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Is Energy Local? Counterintuitive Non-Localness of Energy in General Relativity Can Be Naturally Explained on the Newtonian Level

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From the physics viewpoint, energy is the ability to perform work. To estimate how much work we can perform, physicists developed several formalisms. For example, for fields, once we know the Lagrangian, we can find the energy density and, by integrating it, estimate the overall energy of the field. Usually, this adequately describe how much work this field can perform. However, there is an exception – gravitational field in General Relativity. The known formalism to compute its energy density leads to 0 – and by integrating this 0, we get a counterintuitive conclusion that the overall energy of the gravitational field is 0 – while hydroelectric power stations that produce a significant portion of world's energy show that gravity *can* perform a lot of work. The usual solution to this puzzle is that for gravity, energy is not localized. In this paper, we show: (1) that non-localness of energy can be explained already on the Newtonian level, (2) that the discrepancy between energy as ability to perform work and energy as described by the Lagrangian-based formalism is ubiquitous even in the Newtonian case, and (3) that there is a positive side to this non-localness: it may lead to faster computations.

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1. Introduction

What is energy: a commonsense physical meaning. What is energy? From the commonsense physics viewpoint, energy is the ability to perform work.

How to estimate how much work we can perform: use known techniques. An important part of physics applications is estimating how much work we can perform. For this purpose, physicists have developed several useful formalisms; see, e.g., [1,3,6]. For example, for fields, once we know the Lagrangian (see explanations below), we can find the energy density and, by integrating it over the whole space, estimate the overall energy of the field. For many situations, this leads to useful estimates that adequately describe the amount of work this field can perform.

Gravity is an example when known energy-estimation techniques do not work. However, there is a known exception – gravitational field as described by General Relativity. If we use the known formalism to compute its energy density, we get 0 – and by integrating this 0, we get a counterintuitive conclusion that the overall energy of the gravitational field is 0 [1,4,6,7] – while hydroelectric power stations that use gravity to produce a significant portion of world's energy show that gravity *can* perform a lot of work.

What we do in this paper. First, in Section 2, we describe the gravity-energy problem in detail. In Section 3, we provide a simple physical explanation of this phenomenon – namely, we show that a similar phenomenon can be traced already on the Newtonian level. In Section 4, we go deeper into the corresponding mathematics, and show that gravity is not the only exception – the discrepancy between energy as the amount of work and energy as generated by the corresponding mathematical formalisms is more general that just gravity. All this is about somewhat negative consequences of the discrepancy. But, as we show in the last Chapter 5, this discrepancy may have a positive side: it may lead to faster computations.

2. Non-Localness of Energy In General Relativity: A Brief Reminder

Who are the intended readers of this section. We want to describe this phenomenon to as many readers as possible – including readers who may not be very familiar with post-Newtonian physics. Because of this, we will briefly describe things that may be familiar to many readers. So, readers who are knowledgeable in physics can skip some (or even all) of this section.

How modern physical theories are described. Newton formulated his mechanics in terms of differential equations. Physicists still use differential equations to described physical phenomena, but this is not how new physical theories are formulated. These theories are usually formulated in terms of *action* S – a physical quantity whose minimum describes the system's dynamics. For fields, the action takes the form $S = \int L(x)\,dx$, where the value $L(x)$ at any space-time location x depends on the values of the fields $f_a(x)$ and their spatial and time derivatives

$$f_{a,i}(x) \stackrel{\text{def}}{=} \frac{\partial f_a(x)}{\partial x_i}$$

at the location x. The function L is known as the *Lagrangian*.

Similar to calculus, where we can find the minimum of a function $f(x)$ by looking for locations where small changes in x do not change the value $f(x)$, i.e., where the derivative is equal to 0:

$$\frac{df}{dx} = 0,$$

we can find the minimizing fields $f_a(x)$ by looking for values of the fields for which the corresponding *variational derivative* is equal to 0:

$$\frac{\delta L}{\delta f_a(x)} = 0.$$

Probably the easiest way to consider variational derivatives is to consider a dense grid of space-time points instead of a continuous space-time, and to approximate the derivatives by the corresponding ratios

$$\frac{f_a(x_1,\ldots,x_n)}{\partial x_i} \approx$$
$$\frac{f_a(x_1,\ldots x_{i-1}, x_i+h_i, x_{i+1},\ldots,x_n) - f_a(x_1,\ldots x_{i-1}, x_i, x_{i+1},\ldots,x_n)}{h_i},$$

where h_i is a step in i-th direction. In this case, S is a function of the values $f_a(x)$ at different locations, and variational derivative becomes a usual partial derivative of this function. In the limit, when we tend h_i to 0, we get the variational derivatives.

How can define energy in this formalism. Energy is the easiest to define in the case of a general curved space-time, in which the proper time ds between points x and $x+dx$ is determined by the formula

$$ds^2 = \sum_{i,j} g^{ij} \cdot dx_i \cdot dx_j$$

– the generalization of the formula

$$ds^2 = dx_0^2 - \sum_i dx_i^2$$

in the non-curved space-time of Special Relativity. The values g^{ij} form what is known as the *metric tensor*. In this case, the so-called *energy-momentum tensor* T_{ij} takes the form

$$T_{ij} = \frac{\delta L}{\delta g^{ij}}.$$

The usual physical interpretation of the component T_{00} is that it is energy density, and the integral of energy density over the proper space describe the overall energy at a given moment of time.

This technique leads to reasonable results for many physical fields, but not for gravity. For many physical fields, the above definition leads to a reasonable description of the overall energy – measured by the ability to perform work. However, for gravity, the situation is different. Indeed, in physics, gravity is interpreted as curvature of space-time, so gravity is described by the metric tensor. In line with the general minimal action principle, the differential equations of this field take the form

$$\frac{\delta L}{\delta g^{ij}} = 0.$$

But according to the above technique, this means that the energy of the gravitational field itself is 0. Thus, the energy density of the gravitational field is 0, and hence the overall energy of the gravitational field is 0; see, e.g., [1,4,6].

But we all know that gravity can perform a lot of work. How can we reconcile this with the fact that energy density is 0?

What is the physical meaning of zero energy density. Actually, the fact that the energy density of the gravitational field is 0 make perfect physical sense. Indeed, according to General Relativity – more precisely, according to Einstein's Equivalence Principle – a gravitational field is largely equivalent to an acceleration – and can be compensated by an appropriate acceleration. According to Equivalence Principle, people in a free-falling elevator do not feel any gravity – and this is exactly how the absence of gravity is often simulated, by making an aircraft falling free for some time. If we are floating around in a falling elevator, we cannot use this to perform any work, so locally, the energy is indeed 0.

This means that the overall energy is *not local* – it cannot be computed by integrating energy density for all spatial locations.

Comment: this is not just about General Relativity. In this section, we talked about General Relativity, so a natural question is: what if General Relativity is not the final theory, and some alternative theory, with a different Lagrangian, describes gravity better? It may be so, but in the above arguments, we never used the specific form of the General Relativity Lagrangian. These argument hold for any theory in which gravity is described by the curved space-time: differential equation may differ, but the energy density remains 0.

3. Non-Localness of Energy in Newtonian Physics

At first glance, it may seem that localness is a purely post-Newtonian phenomenon. Since non-localness has been first observed on the example of general relativity – a complex post-Newtonian theory of gravitation, it may seem that non-localness of energy is a purely post-Newtonian phenomenon. However, in this section, we argue that non-localness of energy can be found already in Newtonian physics – moreover, in the simplest Newtonian-type systems.

A simple Newtonian example of non-localness of energy. Let us consider an area of the world that has only two inertial particles moving towards each other along the same line. As long as these particles are by themselves, we cannot make them perform any work – unless, of course, we shoot a projectile towards one of them and get energy from the collision. However, when the two particles meet and collide, this collision releases a lot of energy.

Where was this energy located before the collision? Clearly, not in any of the particles themselves, and there was no field that could contain this energy. The energy can be explained only if we consider both particles at the same time – i.e., if instead of a function $f(x)$ like energy density that describes the energy stored at a single location x, we have a function $f(x,y)$ that described how much work we can do if we use *both* the object at location x and the object at location y.

This is exactly the situation that we have in the gravitational case – we cannot describe the overall ability to perform work by adding up energies corresponding to different locations – i.e., energy is not local.

Non-localness of energy in Newtonian physics: a general case. Let us consider a general Newtonian system, with n particles of masses $m_1,\ldots,m_n$ moving at speeds $\vec v_1,\ldots,\vec v_n$. When these velocities are different, we can release some energy by colliding these particles – until they all start moving at the same speed $\vec V$.

Due to momentum conservation law, this final speed can be found from the equality

$$M\cdot\vec V=\sum_i m_i\cdot\vec v_i,$$

where we denoted

$$M\stackrel{\rm def}{=}\sum_i m_i.$$

So

$$\vec V=\frac{\sum\limits_i m_i\cdot\vec v_i}{M}.$$

The released energy can be computed as the difference between the original kinetic energy of all the particles and their final kinetic energy:

$$E=\frac{1}{2}\cdot\sum_i m_i\cdot(\vec v_i)^2-\frac{1}{2}\cdot M^2\cdot\left(\vec V\right)^2.$$

Substituting the expression for $\vec V$ into this formula, we conclude that

$$E=\frac{1}{2}\cdot\sum_i m_i\cdot(\vec v_i)^2-\frac{1}{2}\cdot\frac{\left(\sum\limits_i m_i\cdot\vec v_i\right)^2}{M},$$

i.e., if we open the parentheses:

$$E = \frac{1}{2} \cdot \sum_i \left(m_i - \frac{m_i^2}{M} \right) \cdot (\vec{v}_i)^2 - \sum_{i<j} \frac{m_i \cdot m_j \cdot (\vec{v}_i \cdot v_j)}{M}.$$

In other words, instead of the usual "local" formula of energy as the sum $\sum f(x)$ of values at different locations x – similar to how the overall energy of a field is the integral of the energy density – we get a more complex formula

$$E = \sum_x f(x) + \sum_{x,y} f(x,y)$$

that includes non-local terms. These terms are proportional to $\vec{v}_i \cdot \vec{v}_j$, where $\vec{v}_i$ and $\vec{v}_j$ are velocities are different locations x and y.

4. This Phenomenon Is Ubiquitous

What we do in this section. After reading the previous sections, one may think that gravity is the only exception, that only for gravity, there is a difference between energy as the ability to perform work and energy as an integral of energy density as computed by Lagrangian techniques. In this section, following [2], we show that this difference is ubiquitous – even for simple multi-particle theories, when no fields are involved.

Lagrangian techniques for multi-particle systems. To describe the state of a multi-particle system at a given moment of time t, instead of the values of the fields at all spatial points, we need to describe the coordinates (and, if appropriate, additional characteristics) x_i^a of all the particles $a, b, \ldots$ Thus, the action takes the form $S = \int L(t)\,dt$, where the Lagrangian L depends on all the values x_i^a and their time derivatives $\dot{x}_i^a$. In this case, the variational equations

$$\frac{\delta L}{\delta x_i^a} = 0$$

take the following form [1,3,6]:

$$\frac{\partial L}{\partial x_i^a} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}_i^a} \right) = 0,$$

and the corresponding energy has the form

$$E = \sum_{a,i} \dot{x}_i^a \cdot \frac{\partial L}{\partial \dot{x}_i^a} - L.$$

Comment. Field-related equations can be obtained from the particle case if we view a field as a swarm of many particles – which, by the way, according to quantum physics, a field actually is – and then tend to a limit.

A simple example. Let us consider the simplest example of a system for which physical energy is not conserved: a 1-D particle with friction, whose dynamics is described by the following differential equation:

$$\ddot{x}(t) = -k_0 \cdot \dot{x}(t).$$

For this particle, its velocity exponentially decreases with time, and thus, its ability to perform any work also exponentially decreases with time.

However, we can come up with a Lagrangian that describes exactly this dynamical system – and thus, with the corresponding value of Lagrangian-based energy. The Lagrangian is

$$L(x,\dot{x})=\frac{C}{k_0}\cdot\dot{x}\cdot\ln(\dot{x})+C_0\cdot\dot{x}-C\cdot x,$$

for some values C and C_0. One can check that for this Lagrangian, the variational equations have the desired form, and the corresponding Lagrangian-based energy has the form

$$E=\frac{C}{k_0}\cdot\dot{x}+C\cdot x.$$

One can easily check that the derivative of this expression is indeed always 0.

So, here, from the physical viewpoint, energy is not conserved, but the Lagrangian-based expression is conserved – which means that the Lagrangian-based expression is clearly different from the ability to perform work.

General case. As shown in [2], the above simple system is not an exception: a Lagrangian – and the resulting preserved Lagrangian-based energy – can be found for any system of differential equations

$$\ddot{x}_i^a=f_i^a(x_1^a,x_2^a,\ldots,x_1^b,x_2^b,\ldots,\dot{x}_1^a,\dot{x}_2^a,\ldots,\dot{x}_1^b,\dot{x}_2^b,\ldots).$$

Thus, in all the cases when the physical energy is not conserved, the Lagrangian-based expression for energy *is* conserved – which means that this expression is different from physical energy defined as the ability to perform work.

5. Possible Positive Side of Non-Localness

So far, we dealt with somewhat negative consequences of non-localness. We started with a simple straightforward picture of energy, where known field theory techniques can estimate how much work we can perform by using this field. At first, we mentioned that there is one exception to this straightforward picture – gravity, but then we showed that a similar discrepancy between a physical meaning of energy and the current energy estimation techniques can appear in many other situations. The fact that we cannot use known techniques for estimating energy, that we still need to come up with new techniques is a somewhat negative consequence of non-localness.

But non-localness also has possible positive consequences. In this section, following [7], we show that non-localness can potentially lead to faster computations. First, we explain why we need faster computations in the first place, then we explain how localness limits the computation speed, and finally, we explain how non-local effects can help.

Why faster computations are needed. While modern computers are extremely fast, there are still practical problems that requires even faster computations.

For example, modern high-performance computers can reasonably accurately predict tomorrow's weather: a few hours of high-performance computations, and we know what will be tomorrow' temperature, when and where it will rain, etc. Algorithms for this prediction, in effect, solve Navier-Stokes equations that describe all atmospheric processes. In principle, similar equations describe tornadoes, so it is desirable to predict their motion – since as of now, all we can do is send an alert to a wide area, and since tornados are frequent, it is not realistic to evacuate people from this wide area every time. As a result, when tornado strikes, it is often a disaster.

Indeed, it turned out that, based on initial conditions, high-performance computers can predict in what direction a tornado will move. Of course, since tornados are much faster than regular weather processes, so in 15 minutes the change in a tornado is of the same relative size as the daily change in wether. So, the current algorithms can predict, with reasonable accuracy, in what direction a tornado will move in the next 15 minutes. The problem is that this prediction takes about the same time as accurate prediction of tomorrow's weather – several hours on a high-performance computer. For predicting weather, this is reasonable, but for predicting tornados, this make computations useless: in 15 minutes we already know in what direction it turned.

There are other practical problems in which faster computations will help.

Parallelization – a natural way to speed up computations. To understand why non-local effects can help, let us recall that one of the main ways to speed up computations is to parallelize them – modern high-performance computers consist of thousands of processors working in parallel, and our own brains consist of trillions of neurons working mostly in parallel.

Localness brings limit to parallelization – and thus, to computation speed. In the usual localized picture of the world, all the processes are localized, and communication speed is limited by the speed of light c. Let us show how localness property – namely, the fact that signal processing speed is limited by the speed of light c – limits our ability to parallelize – and thus, limits the computation speed; see, e.g., [5].

Indeed, suppose that we have a parallel computer that finished computation on some input in time $T_{\rm par}$. This computer may have many processors, but the only processors that could participate in our computations are the ones that are located at a distance $\le c\cdot T_{\rm par}$ from the user – from larger distances, the signal would arrive only after the time $T_{\rm par}$ and thus, cannot affect the computation result. Thus, all the processors involved in our computations are located inside the sphere of radius $R=c\cdot T_{\rm par}$ centered at the user. The volume V of this sphere is equal to

$$V=\frac{4}{3}\cdot\pi\cdot R^3=\frac{4\cdot\pi}{3}\cdot c^3\cdot T_{\rm par}^3.$$

Let ΔV denote the smallest possible volume of a processor, and let N denote the number of processors involved in our computations. The overall volume V_N of these processors is thus at least $V_N\ge N\cdot\Delta V$. On the other hand, since all the processors are located inside the sphere, their total volume cannot exceed the volume of the sphere. Thus, $N\cdot\Delta V\le V_N\le V$, hence $N\cdot\Delta V\le V$ and so

$$N\le\frac{V}{\Delta V}.$$

If we have N processors working in parallel in time $T_{\rm par}$, then we can compute the same thing sequentially in time $N\cdot T_{\rm par}$ if we simulate all the processors one by one:

- first, we simulate first steps of each of N processors;
- then, we simulate the second step of each of N processors, etc.

By using this simulation, we can perform our computations on a sequential machine in time

$$T_{\rm seq}=N\cdot T_{\rm par}\le\frac{V}{\Delta V}\cdot\frac{4\cdot\pi}{3}\cdot c^3\cdot T_{\rm par}^3=\frac{V\cdot 4\cdot\pi}{3\cdot\Delta V}\cdot c^3\cdot T_{\rm par}^4.$$

So, if there is a lower limit L on the amount of time that is needed to perform the desired computations on a sequential machine, we have $L\le T_{\rm seq}$, thus

$$L\le\frac{V\cdot 4\cdot\pi}{3\cdot\Delta V}\cdot c^3\cdot T_{\rm par}^4.$$

If we divide both sides of this inequality by the coefficient at $T_{\rm par}^4$, and extract 4-th order root from both sides, we conclude that

$$T_{\rm par}\ge T_{\rm loc}\stackrel{\rm def}{=}L^{1/4}\cdot\left(\frac{3\cdot\Delta V}{V\cdot 4\cdot\pi}\right)^{1/4}\cdot c^{-3/4}.$$

What happens of we allow non-local effects. Allowing non-local affects means, in effect, that we are no longer bound by the requirement that communication speed v is limited by the speed of light c. For $v > c$, similar arguments lead to

$$T_{\rm par} \geq T_{\rm gen} \stackrel{\rm def}{=} L^{1/4} \cdot \left(\frac{3 \cdot \Delta V}{V \cdot 4 \cdot \pi}\right)^{1/4} \cdot v^{-3/4}.$$

Since $v > c$, we have $T_{\rm gen} < T_{\rm loc}$, so this can indeed reduce the time needed for parallel computing – and thus, we can indeed speed up computations.

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