Why Polynomials Accurately Describe the Shape of a Girus

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Giruses: a brief introduction. While most viruses are small, there are viruses whose size – 300-500 nanometers – is larger than typical bacteria. They are called giant viruses, or giruses, for short. It is important to study giruses because, due to their relatively large size – comparable to the size of bacteria – they play an important role in ecology. It is also important to study them since they are, in many aspects, similar to usual viruses, but, because of their much larger size, it is easier to study their geometric shape.

What is known about their geometry. Similar to many other viruses, the surface of a girus consists of several faces. So, to describe this surface, it is sufficient to describe the shape of these faces. In general, the surface of a girus can be described as \( f(x) = 0 \) for some function \( f(x) \), where \( x = (x_1, x_2, x_3) \). So, to describe a surface, it is sufficient to describe the corresponding function.

A possible explanation of why polynomials are a good approximation. The surface of each face is smooth, so it is reasonable to consider smooth basic functions. Every sufficiently smooth function can be extended in Taylor series:

\[
e^i \cdot \sum c_{i,j} \cdot x_j = e^i \cdot \sum c_{i,j} \cdot x_j = 0 \text{ for some function } c_{i,j}.
\]

It turns out that for many marine giruses, a good description of the shape of their faces is provided by cubic polynomials [1].

A natural question. A natural question is: why polynomials – and not any other functions – provide a good basis for this problem?

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Similar argument shows that the first term in the Taylor expansion of this difference – which is the second non-zero term in the expansion of \( e_i(x) \) – should also be in \( F \). Similarly, we can prove that each non-zero term \( e_{i,j}(x) \) should be in \( F \). In \( F \) we have only \( n \) linearly independent functions. Polynomials of different order are linearly independent, so we can have only a finite number of different non-zero terms \( e_{i,j}(x) \). This means that each \( e_i(x) \) is a sum of finitely many polynomials – and thus, itself a polynomial. This is exactly what we wanted to explain.