

# Why Magenta Is Not a Real Color, and How It Is Related to Fuzzy Control and Quantum Computing

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**Abstract** It is well known that every color can be represented as a combination of three basic colors: red, green, and blue. In particular, we can get several colors by combining two of the basic colors. Interestingly, while a combination of two neighboring colors leads to a color that corresponds to a certain frequency, the combination of two non-neighboring colors – red and blue – leads to magenta, a color that does not correspond to any frequency. In this paper, we provide a simple explanation for this phenomenon, and we also show that a similar phenomenon happens in two other areas where we can find a natural analogy with colors: fuzzy control and quantum computing. Since the analogy with fuzzy control has already led to efficient applications, we hope that the newly discovered analogy with quantum computing will also lead to computational speedup.

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## 1 Formulation of the Problem

**What is so special about magenta.** It is well known that every color can be represented as a combination of three colors: red (R), green (G), and blue (B); see, e.g., [10]. In particular, by combining two of these colors, we get the following:

- a combination of red and green leads to yellow;
- a combination of red and blue leads to magenta; and
- a combination of green and blue leads to cyan.

Both yellow and cyan are real colors, they correspond to certain wavelengths. However, magenta is *not* a real color – in the sense that no wavelength corresponds to magenta. The only way to get a perception of the magenta color is to combine red and blue. But why is magenta different?

**What we do in this paper.** In Section 2 of this paper, we provide a possible simple explanation of why magenta is different. In Sections 3 and 4, we show that this explanation is related to fuzzy control and to quantum computing.

## 2 A possible simple explanation of why magenta is different

In terms of frequencies, red corresponds to the lowest frequency, blue to the highest, and green is in between. From this viewpoint, green is between red and blue. Symbolically, we can describe this situation as follows:



When we combine two neighboring colors, we get a connected range of colors:



However, when we combine two non-neighboring colors – red and blue – we get a disconnected range of colors:

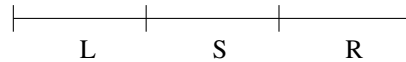


The problem with this is that when we perceive objects, we naturally interpolate; see, e.g., [10]. For example, if we see four corner of a square, we naturally interpolate it to a square. From this viewpoint, if we see two colors, we naturally interpolate and conclude that all intermediate colors should be there as well. But if all colors were present, we would get a white color, and combination of red and blue does not make a white. As a result, this non-neighboring combination is treated differently from the two other cases – cases in which we combine neighboring colors.

### 3 Relation to fuzzy control

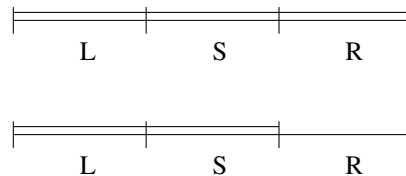
**What is fuzzy control: a very brief reminder.** When experts describe their decision-making process, they rarely use exact numbers – all the rules using exact numbers have already been incorporated into automatic control, there is no need to use experts for that. What experts use in their explanations is natural-language words. For example, when we control the car direction, we can go straight, turn left, or turn right – and we can also add gradations: turn a little bit to the left, turn somewhat to the right, etc. Techniques for translating this natural-language-based knowledge into an exact control strategy are known as *fuzzy control*; see, e.g., [1, 4, 6, 7, 9, 18].

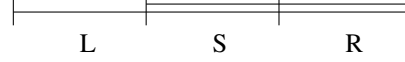
**The simplest case.** Let us consider the simplest case when we have only three recommendations: straight (S), left (L), and right (R). Naturally, straight is between left and right, which can be described symbolically as follows:



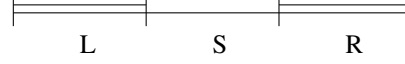
In this case, we have a direct analogy with three basic colors, so that the intermediate recommendation – of going straight – corresponds to the intermediate basic color green (which, by the way, makes perfect sense from the viewpoint of traffic lights :-).

In this case, a resulting recommendation may be either one of these three options, or several of them. As a result, we get some recommended range – which is a union of ranges recommended by an expert (or experts). If we combine all three ranges – or if we combine two neighboring ranges – we get a connected set:





However, if we combine two disconnected ranges, we get a disconnected union:



To translate expert recommendations into an automatic control, we need to transform this union into a single number. A connected range – interval  $[\underline{x}, \bar{x}]$  – does not change if we invert into with respect to the midpoint

$$\tilde{x} \stackrel{\text{def}}{=} \frac{x + \bar{x}}{2},$$

i.e., if we apply a transformation  $x \mapsto 2\tilde{x} - x$ . Thus, it make sense to require that the recommended control value  $x_r$  should also be invariant with respect to this transformation, i.e., that we should have  $x_r = 2\tilde{x} - x_r$ . This leads to  $x_r = \tilde{x}$ , a value which is well inside the recommended range.

But what happens if we have a disjoint set of recommendations, i.e., when we combine recommendations corresponding to colors red and blue? In the simplest case, assuming that all three intervals have the same width  $w$ , we get the union  $[x_0, x_0 + w] \cup [x_0 + 2w, x_0 + 3w]$ . This set is also invariant with respect to an inversion: namely, an inversion with respect to the point  $\tilde{x} = x_0 + 1.5w$ :  $x \mapsto 2\tilde{x} - x$ . Thus, it seems reasonable to select a recommendation  $x_r$  which is also invariant with respect to this transformation, i.e., the value  $x_r = \tilde{x} = x_0 + 1.5w$ . However, in contrast to the connected case, this value is *not* in the recommended range.

**This is a real problem, not just a mathematical exercise.** At first glance, this may seem like a mathematical mumbo-jumbo, but actually, it is an important practical problem. Suppose that you are driving on an empty road and you see a rock (or some other small obstacle) right ahead. Then, it make sense to veer a little bit to the left and it makes equal sense to veer a little bit to the right. So, we get the disjoint recommended set. But what naively applied fuzzy control recommends in this case is – exactly as we described earlier – going straight ahead, which makes no sense in this case.

**The difference between combining neighboring and non-neighboring recommendations is similar to the case of colors.** So, for fuzzy control, the case when we combine non-neighboring recommendations requires a different approach than simpler cases – cases in which we combine neighboring recommendations. From this viewpoint, the fuzzy control situation is similar to the situation involving colors.

**The relation between fuzzy control and basic colors is well known and well used.** While the above relation between fuzzy control and colors may be new, in general, the relation between fuzzy control and colors is rather well known and well studies. It has led to successful applications in both directions:

- fuzzy logic has been used in optics (see, e.g., [3]), and
- color optical devices have been proposed to perform computations related to fuzzy control; see, e.g., [5, 12, 13, 14, 15, 16, 17] and references therein.

## 4 Relation to quantum computing

**The very basics of quantum computing.** The main idea behind quantum computing (see, e.g., [8]) is that using quantum effects (see, e.g., [2, 11]) enables us to speed computations.

One of the specific features of quantum physics is that for each object, in addition to its usual (“classical”) states  $s_1, \dots, s_k$  – which in quantum physics are denoted by  $|s_1\rangle, \dots, |s_k\rangle$ , we can also have *superpositions* of these states, i.e., states of the type  $c_1 \cdot |s_1\rangle + \dots + c_k \cdot |s_k\rangle$ , where  $c_1, \dots, c_k$  are complex numbers for which the sum of the squares of their absolute values is equal to 1:

$$|c_1|^2 + \dots + |c_k|^2 = 1.$$

In particular, for a *bit* – the main computing unit that can be in two states 0 and 1, its quantum analog – known as *qubit* – can be in any state  $c_0 \cdot |0\rangle + c_1 \cdot |1\rangle$ , where  $|c_0|^2 + |c_1|^2 = 1$ .

Another important feature of quantum states is that two expressions that differ only by a constant whose absolute value is 1 actually correspond to the same state. For example, the expressions  $|0\rangle, -|0\rangle, i \cdot |0\rangle$  (and, in general,

$$\exp(i \cdot a) \cdot |0\rangle,$$

where  $a$  is any real number) describe the same classical state 0.

**So how can we describe a general state of a qubit.** Any complex number  $c$  can be represented as  $\rho \cdot \exp(i \cdot a)$  for some real numbers  $\rho \geq 0$  and  $a$ . In these terms, a general state of a qubit can be described as

$$\rho_0 \cdot \exp(i \cdot a_0) \cdot |0\rangle + \rho_1 \cdot \exp(i \cdot a_1) \cdot |1\rangle.$$

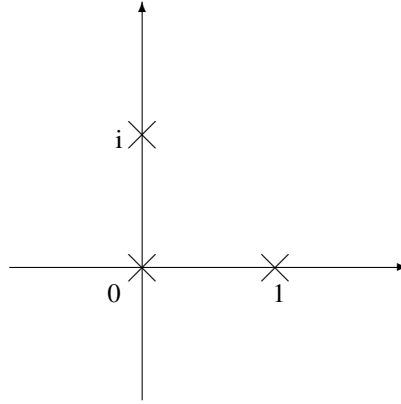
Since multiplication by  $\exp(-i \cdot a_0)$  does not change the state, the same state can be represented as  $\rho_0 \cdot |0\rangle + c'_1 \cdot |1\rangle$ , where we denoted  $c'_1 \stackrel{\text{def}}{=} \exp(-i \cdot a_0) \cdot c_1$ . This way, we get a unique representation of the state of a qubit, since the only way to go from a positive real number  $\rho_1$  to another positive real number by multiplying it by a number whose absolute value is equal to 1 is to multiply it by 1 – i.e., to keep it unchanged.

In this representation, the coefficient at  $|0\rangle$  is a real number, while a coefficient  $c'_1$  at  $|1\rangle$  is, in general, a complex number, i.e., an expression of the type  $c'_1 = a + b \cdot i$  for some real numbers  $a$  and  $b$ . In these terms, a general state of a qubit can be represented as

$$\rho \cdot |0\rangle + (a + b \cdot i) \cdot |1\rangle.$$

**This way, we get a 3-D representation of qubits.** The above expression shows that any state of a qubit can be represented as a real-valued linear combination of the following three basic states:  $|0\rangle$ ,  $|1\rangle$ , and  $i \cdot |1\rangle$ . From this viewpoint, it is somewhat similar to the basis of three colors in the color space.

**What is a natural order between the three basic states?** The three basic states correspond to coefficients at  $|1\rangle$  equal to 0, 1, and  $i$ , see below:



The shortest path that connects all three values has 0 in between 1 and  $i$ . Thus, it is reasonable to conclude that the state  $|0\rangle$  – that corresponds to a 0 coefficient at  $|1\rangle$  – is in between two other states, i.e., in between states  $|1\rangle$  and  $i \cdot |1\rangle$ . From the viewpoint of the analogy between the three basis quantum states and the three basic colors, the state  $|0\rangle$  corresponds to the basic color green (G), which is also in between the two other basic colors: red (R) and blue (B).

**What if we combine two basic quantum states?** If we combine basic states  $|0\rangle$  and  $|1\rangle$ , then we get generic states of the type  $\rho \cdot |0\rangle + a \cdot |1\rangle$ , for real numbers  $\rho > 0$  and  $a$ . To get a legitimate quantum state, we need to make sure that the sum of the squares of the coefficients is 1, i.e., that  $\rho^2 + a^2 = 1$ . One can easily check that all resulting states are different, i.e., that we get a 1-dimensional family of different qubit states – which are, by the way, exactly the states used in most quantum algorithms [8].

Similarly, if we combine basic states  $|0\rangle$  and  $i \cdot |1\rangle$ , then we get generic states of the type  $\rho \cdot |0\rangle + b \cdot i \cdot |1\rangle$ , for real numbers  $\rho > 0$  and  $b$ . To get a legitimate quantum state, we need to make sure that the sum of the squares of the coefficients is 1, i.e., that  $\rho^2 + b^2 = 1$ . One can easily check that in this case, all resulting states are also different, i.e., that we also get a 1-dimensional family of different qubit states.

So, if general, if we combine the state corresponding to green with a state corresponding to another basic color, we get, as expected, 1-D families of combined states.

However, interestingly, if we combine the basic states  $|1\rangle$  and  $i \cdot |1\rangle$  – i.e., states corresponding to basic colors red and blue – then all we get are the states of the type  $(a + b \cdot i) \cdot |1\rangle$ , which are all equivalent to the original state  $|1\rangle$ .

So, in quantum computing case too, the result of combining states corresponding to colors red and blue is very different from the results of combining states corresponding to two other colors.

## 5 Conclusions

In this paper, we explain why magenta – a combination of non-neighboring red and blue basic colors – is different from all other color combinations. We also show that both in fuzzy control and in quantum computing, a combination of states corresponding to non-neighboring basic colors is also different from a combination of two other states.

The analogy between colors and fuzzy degrees is already known – and it has led to the ideas of using colors in fuzzy computations. The fact that there is a similar analogy between color and quantum computing makes us conjecture that this can also be useful in computations. In particular, since most quantum algorithms now only use states with real-valued coefficients – that correspond to the two of three basic colors – we conjecture that using imaginary coefficients may lead to further speed-up of quantum computing.

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