How Can We Explain Empirical Formulas for Shrinkage Cracking of Cement-Stabilized Pavement Layers

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Abstract In pavement construction, one of the frequent defects is shrinkage cracking of the cement-stabilized pavement layer. To minimize this defect, it is important to be able to predict how this cracking depends on the quantities describing the pavement layer and the corresponding environment. Cracking is usually described by two parameters: the average width of the crack and the crack spacing. Empirical analysis shows that the dependence of the width on all related quantities is described by a power law. Power laws are ubiquitous in physics, they describe a frequent case when the dependence is scale-invariant – i.e., does not change if we change the measuring units. However, for crack spacing, the dependence is more complex: namely, the dependence of the logarithm of spacing is described by a power law. In this paper, we provide a possible explanation for this more complex dependence.

1 Formulation of the problem

Description of the practical problem. A typical design of a road pavement consists of the top asphalt layer, the base layer underneath it, then the sub-base, and finally, the soil. To make the pavement stronger, stabilizer agents such as cement are added. When the cement-layer mix solidifies, it shrinks, and this shrinking may cause cracking. These cracks weaken the base layer, leading to the cracking of the

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road surface. Once the cracks appear on the road surface, the constant stress of traffic will make the cracks grow and multiply, eventually creating dangerous potholes.

To avoid this negative effect, it is desirable to be able to predict the amount of cracking based on the parameters of the cement and on the weather conditions – such as temperature and humidity – that affect solidification.

How can we characterize the cracks. A crack is usually practically a straight line. So, to characterize an individual crack, it is necessary and sufficient to describe its length and its width. For the whole set of cracks, it is therefore reasonable to consider the overall length ℓ of the cracks per unit-length road segment and their average width W.

In practice, instead of the overall crack length ℓ per unit road segment, practitioners usually consider a more intuitive characteristic $L \stackrel{\text{def}}{=} W_r/\ell$, where W_r denotes the width of the road – which is usually 12 feet per lane. This characteristic, crudely speaking, describes how far away the cracks are from each other; it is therefore called the *crack spacing*.

Empirical formulas for the crack characteristics. Empirical formulas are known (see, e.g., [4]) that provide a good description of the crack characteristics. These formulas describe the dependence of W and L on all related quantities. To make the description clearer, we will only explicitly describe the dependence on the main quantities:

- calcium content c of the cement;
- mechanical characteristics of the base layer: compressive strength UCS_{28} and tensile strength S_{IDT}; these strengths are usually measured after 28-day curing at 20° C and 100% humidity; and
- the weather characteristics: the average daily maximum temperature variation ΔT and the average relative humidity RH.

The dependence on these characteristics has the following form:

$$W = a_0 \cdot c^{a_1} \cdot UCS_{28}^{a_2} \cdot S_{IDT}^{a_3} \cdot (\Delta T)^{a_4} \cdot RH^{a_5}; \quad (1)$$

$$L = \exp(b_0 \cdot c^{b_1} \cdot UCS_{28}^{b_2} \cdot S_{IDT}^{b_3} \cdot (\Delta T)^{b_4} \cdot RH^{b_5}), \quad (2)$$

for some values a_i and b_i. The equation (2) is usually described in the following equivalent form:

$$\ln(L) = b_0 \cdot c^{b_1} \cdot UCS_{28}^{b_2} \cdot S_{IDT}^{b_3} \cdot (\Delta T)^{b_4} \cdot RH^{b_5}. \quad (3)$$

In other words:

- the dependence of the crack width W on these quantities is described by a power law, while
- for the crack spacing L, the dependence is different: only the dependence of the logarithm $\ln(L)$ is described by a power law, while the dependence of L itself is described by a more complex formula (2).

Comment. In our formulas, we only listed the dependence on the major quantities, but the dependence on all other quantities is described by similar formulas: power laws describe the dependence of both W and $\ln(L)$ on these other quantities as well.

2 Our explanation

Why power law. A power law dependence of W on several quantities is easy to explain; see, e.g., [2]. The usual explanation takes into account that while we are interested in finding the dependence between physical quantities, what we actually describe is the relation between the numerical values of these quantities. The numerical value of a quantity depends not only on the quantity itself, it also depends on the choice of the measuring unit. If we replace the original measuring unit with a one which is λ times smaller, then all the numerical values get multiplied by λ: $x \mapsto x' = \lambda \cdot x$. For example, if we replace meters with centimeters – a unit which is $\lambda = 100$ times smaller than the meter – then 2 m becomes $100 \cdot 2 = 200$ cm.

In many cases, the physics does have any fixed unit, so the dependence $y = f(x)$ of a quantity y on the quantity x should look the same in all measuring units. Of course, if we change the measuring unit for x, then we may need to appropriately change the measuring unit for y. For example, the area of a square is described by the formula $y = x^2$. So, if we have a square with a side of 2 m, then its area is equal to $2^2 = 4$ square meters. If we replace meters with centimeters, the formula remains valid, we still have $y' = (x')^2$, but for this formula to remain valid, we need to replace the measuring unit for area from square meters to square centimeters.

In general, in such situations, it is reasonable to require that for each possible x-re-scaling coefficient $\lambda > 0$, there exists an appropriate y-re-scaling coefficient $\mu(\lambda)$ such that:

- if we have $y = f(x)$ in the original units,
- then we should have $y' = f(x')$ in the new units $x' = \lambda \cdot x$ and $y' = \mu(\lambda) \cdot y$.

One can show that for continuous (and, more generally, measurable) dependencies $f(x)$ this "scale-invariance" requirement implies that the dependence is described by a power law, i.e., that $y = A \cdot x^a$ for some real numbers A and a; see, e.g., [1, 2].

For dependence on several quantities $y = f(x_1, \ldots, x_n)$, it often makes sense to similarly require that this dependence preserve the same form if we use different units for all the inputs x_i. In other words, for each possible combination of x-re-scaling coefficients $\lambda_1 > 0, \ldots, \lambda_n > 0$, there exists an appropriate y-re-scaling coefficient $\mu(\lambda_1, \ldots, \lambda_n)$ such that:

- if we have $y = f(x_1, \ldots, x_n)$ in the original units,
- then we should have $y' = f(x'_1, \ldots, x'_n)$ in the new units $x'_1 = \lambda_1 \cdot x_1$, …, $x'_n = \lambda_n \cdot x_n$ and $y' = \mu(\lambda_1, \ldots, \lambda_n) \cdot y$.

One can show that for continuous (and, more generally, measurable) dependencies $f(x_1, \ldots, x_n)$ this "scale-invariance" requirement implies that this dependence is described by a power law, i.e., that $y = A \cdot x_1^{a_1} \cdot \ldots \cdot x_n^{a_n}$ for some real numbers A, a_1, $\ldots$, a_n; see, e.g., [1, 2].

How can we explain the appearance of logarithm in the formula for L? In a nutshell, a shrinkage crack appears when the stress exceeds the material's strength at a given location. The amount of this cracking is not high: typically, we may have crack spacing of 40 to 60 cm, which means that we have about two cracks per meter. This does not mean that these cracks are not dangerous: when the traffic start going over the road, these cracks expand and deteriorate the road's quality. What we are saying is that cracking is a – relatively – very rare event, in the sense that only a very small portion of the pavement is covered by cracks.

This means that the values of stress at different locations are different, and only in a few places, the stress exceeds the strength. The stress is caused by many different independent factors. In general, if you have a joint effect of a large number of small independent factors, the resulting probability distribution is – under some reasonable assumptions – close to Gaussian; this fact is known as the Central Limit Theorem; see, e.g., [3]. This means that the probability of exceeding the strength is proportional to $\exp(-x^2/(2\sigma^2))$, for some quantities x and σ. As a result, the area covered by the cracks per unit of road length – which is, itself, inverse proportional to the spacing L – is proportional to this exponential term. Because of this, the logarithm $\ln(L)$ of L is proportional to the logarithm of the exponential term, i.e., to x^2/σ^2.

For both quantities x and σ, it makes perfect sense to conclude that their dependence on other quantities is expressed by a power law – similarly to what we concluded about W. And if both x and σ are described by a power law, then the ratio x^2/σ^2 is also described by a power law: since, as one can easily check, the square and the ratio of two power law expressions is also a power law expression. Thus, we have explained why the dependence of $\ln(L)$ on other quantities is well described by a power law.

Acknowledgments

This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), HRD-1834620 and HRD-2034030 (CAHSI Includes), EAR-2225395 (Center for Collective Impact in Earthquake Science C-CIES), and by the AT&T Fellowship in Information Technology.

It was also supported by a grant from the Hungarian National Research, Development and Innovation Office (NRDI).

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