Why Resilient Modulus Is Proportional to the Square Root of Unconfined Compressive Strength (UCS): A Qualitative Explanation

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Abstract The strength of the pavement is determine by its resilient modulus, i.e., by its ability to withstand (practically) instantaneous stresses caused by the passing traffic. However, the resilient modulus is not easy to measure: its measurement requires a special expensive equipment that many labs do not have. So, instead of measuring it, practitioners often measure easier-to-measure Unconfined Compressive Strength (UCS) – that describes the effect of a continuously applied force – and estimate the resilient modulus based on the result of this measurement. An empirical formula shows that the resilient modulus is proportional to the square root of the Unconfined Compressive Strength. In this paper, we provide a possible explanation for this empirical dependence.

1 Formulation of the Problem

What is important for pavement is resilient modulus M_r. We want pavement to be strong enough to withstand a dynamic stress of the traffic. For each piece of pavement, a car or a truck passing by provides a practically instantaneous stress, lasting about 0.1 seconds, followed by a period whether there are no cars over this piece. The modulus measuring the material's reaction to this type of load is called *resilient modulus* M_r. In general, modulus is a ratio of load and deformation. When

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we take the deformation corresponding to instantaneous loads, we get the resilient modulus.

Resilient modulus is difficult and expensive to measure. Directly measuring resilient modulus means simulating instantaneous stress conditions. This requires special expensive equipment that most labs do not have.

What we can easily measure is Unconfined Compressive Strength (UCS). Most labs, however, have equipment that measures the Unconfined Compressive Strength (UCS), i.e., the modulus corresponding to the situation when we compress the material for some time.

It is therefore desirable to estimate M_r based on UCS. In view of the fact that directly measuring M_r is difficult, but UCS is easy to measure, it is desirable to be able to estimate M_r based on the results of measuring UCS.

Empirical dependence. According to the experiments described in [1], the following formula provides a good estimate for M_r:

$$M_r = \text{const} \cdot \sqrt{\text{UCS}}. \tag{1}$$

Remaining problem. A natural question is: how to explain this empirical dependence?

What we do in this paper. In this paper, we provide a possible explanation for this dependence.

2 A Possible Explanation

How force leads to deformation: a brief reminder. When you apply a force to an object, it affects the object, causing its deformation. This effect is not instantaneous: once the force is applied, the deformation process starts and it continues while the object is under stress. Once the stress ends, the object goes back to its original non-deformed shape.

What is the main difference between forces corresponding to M_r and to UCS. To understand the difference between the deformations corresponding to the resilient modulus M_r and to the Unconfined Compressive Strength UCS, let us describe the difference between the corresponding forces. In both cases, the overall action is the same, the difference is in timing:

- The resilient modulus corresponds to the instantaneous force, that acts for a short period of time and then disappears. So, in this case, all the force is applied at one single moment of time. (Then, after a rest period, it is applied again.)
- In contrast, the UCS corresponds to the force that acts for a certain period of time. In this case, the force is spread over time.

Resulting explanation. For simplicity, let us consider the effect of the force F as the joint effect of F unit forces. Each unit force causes some deformation that continues to increase until the object is not longer under stress. Let d denote deformation per time that is caused by a unit force.

In the case of the instantaneous force, we have F unit forces applied at the same time. Each of them causes deformation d, so the overall deformation is equal to $F \cdot d$. This force is only in action during one single moment of time. At the next moment of time, the stress ends, and the object goes back to the original shape. So, in this case, the overall deformation is equal to

$$d_1 = F \cdot d. \tag{1}$$

To analyze the case of the continuous force, let us divide the time T during which the force is applied into F moments of time, so that at each moment of time, we, in effect, apply the effect $F/F = 1$. Then:

- At the first moment of time, we have the effective unit stress that leads to deformation d.
- At the second moment of time, the object is still under stress, so the previous stress continues to cause the additional deformation, leading to another deformation of size d. In addition, the force applied at the second moment of time also causes additional deformation d. So, at the second moment of time, the additional deformation is equal to $2 \cdot d$.
- In general, at each moment t, the object is still under stress, so each of the previous $t-1$ stresses continues to cause the additional deformation, leading to another deformation of size $(t-1) \cdot d$. In addition, the force applied at the t-th moment of time also causes additional deformation d. So, at the t-th moment of time, the additional deformation is equal to $t \cdot d$.

The resulting accumulated deformation can be obtained by adding the original deformation (at the first moment of time) and additional deformations that occurred during moments $t = 2, \ldots, F$. Thus, the overall deformation is equal to

$$d + 2 \cdot d + \ldots + F \cdot d = (1 + 2 + \ldots + F) \cdot d.$$

It is known that

$$1 + 2 + \ldots + F = \frac{F \cdot (F-1)}{2},$$

thus the overall deformation is equal to

$$d_2 = \frac{F \cdot (F-1)}{2} \cdot d,$$

which, for large F, is approximately equal to

$$d_2 \approx \frac{1}{2} \cdot F^2 \cdot d. \tag{2}$$

The square root of the expression (2) is thus equal to

$$\sqrt{d_2} = \text{const} \cdot F.$$

So, we see that the deformation (1) corresponding to the instantaneous case is indeed proportional to the square root of the deformation (2) corresponding to the continuous case.

This explains why the resilient modulus (which is determined by the instantaneous deformation) is proportional to the square root of the Unconfined Compressive Strength (which is determined by the continuous deformation).

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