

Why Inverse Layers in Pavement? Why Zipper Fracking? Why Interleaving in Education? A General Explanation*

Edgar Daniel Rodriguez Velasquez^{1,2}, Aaron Velasco³,
Olga Kosheleva⁴[0000-0003-2587-4209], and
Vladik Kreinovich⁵[0000-0002-1244-1650]

¹ Department of Civil Engineering, Universidad de Piura in Peru (UDEP)
Piura, Peru, edgar.rodriguez@udep.pe

² Department of Civil Engineering, University of Texas at El Paso,
El Paso, TX 79968, USA, edrodriguezvelasquez@miners.utep.edu

³ Department of Earth, Environment, and Resource Sciences,
University of Texas at El Paso, El Paso, TX 79968, USA
aavelasco@utep.edu

⁴ Department of Teacher Education, University of Texas at El Paso,
El Paso, TX 79968, USA, olgak@utep.edu

⁵ Department of Computer Science, University of Texas at El Paso,
El Paso, TX 79968, USA, vladik@utep.edu

Abstract. In many practical situations, if we split our efforts into two disconnected chunks, we get better results: a pavement is stronger if instead of a single strengthening layer, we place two parts of this layer separated by no-so-strong layers; teaching is more effective if instead of concentrating a topic in a single time interval, we split it into two parts separated in time, etc. In this paper, we provide a general explanation for all these phenomena.

Keywords: Pavement engineering · Fracking · Interleaving in education.

1 Formulation of the Problem

General idea. This research was motivated by the fact that in several application areas, there appears a similar empirical phenomenon, a phenomenon that,

* This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), HRD-1834620 and HRD-2034030 (CAHSI Includes), EAR-2225395, and by the AT&T Fellowship in Information Technology.

It was also supported by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478, and by a grant from the Hungarian National Research, Development and Innovation Office (NRDI).

in each of these areas, is difficult to explain. In this paper, we provide a general explanation for this phenomenon. Let us list the examples of this phenomenon.

Pavement engineering. Road pavement must be strong enough to sustain the traffic loads. To strengthen the pavement, usually, the pavement is formed by the following layers (see, e.g., [1]):

- First, on top of the soil, we place compacted granular material; this is called the *sub-base*.
- On top of the sub-base, we place granular material strengthened with cement; this layer is called the *base*.
- Finally, the top layer is the granular material strengthened by adding the liquid asphalt; this layer is called the *asphalt concrete layer*.

In this arrangement, the strength of the pavement comes largely from the two top layers: the asphalt concrete layer and the base.

Empirical evidence shows that in many cases, the inverse layer structure, where the base and sub-base are switched – so that the two strong layers are separated by a weaker sub-base layer – leads to better pavement performance; see, e.g., [3, 6–8, 10, 11, 16–18, 20, 22, 25].

Fracking. Traditional methods of extracting oil and gas leave a significant portion of them behind. They were also unable to extract oil and gas that were concentrated in small amounts around the area. To extract this oil and gas, practitioners use the process called *fracking*, when high-pressure liquid is injected into the underground location, cracking the rocks and thus, providing the path for low-density oil and gas to move to the surface. Usually, several pipes are used to pump the liquid. Empirically, it turned out that the best performance happens when not all the pipes are active at the same time, but when there is always a significant distance between the active pipes. One way to maintain this distance – known as *zipper fracking* – is to activate, e.g., every other pipe, interchanging activations of pipes 1, 3, 5, etc., with activating the intermediate pipes 2, 4, 6, etc. (This particular technique is known as *Texas two-step*.) For more information, see, e.g., [19, 26] and references therein.

Education. In education, best learning results are achieved when there is a pause between two (or more) periods when some topic is studied; this pedagogical practice is known as *interleaving*. Several studies show that interleaving enhances different types of learning, from learning to play basketball [9, 13] to learning art [12] to learning mathematics [14, 23, 24], to training and re-training medical doctors [2, 21]; see also [4, 5, 15].

2 Towards an Explanation

General idea. What is the ideal situation?

- The ideal pavement would mean that all layers are strong.
- The ideal fracking would mean that all the pipes are active all the time.

- The ideal study process would mean that we study all the time.

So, a natural way to compare the quality of different strategies is to see which ones are closer to this ideal case.

A general mathematical description of the problem. Let us formulate this general setting in precise terms.

In general, we have a certain range; this can be the range that describes:

- strength as a function of depth,
- study intensity as a function of time, etc.

From the mathematical viewpoint, we can always change the starting point to be 0. For example, for studying, we can measure time starting with the moment when we started the whole study process. In this case, the range will take the form $[0, T]$ for some $T > 0$. So, for simplicity, let us assume that this range has the form $[0, T]$.

Ideally, we should have full intensity at all points from this range:

- we should have full strength at all depth,
- we should have full study intensity at all moments of time, etc.

Again, from the mathematical viewpoint, we can re-scale intensity by taking this level as a new unit for measuring intensity. After this re-scaling, the value of the high level of intensity will be 1. So, the ideal case (I) is described by a function that takes the value 1 on the whole interval $[0, T]$; see Fig. 1.

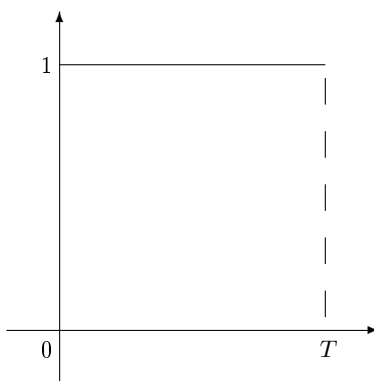


Fig. 1. Ideal case (I)

The problem is that in all the above applications, the ideal case is not realistic. In practice, we can have full strength only over a small portion of this range, a portion of overall size ε .

- We can have this strength portion concentrated on a connected (C) subrange (see Fig. 2) – as is the case, e.g., of the traditional pavement.
- Alternatively, we can divide this portion into two (or more) disconnected (D) subranges, as in Fig. 3.

In both cases, the value of intensity:

- is equal to 1 on a small part of the range, and
- is equal to 0 for all other values from the range.

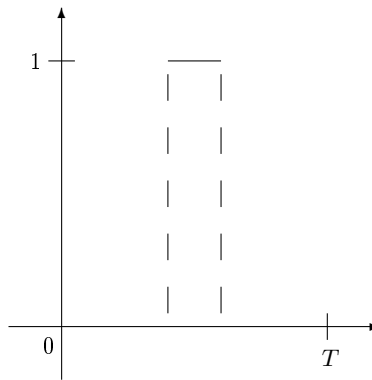


Fig. 2. Connected portion (C)

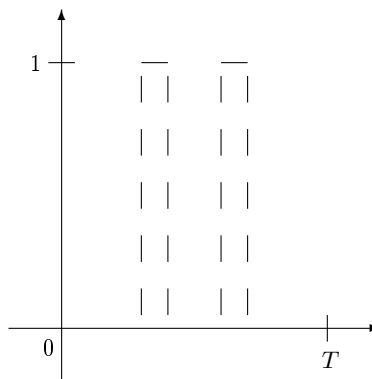


Fig. 3. Disconnected portion (D)

In all the above examples, the performance was better for the disconnected subranges. We will explain this by proving that, in some reasonable sense, the graph D corresponding to the disconnected portion is indeed closer to the graph I of the ideal case than the graph corresponding to the connected portion C , i.e., that $d(D, I) < d(C, I)$. In order to prove this, let us recall what is the natural way to describe distance $d(A, B)$ between two graphs A and B .

From the mathematical viewpoints, graphs are sets in a plane. So, to be able to describe distance between graphs, let us recall how to describe distance $d(A, B)$ between sets A and B .

How to define the distance $d(A, B)$ between two sets A and B : reminder.

Let us start with the simplest case, when both sets are 1-element sets, i.e., when $A = \{a\}$ and $B = \{b\}$ for some points a and b . We assume that for two points a and b , distance $d(a, b)$ is already defined. In this case, it is reasonable to define $d(A, B) = d(\{a\}, \{b\}) \stackrel{\text{def}}{=} d(a, b)$.

A natural idea is to use Euclidean distance here:

$$d((x, y), (x', y')) = \sqrt{(x - x')^2 + (y - y')^2}.$$

Instead, we can use a more general ℓ^p -metric for some $p \geq 1$:

$$d((x, y), (x', y')) = (|x - x'|^p + |y - y'|^p)^{1/p}.$$

It is worth mentioning that our result remains valid whichever value $p \geq 1$ we select.

A slightly more complex case is when only one of the sets is a one-point set, e.g., $A = \{a\}$. In this case, it makes sense to define the distance $d(\{a\}, B)$ in such a way that this distance is 0 when $a \in B$. A reasonable idea is to take

$$d(A, B) = d(\{a\}, B) \stackrel{\text{def}}{=} \inf_{b \in B} d(a, b).$$

Finally, let us consider the general case, when both sets A and B may contain more than one point. In line with the general definition of a metric, we would like to have $d(A, B) = 0$ if and only if the sets A and B coincide, i.e., if and only if:

- every element the set A is also an element of the set B , and
- every element of the set B is also an element of the set A .

In other words, for us to declare that $d(A, B) = 0$:

- we must have $d(\{a\}, B) = 0$ for all $a \in A$, and
- we must have $d(\{b\}, A) = 0$ for all $b \in B$.

The usual way to achieve this purpose is – similarly to how we defined $d(\{a\}, B)$ – to define $d(A, B)$ as the largest of all these values; the resulting “worst-case” expression $d_w(A, B)$ is known as the *Hausdoff distance*:

$$d_w(A, B) \stackrel{\text{def}}{=} \max \left(\sup_{a \in A} d(\{a\}, B), \sup_{b \in B} d(\{b\}, A) \right). \quad (1)$$

In general, the worst case is not always the most adequate description. For example, if we have the set B almost equal to A , but with a very tiny additional part which is far away from the original set, the worst-case distance is huge, but in reality, the sets A and B are almost the same. To better capture the intuitive idea of distance between two sets, it is reasonable to consider not the *worst-case* values of $d(\{a\}, B)$ and $d(\{b\}, A)$, but their *average* values:

$$d_a(A, B) \stackrel{\text{def}}{=} \frac{1}{2} \cdot \frac{\int_A d(\{a\}, B) da}{\mu(A)} + \frac{1}{2} \cdot \frac{\int_B d(\{b\}, A) db}{\mu(B)}. \quad (2)$$

Let us see what these two definitions $d_w(A, B)$ and $d_a(A, B)$ say about the relation between our graphs I , C , and D .

What are the values $d_w(A, B)$ and $d_a(A, B)$ in our case. Both worst-case and average-case definitions are based on the values $d(\{a\}, B)$ and $d(\{b\}, A)$. So, to compute the distances between the corresponding graphs, let us first analyze what are the values $d(\{a\}, B)$ and $d(\{b\}, A)$ for our case.

Without losing generality, let us denote one of the graphs C or D by A , and the ideal graph I by B . Let us first consider the values $d(\{a\}, B) = d(\{a\}, I)$.

- Here, for points $a \in A$ corresponding to the portion of overall length ε , the intensity is equal to 1. So these points also belong to the graph I and thus, $d(\{a\}, I) = 0$.
- For all other points $a \in A$, the intensity is 0, i.e., this point has the form $(x, 0)$ for some $x \in [0, T]$. The set I is the straight line segment. So, the closest element to I is the projection of the point A on this straight line, i.e., the point $(x, 1)$. In this case, the shortest distance $d(\{a\}, I)$ from the point a and points $b \in I$ is equal to 1: $d(\{a\}, I) = 1$.

So, we have

$$\sup_{a \in A} d(\{a\}, I) = 1 \quad (3)$$

and

$$\frac{\int_A d(\{a\}, I) da}{\mu(A)} = \frac{0 \cdot \varepsilon + 1 \cdot (T - \varepsilon)}{T} = \frac{T - \varepsilon}{T}. \quad (4)$$

It should be mentioned that the values (3) and (4) are the same both:

- for the connected portion C and
- for the disconnected portion D ;

these values only depend on the overall length of the portion.

Let us now consider the values $d(\{b\}, A)$, when $b \in I$, i.e., when $b = (x, 1)$ for some $x \in [0, 1]$, and A is C or D . By definition, $d(\{b\}, A)$ is the smallest of the values $d(a, b)$ when a is in the set A , i.e., when a is:

- either in the portion – in which case $a = (x', 1)$ for some $x' \in [0, T]$,
- or not in the portion – in which case $a = (x', 0)$ for some $x' \in [0, T]$.

In the second case, the distance is at least 1 – and can always be made smaller than or equal to 1 if we take the point $(x, \cdot) \in A$. In the first case, the distance is equal to $d(a, b) = d((x, 1), (x', 1)) = |x - x'|$. So:

- for points $b = (x, 1) \in I$ which are at most 1-close to the portion, the shortest distance $d(\{b\}, A)$ is equal to the distance z between x and the portion, while
- for all other points $b = (x, 1) \in I$, we have $d(\{b\}, A) = 1$.

And herein lies the difference between the connected case C and the disconnected case D . In the connected case, we have:

- one connected portion of length ε on which $d(\{b\}, A) = 0$, and
- two nearby intervals for which $d(\{b\}, A) < 1$,

see Fig. 4.

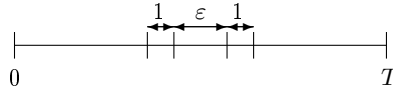


Fig. 4. Case of connected portion (C)

In this case, provided:

- that ε is sufficiently small, and
- that the portion is sufficiently separated from the endpoints 0 and T of the range,

we have

$$\sup_{b \in C} d(\{b\}, I) = 1 \quad (5)$$

and

$$\int_I d(\{b\}, C) db = 0 \cdot \varepsilon + 2 \int_0^1 z dz + (T - 2 - \varepsilon) \cdot 1 = 2 \cdot \frac{1}{2} + T - 2 - \varepsilon = T - 1 - \varepsilon,$$

thus

$$\frac{\int_I d(\{b\}, C) db}{\mu(I)} = \frac{T - 1 - \varepsilon}{T}. \quad (6)$$

In the disconnected case, we have:

- two connected subranges (of length $\varepsilon/2$ each) on which $d(\{b\}, A) = 0$, and
- two pairs of nearby intervals for which $d(\{b\}, A) < 1$,

see Fig. 5.

In this case, provided:

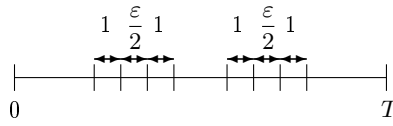


Fig. 5. Case of disconnected portion (D)

- that ε is sufficiently small, and
- that both subranges are sufficiently separated from each other and from the endpoints 0 and T of the range,

we have

$$\sup_{b \in D} d(\{b\}, I) = 1 \quad (7)$$

and

$$\int_I d(\{b\}, D) db = 0 \cdot \varepsilon + 4 \int_0^1 z dz + (T - 2 - \varepsilon) \cdot 1 = 4 \cdot \frac{1}{2} + T - 4 - \varepsilon = T - 2 - \varepsilon,$$

thus

$$\frac{\int_I d(\{b\}, C) db}{\mu(I)} = \frac{T - 2 - \varepsilon}{T}. \quad (8)$$

By combining the formulas (3), (5), and (7), we conclude that

$$d_w(C, I) = d_w(D, I) = 1.$$

Thus, if we only take into account the worst-case distance, then we cannot distinguish between the connected and disconnected cases.

However, if we use a more adequate average distance, then, by combining the formulas (4), (6), and (8), we get

$$d_a(C, I) = \frac{1}{2} \cdot \left(\frac{T - \varepsilon}{T} + \frac{T - 1 - \varepsilon}{T} \right) = \frac{T - 1/2 - \varepsilon}{T}, \quad (9)$$

while

$$d_a(D, I) = \frac{1}{2} \cdot \left(\frac{T - \varepsilon}{T} + \frac{T - 2 - \varepsilon}{T} \right) = \frac{T - 1 - \varepsilon}{T}. \quad (10)$$

Here clearly, $d_a(D, I) < d_a(C, I)$. In other words, the disconnected situation is closer to the ideal case than the connected one – which explains why in all above cases, the disconnected approach indeed leads to better results.

3 Conclusions and Future Work

This paper provides a mathematical explanation for a phenomenon that is observed in reality but runs against our intuition: that a “disconnected” control,

when there is a pause between the two control stages or a gap between two control locations, is often more effective than the “connected” control, with no pauses or gaps. This is true in education, when learning the material during two time intervals, with a pause in between, is often more effective than when these two time intervals immediately follow each other. This is true in pavement engineering, where it is more effective to place two strong layers at some distance from each other rather than placing them next to each other – as it is usually done. This is true in fracking, where keeping a distance between the two active pipes is more effective than activating two neighboring pipes.

In this paper, we provide a general mathematical explanation for this phenomenon – that for a naturally defined distance between settings, the discontinuous setting is closer to the ideal one than the continuous setting. The generality of this explanation makes us conjecture that a similar discontinuous arrangement is worth trying in many other cases, be it automatic control, medical therapy, or influencing people – and our first-approximation mathematical model will hopefully provide a way to compare different approaches.

Acknowledgements The authors are greatly thankful to the anonymous reviewers for valuable suggestions.

References

1. American Association of State Highway and Transportation Officials (AASHTO), *Mechanistic-Empirical Pavement Design Guide: A Manual of Practice*, AASHTO, Washington, D.C., 2008.
2. R. Bachhel and R. G. Thaman, “Effective use of pause procedure to enhance student engagement and learning”, *Journal of Clinical and Diagnostic Research*, 2014, Vol. 8, No. 8, pp. XM01–XM03, doi 10.7860/JCDR/2014/8260.4691
3. R. D. Barksdale, *Performance of Crushed Stone Base Courses*, Transportation Research Record 954, Transportation Research Board, Washington, D.C., 1984.
4. M. S. Birnbaum, N. Kornell, E. L. Bjork, and R. A. Bjork. “Why interleaving enhances inductive learning: the roles of discrimination and retrieval”, *Memory Cognition*, 2013, Vol. 41, pp 392–402.
5. L. Bokati, J. Urenda, O. Kosheleva, and V. Kreinovich, “Why immediate repetition is good for short-term learning results but bad for long-term learning: explanation based on decision theory”, In: M. Ceberio and V. Kreinovich (eds.), *How Uncertainty-Related Ideas Can Provide Theoretical Explanation for Empirical Dependencies*, Springer, Cham, Switzerland, 2021, pp. 27–35.
6. S. Buchanan, “Inverted pavements-what, why, and how?”, *Proceedings of the AFTR Industry Education Webinar*, Aggregates Foundation for Technology, Research, and Education, 2010, available at <https://www.vulcaninnovations.com/public/pdf/4-Inverted-Pavement-Systems.pdf>
7. T. Georges, *Falling Weight Deflectometer (FWD) Test Results: Entrance Road of the Lafarge Quarry in Morgan County, Georgia*, Final Report, Georgia Department of Transportation, 2007.

8. R. W. Grau, *Evaluation of Structural Layers of Flexible Pavements*, Miscellaneous Paper S-73-26, U.S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi, 1973.
9. K. G. Hall, D. A. Dominguez, and R. Cavazos, "Contextual interference effects with skilled basketball players", *Perceptual and Motor Skills*, 1994, Vol. 78, pp. 835–841.
10. B. E. Hoskins, B. F. McCullough, and D. W. Fowler, *The Development of a Long-Range Rehabilitation Plan for US-59, in District 11*, Research Report No. 987-1, Austin, Texas, 1991.
11. C. W. Johnson, *Comparative Studies of Combinations of Treated and Untreated Bases and Subbase for Flexible Pavements*, Highway Research Board, Bulletin 289, National Research Council, Washington, D.C., 1960.
12. N. Kornell and R. A. Bjork, "Leaning concepts and categories: is spacing the 'enemy of induction'?", *Psychological Science*, 2008, Vol. 19, pp. 585–592.
13. D. K. Landin, E. P. Hebert, and M. Fairweather, "The effects of variable practice on the performance of a basketball skill", *Research Quarterly for Exercise and Sports*, 1993, Vol. 64, pp. 232–236.
14. K. Le Blanc and D. Simon, "Mixed practice enhances retention and JOL accuracy for mathematical skills", *Proceedings of the 49th Annual Meeting of the Psychonomic Society*, Chicago, Illinois, November 2008.
15. O. Lerma, O. Kosheleva, and V. Kreinovich, "Interleaving enhances learning: a possible geometric explanation", *Geombinatorics*, 2015, Vol. 24, No. 3, pp. 135–139.
16. D. E. Lewis, *Inverted Base Pavement at Lafarge Quarry Entrance Road: 445 Five-Year Field Evaluation*, Final Report. Georgia Department of Transportation (GDOT), Georgia, 2006.
17. D. E. Lewis, K. Ledford, T. Georges, and D. M. Jared, "Construction and performance of inverted pavements in Georgia", *Proceedings of the 91st Annual Meeting of the Transportation Research Board*, Washington, D.C., January 22–26, 2012, Paper No. 12-1872, Poster Presentation in Session 639.
18. E. D. Moody, *Field Investigations of Selected Strategies to Reduce Reflective Cracking in Asphalt Concrete Overlays Constructed over Existing Jointed Concrete Pavements*, Transportation Research Record 1449, Transportation Research Board, Washington, D.C., 1994.
19. B. Qian, J. Zhang, J. Zhu, Z. Fang, S. Kou, and R. Chen, "Application of zipper-fracturing of horizontal cluster wells in the Changning shale gas pilot zone, Sichuan Basin," *Natural Gas Industry B*, 2015, Vol. 2, No. 2–3, pp. 181–184.
20. M. Rasoulia, B. Becnel, and G. Keel, *Stone Interlayer Pavement Design*, Transportation Research Record 1709, Transportation Research Board, Washington, D.C., 2000.
21. L. W. Richards, A. T. Wang, S. Mahapatra, S. M. Jenkins, N. M. Collins, T. J. Beckman, and C. M. Wittich, "Use of the pause procedure in continuing medical education: A randomized controlled intervention study", *Medical Teacher*, 2017, Vol. 39, No. 1, pp. 74–78.
22. E. D. Rodriguez Velasquez, *Characterization and Modeling of Unbound and Cementitiously Stabilized Materials for Structural Analysis of Multilayer Pavement Systems*, PhD Dissertation, Department of Civil Engineering, University of Texas at El Paso, El Paso, Texas, USA, May 2023.
23. D. Rohrer and K. Taylor, "The shuffling of mathematics practice problems boosts learning", *Instructional Science*, 2007, Vol. 35, pp. 481–498.
24. K. Taylor and D. Rohrer, "The effects of interleaved practice", *Applied Cognitive Psychology*, 2010, Vol. 24, pp. 837–848.

25. H. Titi, M. Rasoulian, M. Martinez, B. Becnel, and G. Keel, "Long-term performance of stone interlayer pavement", *Journal of Transportation Engineering*, 2003, Vol. 129, No. 2, pp. 118–126.
26. W. Zhen, Y. Lifeng, G. Rui, X. Guanshui, L. Zhe, M. Shaoyuan, F. Meng, and W. Xin, "Numerical analysis of zipper fracturing using a non-planar 3D fracture model", *Frontiers in Earth Science*, 2022, Vol. 10, DOI 10.3389/feart.2022.808183.