7 Plus Minus 2 Law Revisited: Alternative Geometric Explanation, Mayan Arithmetic, and Using 9- and 18-Based Numbers in Jewish Tradition

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Abstract A recent paper showed that to make sure that the movements in the crowd are not chaotic, the directions of all the motions should deviate from some fixed direction by no more than 13 degrees. We show that this results provides a new geometric explanation for the seven plus minus two law in psychology, according to which we can keep in mind no more than 7 plus minus 2 items. We also show that all this is related to the somewhat mysterious appearance of 9- and 18-based number systems in Jewish and Mayan traditions.

1 7 plus minus 2 law: a brief reminder

Empirical law. It is known that in general, we can immediately have in mind at most between 5 and 9 different objects – how many depends on the individual person; see, e.g., [5, 6]. For most people, this number is 7, but for different people, it can take any value between $7-2=5$ and $7+2=9$.

Related challenges. There are two important challenges related to this law.

- First, this law is purely empirical. There are some theoretical explanations for this law, but they are not 100% convincing, so additional theoretical explanations are desirable.

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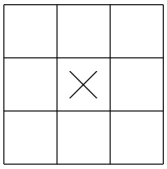
- Second, the law itself is rather vaguely formulated, and it is not always clear how to apply it to different situations. From this viewpoint, it is desirable to have a clearer understanding of this law – and for that, it is desirable to accumulate more examples of the use of this law.

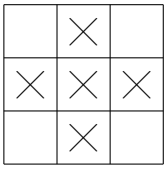
What we do in this paper. In this paper, we deal with both challenges. In Section 2, we provide the current geometric explanation for the 7 plus minus 2 law, and we explain the limitations of this explanation. A new alternative geometric explanation is then provided in Section 3. In Section 4, we provide a new example of the use of this law: namely, we show this law naturally leads to 9-ary number systems, and we cite historical and cultural evidence that such systems were indeed used all over the world – from pre-Biblical Jewish tribes to the Mayans.

2 Current geometric explanation, its natural extension, and its limitations

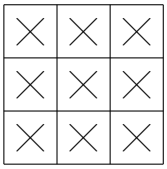
Current geometric explanation. A possible geometric explanation for this law is described in [2]. The idea is that to avoid dangers, to survive in a harsh environment, our ancestors needed to be aware of the immediate surroundings. To describe this is precise terms, let us form a grid in which a person in the central cell:



At a minimum, the person should be aware of what is happening not only in his/her cell, but also in the directly neighboring cells:



This requires keeping track of exactly five cells. An even better strategy is to take into account not only directly neighboring cells, but also cells which are attached to the cell-where-we-are even by a single point – i.e., to take into account even the diagonally connected cells:



This requires keeping track of exactly nine cells. So, this scheme indeed explains why we need to be able to deal with 5 to 9 objects. Thus, we do have a geometric explanation of the 7 plus minus 2 law.

A natural extension of the current geometric explanation. The current geometric explanation explains the endpoints 5 and 9 of the 7 ± 2 range. It is desirable to also explain 7, the most frequently used value. Here is the desired explanation.

When we explained 5, we only took into account potential dangers at 5 locations: at our location, directly in front, directly behind, close by to the left, and close by to the right. This only takes into account the 2D view. But dangers can also come from the third dimension: from above – e.g., predatory birds attacking – or from below – e.g., snakes. To take these dangers into account, instead of a 2D grid of small squares, we should consider 3D grid of small cubes. In such description, for each cube, there are 6 cubes directly neighboring the given one. With the given cube itself it makes exactly 7.

This idea can be naturally described in precise terms. Let us select one of the vertices of one the cubes from the grid as the starting point of the coordinate system, let us use the sides of one of the cubes containing this starting point as the coordinate axes, and let us the length of the side of each cube as the unit of length. Then, each cube from the grid gets the following form $[i,i+1] \times [j,j+1] \times [k,k+1]$ for some integers i, j, and k. In these terms, each such cube has 6 direct neighbors:

$$[i+1,i+2] \times [j,j+1] \times [k,k+1], [i-1,i] \times [j,j+1] \times [k,k+1],$$

$$[i,i+1] \times [j+1,j+2] \times [k,k+1], [i,i+1] \times [j-1,j] \times [k,k+1],$$

$$[i,i+1] \times [j,j+1] \times [k+1,k+2], [i,i+1] \times [j,j+1] \times [k-1,k].$$

With the central cell, this makes exactly 7 cells – which explains why 7 is the most frequent.

Limitations of the current geometric explanation. While the above explanation makes sense, it is not fully convincing: e.g., why not take into account even further cells and end up with $4 \times 4 = 16$ cells instead of $3 \times 3 = 9$?

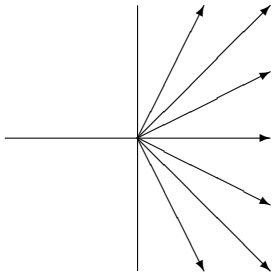
3 An alternative geometric explanation

A new result on which this explanation is based. Our new explanation is based on the recent result about crowd walking [1]. It is well known that in some cases,

when everyone is moving in the same direction, it is relatively easy to navigate a crowded street. However, in some other case, when people move in different direction, walking becomes very difficult. The paper [1] provided a theoretical analysis of this situation and showed that there is a threshold angle – or about 13 degrees – so that:

- if everyone's direction is within this range from the main axis, the movement is reasonably smooth, while
- when some people's directions deviate from the main axis by more than this threshold value, the movement becomes chaotic.

Resulting explanation. Suppose that we want to move forward – and we do not mind also simultaneously moving in an orthogonal direction. In this case potentially, we have infinitely many possible direction, and these directions form a 180 degrees angle.



According to the above empirical result, all directions that different from a selected direction by more than 13 degrees are, in effect, indistinguishable from each other – in the sense that if we combine these two motions, we will not see any interference between them. Two 13-degree zone on each side of a direction form a 26-degree section within which all directions are indistinguishable.

The whole 180-degree section is divided into such 26-degree sections. The number of such sections can be estimated if we divide 180 degrees by 26 – which makes 7 sections. So, all direction are divided into 7 sections – within each of which all directions are indistinguishable from each other. This is in perfect accordance with the fact that, according to the 7 ± 2 law, if we divide people into categories depending on into how many classes this person divides all objects, the largest category will be the one corresponding to 7 classes – and moreover, we get a new geometric explanation of the 7 ± 2 law.

4 How is all this related to ancient number systems

Why we need number systems different from 10-based. At present, most of the world uses a 10-based system. In this system, for example, any number between

10 and 100 – such as 86 – is represented as several groups of 10 objects in each + possibly one group with fewer than 10 objects. For example, 86 means that we have 8 groups of 10 and one more group with 6 elements in it.

However, according to the 7 plus minus 2 law, we cannot easily deal with 10 objects. The largest number of objects that we can easily deal with is $7+2=9$ – and only some people can deal with 9 objects at the same time, everyone can deal with 5 and most can deal with 7. So, not surprisingly, in the ancient times, when we had to reply on keeping everything in mind, people uses different number systems, systems that were more appropriate for such immediate in-your-mind processing.

Base-5 and base-9 systems. The most well-known base-5 system was used by the ancient Mayas; see, e.g., [3]. Interestingly, the Mayan also used multiples of 18 – which is closely related to the 9-based system [3].

There is also evidence of using base-9 and base-18 number systems in the Jewish tradition. In the Biblical Hebrew, while most numbers were represented in the base-10 form, 15 and 16 were represented as, correspondingly, $9+6$ and $9+7$, i.e., in affect, these twi numbers were represented in a base-9 number system; see, e.g., [4]. Also, in Jewish tradition, when one makes a gift or a donation, the usual way is to do it by multiples of 18 – i.e., in effect, by using base-18 number system.

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