When Is It Beneficial to Merge Two Companies? When Is it Beneficial to Start a Research Collaboration?

Miroslav Svítek, Olga Kosheleva, and Vladik Kreinovich

Abstract Merging two companies or splitting a company into two, teaming of two researchers or two research groups – or splitting a research group into two – these are frequent occurrences. Sometimes these actions lead to increased effectiveness, but sometimes, contrary to the optimistic expectations, the overall effectiveness decreases. To minimize the possibility of such failures, it is desirable to replace the current semi-intuitive way of making the corresponding decisions with a more objective approach. In this paper, we propose such an approach.

1 Formulation of the Problem

When joining forces is effective and when it is not. Sometimes, joining two systems – be it companies of research teams – makes them more effective, but sometimes, they end up fighting and decreasing the overall productivity. In this paper, we provide a simple model for analyzing this phenomenon.

Sometimes, mergers and splits work, but sometimes, they don't. Mergers and splits are an important part of business strategies. The problem is that sometimes, they work, but sometimes, they don't.

For mergers:

- sometimes, the merger of two companies is beneficial, it increases the overall cost and the overall productivity;

Miroslav Svítek
Faculty of Transportation Sciences, Czech Technical University in Prague, Konviktska 20
CZ-110 00 Prague 1, Czech Republic, e-mail: svitek@fd.cvut.cz

Olga Kosheleva and Vladik Kreinovich
University of Texas at El Paso, 500 W. University, El Paso, Texas 79968, USA
e-mail: olgak@utep.edu, vladik@utep.edu

- however, sometimes, the expected gains never materialize and the merger lowers the overall value of the two companies.

  Similarly, for splits:

- sometimes, splitting a company into two is beneficial, it increases the overall cost and the overall productivity;
- however, sometimes, the expected gains never materialize and the split lower the overall value of the two resulting companies.

But why? The fact that frequently billion-dollar companies, companies that invest millions of dollars in the analysis of possible consequences of merger or split, end up with wrong decisions – is an indication that predicting success of a merger or of a split is still a challenge.

Sometimes, research collaboration works, but sometimes, it does not. Similarly:

- sometimes, when two researchers – or two groups of researchers – start working together, they productivity increases, but
- sometimes, this merger does not work.

Also:

- sometimes, splitting a department of research group into two boosts their productivity, but
- sometimes, it does not.

Here, billions of dollars are not at stake, but long-term, creative productivity of research is. This is, probably, even more important for the future of humanity than company mergers.

General problem. It is desirable to predict whether a planned merger or split will increase productivity – and, if the proposed split does increase productivity, to find out what is the optimal split (i.e., the split that will lead to the most effective results).

What we do in this paper. In this paper, we provide a natural simple model of this phenomenon, and we use this model to translate the above questions into precise mathematical form.

This translation does not necessarily mean that this model will always provide the optimal solution: the corresponding mathematical problems are a particular case of difficult (NP-hard) computational problems. However, since there are many tools for solving some classes of such problems, we hope that our model will be useful.

2 How to Approach This Problem

Towards a model: general idea. The effectiveness y of a company or of a research group is determined by the values $x_1, \ldots, x_n$, of different resources (generally understood) that are available to this company or group:

- For a company, this may include the available industrial capacity, the available line of credit, the numbers of specialists in different areas – as well as less objective characteristics like the number of enthusiastic workers, the number of people who have a negative attitude to changes, etc.
- For a research group, this may be number of specialists in different areas, numbers of people with different research styles, etc.

Let us describe the dependence of the effectiveness y on these values x_i by

$$y = f(x_1, \ldots, x_n).$$

Towards a model: specifics. Intuitively, small changes in the values x_i should not lead to drastic changes in the effectiveness – we do not expect that, e.g., hiring one more worker would drastically change the effectiveness of a large company. So, it is reasonable to assume that the dependence $y = f(x_1, \ldots, x_n)$ is smooth.

We do not know the exact shape of this dependence. A typical physics approach to such situations is to take into account that smooth functions can usually be expanded in Taylor series:

$$f(x_1, \ldots, x_n) = a_0 + \sum_{i=1}^{n} a_i \cdot x_i + \sum_{i,j=1}^{n} a_{ij} \cdot x_i \cdot x_j + \ldots \tag{1}$$

So:

- to provide the first approximation, we can keep only the linear terms in this expansion;
- if we want a more accurate description, we can also take into account quadratic terms, etc.;

see, e.g., [1, 3].

Let us apply this general idea to the merger problem.

For the merger problem, linear approximation is not enough. In line with the above general idea, let us start with the linear approximation, where we only keep linear terms in the expansion (1), i.e., where we consider the following approximate expression for the desired dependence:

$$f(x_1, \ldots, x_n) = a_0 + \sum_{i=1}^{n} a_i \cdot x_i. \tag{2}$$

If the company or a research group has no resources at all, that it cannot have any effectiveness. So, for $x_1 = \ldots = x_n = 0$, we have $y = 0$. Substituting the values $x_1 = \ldots = x_n = 0$ into the formula (2), we conclude that $a_0 = 0$. So, the linear expression takes the form

$$f(x_1,\ldots,x_n)=\sum_{i=1}^n a_i\cdot x_i. \tag{3}$$

Let us try to apply this model to analyze where merging two companies of two research groups makes sense. Let us denote:

- the values corresponding to the first entity by $x'_1,\ldots,x'_n$, and
- the values corresponding to the second entity by $x''_i,\ldots,x''_n$.

In these terms, the values corresponding to the merged entity are equal to $x_i=x'_i+x''_i$. Substituting these values into the formula (3), we get the following expressions for the effectiveness y' and y'' of the two original entities and for the effectiveness y of the merged entity

$$y'=\sum_{i=1}^n a_i\cdot x'_i,\quad y''=\sum_{i=1}^n a_i\cdot x''_i,\quad y=\sum_{i=1}^n a_i\cdot(x'_i+x''_i).$$

One can easily see that in this approximation, we have $y=y'+y''$, i.e., the effectiveness y of the merged entity is simply equal to the sum $y'+y''$ of the effectivenesses of the original entities. In other words, in this approximation, merger will not lead to any increase in effectiveness. If we take into account that the merger process itself require some resources, in this approximation, mergers make no sense.

So, to analyze effectiveness of mergers, we cannot use linear approximation, we need to consider at least the next – quadratic – terms in the Tayalor expansion. Thus, we arrive at the following model.

Quadratic approximation. In the quadratic approximation, the effectiveness of a company is determined by the formula

$$f(x_1,\ldots,x_n)=a_0+\sum_{i=1}^n a_i\cdot x_i+\sum_{i,j=1}^n a_{ij}\cdot x_i\cdot x_j.$$

Similarly to the linear case, if the entity has no resources, it cannot have any effectiveness. So, we have $a_0=0$ and the formula for describing effectiveness takes the following form:

$$f(x_1,\ldots,x_n)=\sum_{i=1}^n a_i\cdot x_i+\sum_{i,j=1}^n a_{ij}\cdot x_i\cdot x_j. \tag{4}$$

In this model the effectiveness of each of the two original entities has the form

$$y'=\sum_{i=1}^n a_i\cdot x'_i+\sum_{i,j=1}^n a_{ij}\cdot x'_i\cdot x'_j, \tag{5}$$

$$y''=\sum_{i=1}^n a_i\cdot x'_i+\sum_{i,j=1}^n a_{ij}\cdot x''_i\cdot x''_j, \tag{6}$$

and the effectiveness of the merged entity is equal to

$$y = \sum_{i=1}^{n} a_i \cdot (x'_i + x''_i) + \sum_{i,j=1}^{n} a_{ij} \cdot (x'_i + x''_i) \cdot (x'_j + x''_j). \tag{7}$$

Quadratic approximation provides an effective description of mergers. In the above quadratic approximation, the effectiveness y of the merged entity is, in general, different from the sum of the two original entities:

$$y = y' + y'' + 2 \sum_{i,j} a_{ij} \cdot x'_i \cdot x''_j. \tag{8}$$

Thus, in contrast to the linear approximation, the quadratic approximation already provides a reasonable description of the effect of merger on effectiveness.

Simple examples. Let us describe two simple examples: when merger works well, and when merger is counterproductive. For simplicity, let us consider the case when we have only one resource x_1. In this case, the general quadratic model (4) takes the form

$$y = a_1 \cdot x_1 + a_{11} \cdot x_1^2.$$

As we have mentioned, when $a_{11} = 0$, i.e., when the model is linear, the merger will not change the effectiveness at all. Let us therefore consider two remaining options: $a_{11} > 0$ and $a_{11} < 0$.

A simple example when the merger works well. Let us first consider the case when $a_{11} > 0$. In this case,

$$y = y' + y'' + 2 \cdot a_{11} \cdot x'_1 \cdot x''_1.$$

Since $a_{11} > 0$, we conclude that $y > y' + y''$. So, in this example, the merger works well – it increases the overall effectiveness.

For example, when $a_1 = 0$, $a_{11} > 0$, and we merge two equal-size entities $x'_1 = x''_1$, then the effectiveness y of the merged entity is equal to

$$y = 4 \cdot a_{11} \cdot (x'_1)^2$$

and is, thus, twice larger than the sum of the original effectivenesses

$$y' + y'' = 2 \cdot a_{11} \cdot (x'_1)^2.$$

For $a_1 = 0$, if we merge k similar-size entities, the effectiveness

$$y = a_{11} \cdot (k \cdot x'_1)^2 = k^2 \cdot a_{11} \cdot (x'_1)^2$$

of the merged entity is k times larger than the overall effectiveness $k \cdot a_{11} \cdot (x'_1)^2$ of all original entities. For large k, we can have a drastic increase in effectiveness.

A simple example when the merger is counterproductive. Let us now consider the case when $a_{11} < 0$. In this case,

$$y = y' + y'' + 2 \cdot a_{11} \cdot x_1' \cdot x_1''.$$

Since $a_{11} < 0$, we conclude that $y < y' + y''$. So, in this example, merger does not make sense: it will only decrease the overall effectiveness.

So what do we recommend. The above analysis leads to the following recommendation on when to merge two entities and when not to merge.

3 Resulting Recommendation: How to Decide When to Merge and When Not to Merge

Situation. Suppose that we have two entities that are thinking of merging:

- the first entity is characterized by the values $x_1', \ldots, x_n'$, while
- the second entity is characterized by the values $x_1'', \ldots, x_n''$.

When should we recommend the merger?

Recommendation: preliminary stage.

- First, we need to analyze what quantities $x_1, \ldots, x_n$ determine the entity's effectiveness.
- Then, we need to gather the effectiveness values $y^{(k)}$ of different similar entities $k = 1, \ldots, K$, entities that are characterized by the values
$$x_1^{(k)}, \ldots, x_n^{(k)}.$$
- We then need to use these values to find the coefficients a_i and a_{ij} that provide the best fit for the following equalities:
$$y_k^{(k)} \approx \sum_{i=1}^{n} a_i \cdot x_i^{(k)} + \sum_{i,j=1}^{n} a_{ij} \cdot x_i^{(k)} \cdot x_j^{(k)}.$$
  To find these values, we can, for example, use the Least Squares approach.

As a result of this preliminary stage, we get the quadratic function (4) that describes the effectiveness of entities of the given type.

Recommendation: final stage. Once the quadratic function (4) is known, we compute the values $y' = f(x_1', \ldots, x_n')$, $y'' = f(x_1'', \ldots, x_n'')$, and

$$y = f(x_1' + x_1'', \ldots, x_n' + x_n'').$$

Then:

- if $y > y' + y''$, we *recommend* the merger – since it is expected to increase the overall effectiveness;
- if $y \le y' + y''$, we *do not* recommend the merger – since it is *not* expected to increase the overall effectiveness.

Comment. If on the preliminary stage, it turns out that the quadratic approximation is not a very accurate description of the actual effectiveness, then a natural idea is to consider cubic approximations – or, if needed, approximations of higher order.

4 What About Splits?

Shall we perform a planned split? A similar approach can be used to decide whether a planned split of an entity with values $x_1, \ldots, x_n$ into two entities with values x'_i and $x''_i = x_i - x'_i$ makes sense:

- similarly to the previous section, we compute the values $y' = f(x'_1, \ldots, x'_n)$, $y'' = f(x''_1, \ldots, x''_n)$, and $y = f(x_1, \ldots, x_n)$;
- if $y < y' + y''$, we recommend the split; otherwise, if $y \ge y' + y''$, the split is not recommended.

What is the optimal way to split: description of the problem. Sometimes:

- it seems beneficial to split the given entity (characterized by the values $x_1, \ldots, x_n$),
- but it is not clear which split would be the most effective.

In such situation, it is desirable to come up with the split that leads to the largest overall effectiveness.

What is the optimal way to split: formulation of the problem in precise terms. Let us formulate the problem of finding the optimal split in precise terms. We are looking for the values $x'_1, \ldots, x'_n$ that satisfy the constraints $0 \le x'_i \le x_i$. Under these constraints, we want to maximize the overall effectiveness

$$f(x'_1, \ldots, x'_n) + f(x_1 - x'_1, \ldots, x_n - x'_n), \tag{9}$$

where the function $f(x_1, \ldots, x_n)$ is described by the formula (4).

What is known about the resulting optimization problem? We have a problem of maximizing a quadratic function (9) under interval constraints $x'_i \in [0, x_i]$. In general, this problem is known to be NP-hard; see, e.g., [2].

This means that – unless it turns out that P = NP, which most computer scientists do not believe to be possible – no feasible algorithm is possible that would always find the exact maximum. However, there are many optimization algorithms that provide reasonably good solutions to such optimization problems, so we hope that our model will be useful.

Acknowledgments

This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), HRD-1834620 and HRD-2034030 (CAHSI Includes), EAR-2225395 (Center for Collective Impact in Earthquake Science C-CIES), and by the AT&T Fellowship in Information Technology.

It was also supported by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478, and by a grant from the Hungarian National Research, Development and Innovation Office (NRDI).

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