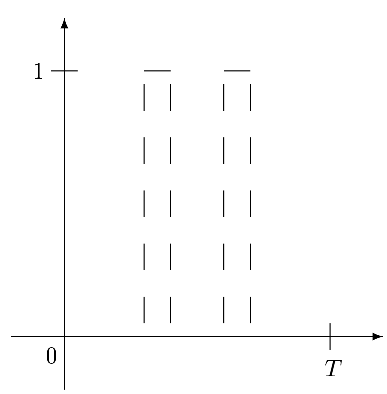

Fig. 3. Disconnected portion (D)

In all the above examples, the performance was better for the disconnected subranges. We will explain this by proving that, in some reasonable sense, the graph D corresponding to the disconnected portion is indeed closer to the graph I of the ideal dependence than the graph corresponding to the connected portion C, i.e., that $d(D,I) < d(C,I)$. In order to prove this, let us recall what is the natural way to describe distance $d(A,B)$ between two graphs A and B.

From the mathematical viewpoints, graphs are sets in a plane. So, to be able to describe distance between graphs, let us recall how to describe distance $d(A,B)$ between sets A and B.

How to define the distance $d(A,B)$ between two sets A and B: reminder. Let us start with the simplest case, when both sets are 1-element sets, i.e., when $A = \{a\}$ and $B = \{b\}$ for some points a and b. We assume that for two points a and b, distance $d(a,b)$ is already defined, In this case, it is reasonable to define $d(A,B) = d(\{a\},\{b\}) \stackrel{\text{def}}{=} d(a,b)$.

A natural idea is to use Euclidean distance here:

$$d((x,y),(x',y')) = \sqrt{(x-x')^2 + (y-y')^2}.$$

Instead, we can use a more general ℓ^p-metric for some $p \ge 1$:

$$d((x,y),(x',y')) = ((x-x')^p + (y-y')^p)^{1/p}.$$

It is worth mentioning that our result remains valid whiever value $p \ge 1$ we select.

A slightly more complex case is when only one the sets is a one-point set, e.g., $A = \{a\}$. In this case, it makes sense to define the distance $d(\{a\},B)$ is such a way that this distance is 0 when $a \in B$. A reasonable idea is to take

$$d(A,B) = d(\{a\},B) \stackrel{\text{def}}{=} \inf_{b\in B} d(a,b).$$

Finally, let us consider the general case, when both sets A and B may contain more than one point. In line with the general definition of a metric, we would like to have $d(A,B) = 0$ if and only if the sets A and B coincide, i.e., if and only if:

- every element the set A is also an element of the set B, and
- every element of the set B is also an element of the set A.

In other words, for us to declare that $d(A,B) = 0$:

- we must have $d(\{a\},B) = 0$ for all $a \in A$, and
- we must have $d(\{b\},A)$ for all $b \in B$.

The usual way to achieve this purpose is – similarly to how we defined $d(\{a\},B)$ – to define $d(A,B)$ as the largest of all these values; the resulting "worst-case" expression $d_w(A,B)$ is known as the *Hausdoff distance*:

$$d_w(A,B) \stackrel{\text{def}}{=} \max\left(\sup_{a\in A} d(\{a\},B), \sup_{b\in B} d(\{b\},A)\right). \tag{1}$$

In general, the worst case is nowalways the most adequate description. For example, if we have the set B almost equal to A, but with a very tiny additional part which is far away from the original set, the worst-case distance is huge, but in reality, the sets A and B are almost the case. To better capture the intuitive idea of distance between two sets, it is reasonable to consider not the *worst-case* values of $d(\{a\}, B)$ and $d(\{b\}, A)$, but their *average* values:

$$d_a(A, B) \stackrel{\text{def}}{=} \frac{1}{2} \cdot \frac{\int_A d(\{a\}, B)\, da}{\mu(A)} + \frac{1}{2} \cdot \frac{\int_B d(\{b\}, A)\, db}{\mu(B)}. \tag{2}$$

Let us see what these two definitions $d_w(A, B)$ and $d_a(A, B)$ say about the relation between our graphs I, C, and D.

What are the values $d_w(A, B)$ and $d_a(A, B)$ in our case. Both worst-case and average-case definitions are based on the values $d(\{a\}, B)$ and $d(\{b\}, A)$. So, to compute the distances between the corresponding graphs, let us first analyze what are the values $d(\{a\}, B)$ and $d(\{b\}, A)$ for our case.

Without losing generality, let us denote one of the graphs C or D by A, and the ideal graph I by B. Let us first consider the values $d(\{a\}, B) = d(\{a\}, I)$.

- Here, for points $a \in A$ corresponding to the portion of overall length ε, the intensity is equal to 1. So these points also belong to the graph I and thus, $d(\{a\}, I) = 0$.
- For all other points $a \in$, the intensity is 0, i.e., this point has the form $(x, 0)$ for some $x \in [0, T]$. The set I is the straight line segment. So, the closest element to I is the projection of the point A on this straight line, i.e., the point $(x, 1)$. In this case, the shortest distance $d(\{a\}, I)$ from the point a and points $b \in I$ is equal to 1: $d(\{a\}, I) = 1$.

So, we have

$$\sup_{a \in A} d(\{a\}, I) = 1 \tag{3}$$

and

$$\frac{\int_A d(\{a\}, I)\, da}{\mu(A)} = \frac{0 \cdot \varepsilon + 1 \cdot (T - \varepsilon)}{T} = \frac{T - \varepsilon}{T}. \tag{4}$$

It should be mentioned that the values (3) and (4) are the same both:

- for the connected portion C and
- for the disconnected portion D;

these values only depend on the overall length of the portion.

Let us now consider the values $d(\{b\}, A)$, when $b \in I$, i.e., when $b = (x, 1)$ for some $x \in [0, 1]$, and A is C or D. By definition, $d(\{b\}, A)$ is the smallest of the values $d(a, b)$ when a is in the set A, i.e., when a is:

- either in the portion – in which case $a = (x', 1)$ for some $x' \in [0, T]$,
- or not in the portion – in which case $a = (x', 0)$ for some $x' \in [0, T]$.

In the second case, the distance is at least 1 – and can always be made smaller than or equal to 1 if we take the point $(x, \cdot) \in A$. In the first case, the distance is equal to $d(a, b) = d((x, 1), (x', 1)) = |x - x'|$. So:

- for points $b = (x, 1) \in I$ which are at most 1-close to the portion, the shortest distance $d(\{b\}, A)$ is equal to the distance z between x and the portion, while
- for all other points $b = (x, 1) \in I$, we have $d(\{b\}, A) = 1$.

And herein lies the difference between the connected case C and the disconnected case D. In the connected case, we have:

- one connected portion of length ε on which $d(\{b\}, A) = 0$, and
- two nearby intervals for which $d(\{b\}, A) < 1$,

see Fig. 4.

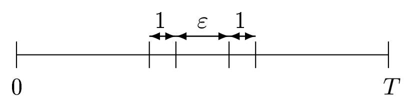

Fig. 4. Case of connected portion (C)

In this case, provided:

- that ε is sufficiently small, and
- that the portion is sufficiently separated from the endpoints 0 and T of the range,

we have

$$\sup_{b \in C} d(\{b\}, I) = 1 \tag{5}$$

and

$$\int_I d(\{b\}, C)\, db = 0 \cdot \varepsilon + 2 \int_0^1 z\, dz + (T - 2 - \varepsilon) \cdot 1 = 2 \cdot \frac{1}{2} + T - 2 - \varepsilon = T - 1 - \varepsilon,$$

thus

$$\frac{\int_I d(\{b\}, C)\, db}{\mu(I)} = \frac{T - 1 - \varepsilon}{T}. \tag{6}$$

In the disconnected case, we have:

- two connected subranges (of length $\varepsilon/2$ each) on which $d(\{b\}, A) = 0$, and
- two pairs of nearby intervals for which $d(\{b\}, A) < 1$,

see Fig. 5.

In this case, provided:

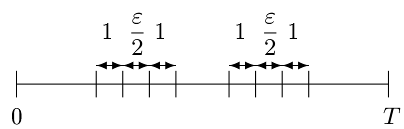

Fig. 5. Case of disconnected portion (D)

- that ε is sufficiently small, and
- that both subranges are sufficiently separated from each other and from the endpoints 0 and T of the range,

we have

$$\sup_{b\in D} d(\{b\},I)=1 \tag{7}$$

and

$$\int_I d(\{b\},D)\,db=0\cdot\varepsilon+4\int_0^1 z\,dz+(T-2-\varepsilon)\cdot 1=4\cdot\frac{1}{2}+T-4-\varepsilon=T-2-\varepsilon,$$

thus

$$\frac{\int_I d(\{b\},C)\,db}{\mu(I)}=\frac{T-2-\varepsilon}{T}. \tag{8}$$

By combining the formulas (3), (5), and (7), we conclude that

$$d_w(C,I)=d_w(D,I)=1.$$

Thus, if we only take into account the worst-case distance, then we cannot distinguish between the connected and disconnected cases.

However, if we use a more adequate average distance, then, by combining the formulas (4), (6), and (8), we get

$$d_a(C,I)=\frac{1}{2}\cdot\left(\frac{T-\varepsilon}{T}+\frac{T-1-\varepsilon}{T}\right)=\frac{T-1/2-\varepsilon}{T}, \qquad (9),$$

while

$$d_a(D,I)=\frac{1}{2}\cdot\left(\frac{T-\varepsilon}{T}+\frac{T-2-\varepsilon}{T}\right)=\frac{T-1-\varepsilon}{T}. \tag{10}$$

Here clearly, $d_a(D,I)<d_a(C,I)$. In other words, the disconnected situation is closer to the ideal case than the connected one – which explains why in all above cases, the disconnected approach indeed leads to better results.

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