Egyptian Triangle and Geometry of Airplane Wings: A Simplified Explanation

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Abstract In historically first planes, wings were orthogonal to the fuselage. However, later it turned out that from the aerodynamic viewpoint, it is most efficient to place the wings at about 37 degrees from this orthogonal direction – and this is where wings are placed in most modern planes. There exist theoretical explanations for this optimality – explanations based on solving the equations of aerodynamics. In such situations when only a complex not-very-intuitive explanation exists, it is desirable to come up with a simpler more intuitive explanation. For the wing angles, such an explanation is provided in this paper. Namely, we show that, somewhat surprisingly, this is all related to the so-called Egyptian triangle – a right triangle with sides 3, 4, and 5. The name for this triangle comes from the fact that already the ancient Egyptians were very familiar with this triangle – they used it to accurately reproduce the right angle.

1 Formulation of the Problem

Geometry of airplane wings: an empirical fact and its current explanation. In the first airplanes, wings were orthogonal to the fuselage. However, as the airplane speeds increased, it turned out that this configuration was not the best from the viewpoint of aerodynamics. It turned out to be more efficient to place the wings at

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some angle from the orthogonal direction. This angle was called *wing sweep angle*. Experiments in the wind tunnel showed that the most efficient wing sweep angle is about 37 degrees. This experimental result later received a theoretical explanation – based on the corresponding equations of aerodynamics.

This angle was first implemented in the design of the Boeing B-47 bomber, and it has been implemented in the design of the commercial jets based on this design, such as Boeing 747. This wing sweep angle is the most efficient for the usual commercial jet cruise speeds of 0.84-0.88 Mach speed (= speed of sound). Similar angles are used on many other commercial jets and bomber planes; see, e.g., [1].

Problem: can we have a simpler explanation? The current explanation for the efficiency of the 37 degrees wing sweep angle is based on the complex computations. From the physical viewpoint, it is desirable to have a simpler and more intuitive explanation.

What we do in this paper. In this paper, we provide a simplified explanation. Somewhat surprisingly, this explanation is related to what mathematicians call an *Egyptian triangle* – a right triangle with sides 3, 4, and 5 that already ancient Egyptians used to form right angles; see, e.g., [3].

Comment. Of course, as usual, simplification comes at a price: while our ideas simply explain why the 37 degrees angle is the most efficient, the current more complex explanation goes much further: it also explains, numerically, how exactly the effectiveness changes with the change in the wing sweep angle.

2 Our Explanation

Our explanation is based on synchronization ideas. Our explanation is based on the physical phenomenon known as *synchronization*. In general, any physical process can be described, via Fourier transform, as a linear combination of periodic processes. At some frequencies – eigen frequencies of the corresponding dynamical system – we have a resonance, and the corresponding signal is amplified.

If we have several nonlinearly interacting dynamical systems, then usually, their frequencies become synchronized: i.e., they either become equal or their ratio becomes equal to the ratio of two small natural numbers; see, e.g., [2, 5, 6]. Many examples of synchronization can be found in planetary systems; see, e.g., [4].

Comment. Also, the larger the numbers, the smaller the synchronization effect – and when the numbers get to 10 or so, the effect disappears.

How is synchronization related to aerodynamic efficiency. In general, a systematic well organized flow – and non-turbulent flow is an extreme example – provides less resistance than a chaotic one. From this viewpoint, synchronized flows are more organized and thus, lead to a more efficient flights.

Let us apply this idea to the plane wings. Frequencies are determined by the forces. There are air resistant forces acting on the plane. These forces are oriented in the direction in which the plane goes – i.e., aligned with the symmetry axis of the fuselage. From the physics viewpoint, it makes sense to distinguish between the two components of this force:

- the component which is orthogonal to the wing, and
- the component which is parallel to the wing axis.

If we denote by F the value of the original force, then the forces acting along the wing and orthogonal to the wing are equal, accordingly, to $F \cdot \sin(\varphi)$ and $F \cdot \cos(\varphi)$ for an appropriate angle φ.

The ratios between these these three forces are $\sin(\varphi)$ and $\cos(\varphi)$. The frequencies depend on forces. It makes sense to assume, in the first approximation, that frequencies linearly depend on forces. Under this assumption, frequencies are proportional to forces. In this case, the ratios between frequencies are also equal to $\sin(\varphi)$ and $\cos(\varphi)$. Thus, the most efficient angle corresponds to the case when the frequencies are synchronized, i.e., when both $\sin(\varphi)$ and $\cos(\varphi)$ are ratios of two small natural numbers:

$$\sin(\varphi) = \frac{n_s}{d_s} \text{ and } \cos(\varphi) = \frac{n_c}{d_c}. \tag{1}$$

We always have $\sin^2(\varphi) + \sin^2(\varphi) = 1$. Due to (1), this leads to

$$\left(\frac{n_s}{d_s}\right)^2 + \left(\frac{n_c}{d_c}\right)^2 = 1. \tag{2}$$

It is convenient to bring both fraction to a common denominator D. This way, we have

$$\sin(\varphi) = \frac{N_s}{D} \text{ and } \cos(\varphi) = \frac{N_c}{D} \tag{3}$$

for appropriate values N_s and N_c. In these terms, the equality (2) takes the form

$$\left(\frac{N_s}{D}\right)^2 + \left(\frac{N_c}{D}\right)^2 = 1, \tag{4}$$

i.e., equivalently,

$$N_s^2 + N_c^2 = D^2 \tag{5}$$

for reasonably small integers N_s, N_c, and D. If the numbers N_s, N_c, and D have a common deniminator, then we can divide all three numbers by this denominator without changing the ratios. Thus, we can safely assume that three numbers do not have a common denominator.

How can we describe all such triples? These triples provide the numerical meaning of the Pythagoras Theorem and are, therefore, known as *Pythagorean triples*. Already Euclid came up with a formula that described all possible Pythagorean triples in which the three numbers do not have a common denominator: such triples can be described as

$$N_s = m^2 - n^2,\ \ N_c = 2m\cdot n,\ \text{and}\ D = m^2 + n^2,$$

for some integers $m > n > 0$ that do not have any common divisor. The smallest triples comes from the smallest possible pair $(m,n) = (2,1)$ for which

$$N_s = 2^2 - 1^1 = 4 - 1 = 3,\ \ N_c = 2\cdot 2\cdot 1 = 4,\ \text{and}$$

$$D = 2^2 + 1^2 = 4 + 1 = 5.$$

For this triple, the ratios become $3/5 = 0.6$ and $4/5 = 0.8$. Interestingly, the corresponding angle φ with $\sin(\varphi) = 3/5$ is $\varphi \approx 37^\circ$ – this explains why this angle leads to the maximal efficiency!

Comment. We can show that $(3,4,5)$ is the only Pythagorean triple which is relevant to our applied problem. Indeed, the next smallest pair that has no common divisor is $(m,n) = (3,2)$ for which

$$N_s = 3^2 - 2^2 = 9 - 4 = 5,\ \ N_c = 2\cdot 3\cdot 2 = 12,\ \text{and}$$

$$D = 3^2 + 2^2 = 9 + 4 = 13.$$

These numbers are already larger than 10, so we do not have any physical synchronization effect.

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