Gurevich's Quizani Dialogs as an Example of Explainable Mathematics, and How This Is Related to Quantum Space-Time Ideas that Can Speed Up Computations

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Abstract. Everyone talks about the need for Explainable AI – when, to supplement a long difficult-to-understand sequence of computational steps leading to AI's decision, we are looking for a shorter and understandable more-informal explanation for this decision. In this paper, we argue that this need is a particular case of what we call Explainable Mathematics – when we want to supplement a long sequence of arguments and/or computations with a shorter and understandable more-informal explanation. Important instances of Explainable Mathematics are Yuri Gurevich's Quizani dialogs that help explain complex results from theoretical computer science and physicists' more-informal explanations of complex physical phenomena. We explain that in the physics' case, since – according to most physicists – all physical theories are approximate, the use of approximate more-informal methods often makes more sense that the use of rigorous methods that implicitly assume that the current theories are absolute correct. We then apply this argument to one of the common uses of physics in theory of computation – that limitation by the speed of light limits the computation speed. Specifically, we show that quantum space-time ideas potentially allow computations at the micro-level speed of light which can be higher than its usual macro-level value. This potential increase in possible communication speed can speed up computations.

Keywords: Explainable AI· Explainable Mathematics · Yuri Gurevich· Quantum space-time · Speeding up computations.

1 Need for Explainable Mathematics

Need for explainable AI: a brief reminder. Everyone knows about spectacular successes of Large Language Models – starting with ChatGPT – and other AI models based on deep learning. While these models are very good, they are not perfect, sometimes they make mistakes. At first glance, this should not be a serious problem: human decision makers also make mistakes: human bankers sometimes miscalculate a situation and deny a loan to a worthy potential customer, human medical doctors sometimes misdiagnose diseases, etc. So why are AI's mistakes not as tolerable?

The answer to this question is that there is a difference. If we are not 100% sure about the doctor's diagnosis, we can ask the doctor for explanations – and either become more convinced or, vice versa, become less convinced and look for a second opinion. The possibility of such explanations decreases the negative effect of mistakes by human experts. In contrast, modern AI systems provide recommendations without providing any explanation whatsoever, we can either take their advice or reject it.

In some cases, the systems are proprietary, we do not know what is happening inside these systems. However, even open-code AI systems are not explanatory. Yes, we can trace all millions and billions of neurons and follow each step of the corresponding computations – but this does not make the recommendation any more explainable.

This absence of explanations is one of the main reasons why many researchers are trying to make AI systems explainable.

There are other areas where there is a similar need for explainability. Modern AI systems are not the only case when we have the results, but we lack a commonsense explanation of these results. This often happens when we use mathematics – we may have a long and complex computations or a long and complex proof. These computations and proofs can be about applications to computing, about applications to physics, about abstract mathematics. In such cases, we may be able to formally check every single step, but it is still desirable to have a more commonsense understanding of the computations or proofs – we need to understand the main ideas behind them.

Yes, in mathematics – and theoretical computer science – the ultimate goal is to have a proof. This is what prizes are given for, this is the main source of prestige – and if later someone finds a clearer proof of the same result, it will not be easy to publish this new proof in a prestigious journal – unless the new proof method leads to new results as well. However, everyone appreciates an explanation. This is one of the perks of personally attending a conference on theoretical computer science – yes, you can always read the proceedings and confirm the proofs, but in the talks, you hear more motivations and explanations – and moreover, you can usually clarify all this in a person-to-person dialog with the author.

Ideally, we want to have – as Paul Erdos said – a proof *from the Book*, where all steps are natural and all why-questions are already answered; see, e.g., [4].

Summarizing: everyone talks about explainable AI, but what we need is *explainable* mathematics as well. Here, by mathematics, we mean – as professional mathematicians do – the application of absolutely rigorous methods.

How is this related to Yuri Gurevich? At this point, readers who are not familiar with all the aspects of Yuri Gurevich's activities may wonder: this all may be good, but what does it have to do with Yuri Girevich whose birthday we are celebrating? Those who are more familiar with Yuri's activities do not have to ask this question: they know that, in addition to proving many interesting (and often technically difficult) results, Yuri has been spending a lot of time and efforts making these results – and results of others – more explainable. The best evidence for this is his regular column in the Bulletin of European Association of Theoretical Computer Science (EATCS), where – especially in his Quisani role – he explains and clarifies complex issues. From this viewpoint, we can truly view him as one of the (or even "the") founding fathers of Explainable Mathematics.

2 Explainable Mathematics: Preliminary Ideas

Main idea. A mathematical proof is a sequence of absolutely rigorous steps. Proofs can be long and thus, not easy to understand. In such cases, it is probably not possible to have a shorter rigorous proof. The only way to have a shorter explanation is not to require absolute rigor, but instead of have a sequence of reasonably convincing steps.

This is, in principle, possible. Such convincing "proofs" are not just a pipe dreams: they are normal in physics (see, e.g., [3, 9]). For example, physicists often provide somewhat informal arguments of why some terms in an equation can be ignored. Studying their "proofs" may help come up with understandable mathematics in other situations as well.

How do physicists do it: a possible ideas. From the purely mathematical viewpoint, what physicists do often sounds weird. They apply differentiation to functions that are known to be not differentiable, they ignore small terms in a sum – i.e., in effect, replace a number a with $a+\varepsilon$ with some small ε, etc. From the mathematical viewpoint, this sounds as heresy: it is well known that if add a single false statement, then we can derive any statement at all. Similarly, if we assume that $a \approx a+\varepsilon$ for all sufficiently small ε – e.g., for all ε whose absolute value is smaller than some threshold value ε_0, and still require transitivity of $\approx$ – as physicists do – then we can, by induction, conclude that $a \approx b$ is true for all possible pairs of real numbers – which is not what physicists claim.

How do they avoid such paradoxical results? One reason is that when they make conclusions, they do not just rely on mathematics – they use physical meaning. In other words, they use *explanations* to filter out physically meaningless results – which is similar to what we want from explainable AI.

Another reason is that the physicists limit the size of the "proof" chains. Yes, if $\varepsilon_0 = 10^{-10}$, then we can conclude, by induction, that $0 \approx 1$, but this conclusion would require more than 10^{10} steps – so, if we limit ourselves to chains with only dozens of step, we will never get this counterintuitive result.

How can we describe this in precise terms? A natural idea is to assign a degree of confidence to each derived statement – similarly to how it is done in fuzzy logic (see, e.g., [1, 5–8, 10]): we assign a certain degree of confidence to 1-step derivations, smaller one to derivations that require 2 steps, etc. – and this degree reaches 0 (or almost 0) after a reasonable number of steps. For example, if we view these degrees as probabilities and assume that all these probabilities are independent, then if p is the degree of confidence in a statement obtained by a single step, then after 2 steps we get degree of confidence p^2, after 3 steps p^3, etc. – and the value p^k tends to 0 very fast.

3 This May Be Also Helpful for Physics

From rigorous derivations to commonsense explanations: a brief reminder. In the previous section, we talked about a situation when we already have a rigorous proof, and we want to make it explainable. As an example, we have mentioned situations in which we have a longer rigorous proof, and we also have a more intuitive and less rigorous explanation of this result that is provided in physics textbooks. This physics example shows that such explanations are, in principle, possible – and a study of such existing explanations can help us come up to come up with similar explanations in situations when only have a rigorous proof and no intuitive explanation.

Sometimes, there is no rigorous derivation. A study of existing explanations may also help in situations when we do not have a rigorous proof, when we only have an informal explanation – and such situations are ubiquitous in physics. For example, there is still no acceptable consistent quantum field theory. In many cases, straightforward computations lead to physically meaningless infinities – and then special tricks known as *renormalization* are used to get meaningful finite results.

Probably the simplest example is computing the overall energy of an electron – including energy $E = m_0 \cdot c^2$ coming from its rest mass and the energy of its electric field; see, e.g., [3, 9]. In relativity theory, each elementary particle is a point-wise particle – otherwise, since all communications are limited by the speed of light, different spatial parts of the particle would not be able to communicate with each other right away and would, thus, act as interacting sub-particles. For a pointwise particle with electric charge q, the amplitude of the electric field $\boldsymbol{E}$ is proportional to q/r^2. Its energy density ρ is proportional to $(\boldsymbol{E})^2$, i.e., to q^2/r^4. So, the overall energy E of the electric field can be computed as the integral of this energy density over the whole 3-D space, i.e., as

$$E = \int_x \frac{q^2}{r^4}\,dx = \int_{r=0}^{\infty} \frac{q^2}{r^4} \cdot 4 \cdot \pi \cdot r^2\,dr = \text{const} \cdot \int_0^{\infty} r^{-2}\,dr =$$

$$-\text{const} \cdot r^{-1}|_0^{\infty} = \frac{1}{0} - \frac{1}{\infty} = \infty.$$

This example is about non-quantum physics, but in the quantum case, we get the same infinity.

To avoid this paradox, we can assume that the particle has finite size ε – in which case the overall energy $E(\varepsilon)$ of the electric field is finite – and that it has rest mass $m_0(\varepsilon)$. Then we select $m_0(\varepsilon)$ in such way that in the limit $\varepsilon \to 0$, the overall energy $m_0(\varepsilon) \cdot c^2 + E(\varepsilon)$ tends to the actually measured value.

Another example is the general case of quantum electrodynamics, where, in principle, we can predict any observation results by summing an infinite series – similar to Taylor series – that correspond to so-called Feynman diagrams with different number N of virtual particles taken of account. This works for reasonable number of virtual particles, but it is known that starting with $N \approx 137$, this series diverges; see, e.g., [2].

Comment. Since most readers of this volume are specialists in theoretical computer science, it is worth mentioning that, in contrast to problems like "is P equal to NP" – that have a precise mathematical formulation, a natural question of whether feasible-time computations that use quantum effects allows up go beyond P is not even properly formalized – because there is still no precise formulation of quantum physics. So, while the fact that we cannot prove whether P = NP is a challenge to interested mathematicians, the quantum question may have to wait until physicists come up with a rigorous theory that would allow this question to be formulated in mathematical terms.

In such "reverse explainable mathematics" cases, explainable mathematics may help. In such situations when we only have an explanation but no rigorous proof is known, analysis of the relation between existing rigorous proofs and their explanations may be able to help in finding the desired rigorous proof.

4 This May Be Also Help to Speed Up Computations

There is a good reason for physicists to use approximate, non-rigorous methods. As we have mentioned, at first glance, from the mathematical viewpoint, what physicists sometimes do with non-rigorous proofs sounds like a sign of weakness: since they cannot prove their results rigorously, they come up with informal explanations.

However, this paternalistic approach to physics implicitly assumes that we know the final equations of physics. History of physics teaches us otherwise. For example, Newton's physics – which seemed to be perfectly correct for several centuries – turned out to be only a good approximation to reality. To get a more accurate description of real-life phenomena, we need to take into account effects of special and general relativity theory and of quantum physics. Based on this history, most physicists believe that the current state-of-the-art in physics is not final: that eventually, a more accurate theory will emerge.

From this viewpoint, since the current physical theory is – most probably – only approximately true, it does not make much sense to make rigorous conclusions based on the assumption that the current theory is absolutely true. In particular, when the current theory predict some value a of a physical quantity, its actual value may well be $a + \varepsilon$ for some small ε. Because of this, both the predicted value a and a nearby value $a + \varepsilon$ provided by an approximate method are both physically possible – so there is a reason to use the value produced by the approximate method.

From this viewpoint, let us look at how we use physics in theoretical computer science: this can help us perform computations faster. One of the physical facts used in the analysis of what can be, in principle, computed in reasonable time, is the fact that, according to relativity theory, all communications are limited by the speed light.

This limitation is absolutely true in special relativity theory – which is a good approximation to reality. To get a better approximation, we need to use General Relativity, where the correct limitation is formulated in terms of the space-time metric g^{ij} – that generalized the metric $\eta^{ij} = \text{diag}(c^2, -1, -1, -1)$ of Special Relativity; see, e.g., [3, 9]. In these terms, the restriction on the velocity $v_i = dx_i/dt$ takes the form

$$g^{00} \cdot dt^2 + 2\sum_i g^{0i} \cdot dx_i \cdot dt + \sum_{i,j} g^{ij} \cdot dx_i \cdot dx_j \geq 0.$$

If we divide both sides by dt^2, we get an equivalent formulation in terms of the velocity components v_i:

$$\sum_{i,j} |g^{ij}| \cdot v_i \cdot v_j - 2\sum_i g^{0i} \cdot v_i \leq g^{00}.$$

In particular, if we only consider motion in the i-th direction, then this constraint takes the form

$$|g^{ii}| \cdot v_i^2 - 2g^{0i} \cdot v_i \leq g^{00}.$$

This quadratic inequality is easy to solve: it is equivalent to:

$$\frac{g^{0i} - \sqrt{(g^{0i})^2 + 4 \cdot |g^{ii}| \cdot g^{00}}}{2 \cdot g^{ii}} \leq v_i \leq \frac{g^{0i} + \sqrt{(g^{0i})^2 + 4 \cdot |g^{ii}| \cdot g^{00}}}{2 \cdot g^{ii}}.$$

In particular, in the case of Special Relativity, when $g^{00} = c^2$, $g^{ii} = -1$, and $g^{0i} = 0$, we get the usual constraints $-c \leq v_i \leq c$. In general, photons – and other particles of 0 rest mass – have the velocities corresponding to the endpoints of the above interval.

Of course, General Relativity is also an approximate theory. To get a more accurate description of reality, we need to take into account quantum effects. According to quantum physics, there are always random fluctuations in the values of the physical field – and the smaller spatial size area we consider, the larger these fluctuations. Thus, even when on the macro-level we have the space-time metric corresponding to Special Relativity, on the micro-level, we have random deviations from this metric. Due to uncertainty principle, a photon with energy E occupies spatial region of size $(h/E) \cdot c$, where h is Planck's constant. A photon with a sufficiently large energy will feel the metric corresponding to its size, i.e., the micro-level metric affected by quantum fluctuations.

Because of the fluctuations, in some locations, the local speed of light is smaller than its macro-value c, while in some other locations it is larger than the macro-level speed of light. Quantum fluctuations at different spatial points are not correlated – because, due to relativity, they cannot affect each other. So in a small vicinity of each spatial point, we can find a nearby point in which the speed of light is larger than the macro-level speed of light c. Thus, by a minor modification of the original straight-line trajectory, we can find a photon that travels with a velocity larger that c.

So, in principle, we can reach communication speeds faster than the macro-level speed of light – and the higher-energy photons we use for communication, the larger will be the fluctuations and thus, the larger the actual communication speed. And since we will be able to communicate faster, we will thus be able to perform computations faster as well!

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