Economic Decision Making under Interval Uncertainty: Hurwicz Optimism-Pessimism Criterion Revisited

Julio Urenda[1] and Vladik Kreinovich[2]
Departments of [1]Mathematical Sciences and [2]Computer Science
University of Texas at El Paso, jcurenda@utep.edu, vladik@utep.edu

Hurwicz criterion: reminder. If there are several options of investing money, and we know exactly how much we will get with each option, then it is natural to select the option with the largest possible gain. In practice, we only know the amounts with some uncertainty. Often, we only know the interval $[\underline{x},\overline{x}]$ of possible values of the gain. To make a decision, we need to decide how much this option is worth to us, i.e., what is the monetary amount $f([\underline{x},\overline{x}])$ that we are willing to pay for this interval option.

This problem was handled by a future Nobelist Leo Hurwicz, who showed that reasonable requirements lead to $f([\underline{x},\overline{x}])=\alpha\cdot\overline{x}+(1-\alpha)\cdot\underline{x}$ for some $\alpha\in[0,1]$. The value $\alpha=1$ corresponds to pure optimism – when we only take into account the best case. The value $\alpha=0$ corresponds to pure pessimism – when we only take into account the worst case. Values $\alpha\in(0,1)$ correspond to realistic attitude, when we take different possibilities into account.

From what requirements is Hurwicz criterion derived. These requirements are: that $\underline{x}\le f([\underline{x},\overline{x}])\le\overline{x}$, that $f([\underline{x},\overline{x}])$ is (non-strictly) increasing in each of its variables, and additivity: If we have two options $[\underline{x}_1,\overline{x}_1]$ and $[\underline{x}_2,\overline{x}_2]$, then we can consider them separately and pay for each, or we can view them as a single option in which the interval of possible gains is $[\underline{x}_1+\underline{x}_2,\overline{x}_1+\overline{x}_2]$. It is reasonable to require that the amount that we are willing to pay should not depend on how we consider this situation, so we must have $f([\underline{x}_1,\overline{x}_1])+f([\underline{x}_2,\overline{x}_2])=f([\underline{x}_1+\underline{x}_2,\overline{x}_1+\overline{x}_2])$.

Similar requirement imply that if we have a general bounded set B instead of the interval, then $f(B)=\alpha\cdot\sup B+(1-\alpha)\cdot\inf B$.

Remaining problem. The above expression for the overall gain from two options implicitly assumes that the gains of the two options are not related. In practice, they may be related. How can we take this possible relation into account?

Our result. All we know about the set S of possible values of (x_1,x_2) is that its projection $\pi_i(S)$ on each axis coincides with $[\underline{x}_i,\overline{x}_i]$. Let $\mathcal{S}$ denote the class of all such sets. For each $S\in\mathcal{S}$, the price is equal to $m(S)\stackrel{\text{def}}{=}\alpha\cdot\sup B(S)+(1-\alpha)\cdot\inf B(S)$, where we denoted $B(S)\stackrel{\text{def}}{=}\{x_1+x_2:(x_1,x_2)\in S\}$. Since all sets $S\in\mathcal{S}$ are possible, the set of all possible outcomes has the form $s\stackrel{\text{def}}{=}\{m(S):S\in\mathcal{S}\}$.

In this case, a natural idea is to value the joint participation in two situations by the value $v\stackrel{\text{def}}{=}\alpha\cdot\sup s+(1-\alpha)\cdot\inf s$. Our result is that this value v is exactly the same as the value $\alpha\cdot(\overline{x}_1+\overline{x}_2)+(1-\alpha)\cdot(\underline{x}_1+\underline{x}_2)$ coming from the straightforward application of Hurwicz criterion. This provides an additional argument in favor of the Hurwicz criterion.