How Sense of Belonging Affects Student Success

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Abstract. Empirical studies show that a proper sense of belonging – to the university, to the department, to the profession – is important for students to succeed. To help students develop this sense, it is desirable to have quantitative models describing how sense of belonging affects student success. In this paper, we show that while we can come up with a reasonable quantitative model by using general model building techniques, we can get a more adequate model if we use fuzzy techniques, techniques that take into account expert's degrees of certainty.

Keywords: student success, sense of belonging, fuzzy techniques

1 Formulation of the problem

Challenge: retention rates are lower than we would like. In many disciplines including Computer Science, retention remains a big problems. After the first semester, almost half of the students change majors.

This cannot be explained by the students' unpreparedness – at most universities, to sign for the Introduction to Computer Science class, students need to successfully pass several math-related classes that make them well prepared. This cannot be fully explained by the students suddenly realizing that Computer Science is not what they thought it would be: Computer Science and AI are all over the media nowadays. Of all possible disciplines, Computer Science is arguably the most popularized of all disciplines. And it is not popularized as supposedly an easy profession: most popular articles are written by journalists who do not fully understand the current successes and who therefore emphasize how difficult this subject is.

So what is the explanation for lower-than-expected retention rates?

One of the most important factors is sense of belonging. While retention rate is somewhat similar in most universities, it still differs a lot. Researchers performed comparative survey studies to find out what are the main factors that affect student retention – and student success in general. Most of these studies show that one of the most important factors is whether the students gain the *sense of belonging* – belonging to the college, belonging to the department, belonging to the profession, etc.; see, e.g., [5] and references therein.

Resulting problem: it is desirable to have quantitative models. Since the sense of belonging is very important, it is desirable to better cultivate this sense. This way, we will increase student retention and student success. Researchers and practitioners have been working on ways of doing that [5].

However, up to now, this is mostly done on a qualitative semi-empirical level. General experience of science shows that quantitative models usually lead to better performance. It is therefore desirable to have a quantitative model of this phenomenon.

What we do in this paper. In this paper, we provide the first approximations to the desired quantitative model.

The structure of this paper. In Section 2, we provide a more detailed description of the above phenomenon. In Section 3, we use general model-building techniques to build the first quantitative model. In Section 4, we explain the limitations of the first model, and we use fuzzy techniques – techniques that take into account the uncertainty (fuzziness) of expert knowledge – to come up with a more adequate quantitative model.

2 Analysis of the problem

Too strong sense of belonging can be counterproductive: general idea and examples. In the previous section, we focused on situations where students are not as successful as they can be because of their low sense of belonging. To design an adequate mathematical model describing how student success depends on the sense of belonging, we need to take into consideration not only cases of low sense of belonging, but the whole range of possible values of this sense, from low to normal to high.

And for high sense of belonging, empirical evidence shows the opposite effect: when the sense of belonging is too strong, the success level decreases. Let us give a few examples.

In our city of El Paso, Texas, many students have a very strong sense of belonging to the community. Because of this, frequently, when a smart high school student gets accepted to one of the out-of-town top schools like MIT, this student either rejects this offer right away, or goes out to this school, studies there (reasonably successfully) for a year or even for a semester, and then transfers to a school back home. In this case, clearly, this student's strong sense of belonging is counterproductive for the student's success. And when this student graduates from a local university, he/she does not want to leave town to get a good job elsewhere, so this student agrees to a not so interesting (and a much-lower-paying) job in the city.

Similarly, because of an unhealthily strong sense of belonging to a family, people stay for years and even decades in an unhappy – and often abusive – relationship. Because of too strong sense of belonging to old ways of life, many people resist changes, even changes that would make everyone more successful and more happy.

In general, human progress is driven by creativity: whenever the humanity faces a challenge, we find a creative solution to the problem. And creative means different from what was common before. So, to be creative, we cannot have a complete sense of belonging to all our communities and all their traditions and ways of life, we need to be open to new ideas, even idea that are different from what the community currently believes.

Also, it is always good to retain a healthy level of caution (and even mistrust). For example, when coding, it is good to feel a member of the community and thus, to believe in the ways of programming that are recommended and trusted by the community, but it is always a good idea to still check and test your code – since sometimes, the community beliefs miss a point. There is even anecdotal evidence that programmers who grew up in not perfectly safe communities – e.g., in big cities where there are unsafe areas that need to be avoided and where pickpockets or even robberies are common – get a healthy habit of checking everything, and that habit naturally extends to their coding – which makes their code more reliable.

These examples show that too much sense of belonging can be detrimental to success. Vice versa, if you analyze successful people, many of them have a limited sense of belonging, they never felt fully at home – examples include Albert Einstein (see, e.g., [6]) and Elon Musk (see, e.g., [7]). This is also reflected in the fact that many successful people have impostor syndrome – when while they are actually well accepted by their communities, they do not feel that they fully belong, they feel that they are impostors who are accepted largely by mistake.

Comments. Since the objectives of successful people are somewhat different from the objectives of the corresponding community,

- sometimes, their activity leads to successes – and
- sometimes, it leads to a failure, when their unorthodox solutions – which make perfect sense from their somewhat different viewpoint – turn out to be detrimental to the community; there are many such examples in history, when a charismatic leader led his community to a disaster.

Since their objectives are somewhat different, successful people often do not see the difference between ideas that will be beneficial to their community and ideas that will be detrimental to this community. This can be seen on the example of creative people.

For many writers, poets, and musicians, what they value most from their artistic heritage is often different from what the community perceives as their best work. For example, the Russian poet Mayakovski hated it when many people asked him to recite his poem "Oblako v shtanah" ("A cloud in pants") that many people perceived as his best work – while he himself considered his other poems as much better ones.

Same phenomenon can be observed with scientists: what a scientist considers his/her best paper is often not what has the largest number of citations – and is, thus, considered the best by the research community. Modern Hollywood is another good example of this phenomenon: movie companies try their best to predict which movie project will succeed, but very frequently:

- a multi-million-dollar movie that everyone hoped would be a mega-success turns out to be a failure, while
- a low-budget independent film becomes a mega-hit.

The Nobel-winning Russian poet Boris Pasternak – of the "Doctor Zhivago" fame – described this phenomenon poetically, than a creative person is usually not able to predict which of his works will be a success and which a failure: "Porazen'ya ot pobedy ty sam ne dolzhen otlichat'".

This phenomenon may be the reason why in the original version of Bloom's taxonomy, a widely framework for categorizing educational goals, ability to evaluate was considered the highest level of knowledge [2] – because even most creative persons are deficient in this ability.

The desired dependence is one of the examples of inverted U-shape dependencies. As we have mentioned earlier, when there is no sense of belonging, students are, in general, not every. As the sense of belonging becomes stronger, students become more and more successful – until the value of this sense reaches a certain threshold, after which any further increase in sense of belonging makes the success rate lower. Such dependencies – when a function first increases and then decreases – are ubiquitous. They are known as *inverted U-shaped dependencies*, since their graphical representation reminds an inverted letter U.

The first example of an inverted U-shape dependence was obtained when researchers analyzed how success depends on the effort or on the motivation; see, e.g., [3, 14].

3 First quantitative model

A usual strategy for designing a model: a brief reminder. Situations in which we do not know the exact form of the dependence between two quantities are ubiquitous in physics. In such situations, a usual strategy (see, e.g., [4, 13]) is to take into account that dependencies are usually smooth – even analytical, so the function describing the dependence can be expanded in Taylor series. This means, in effect, that as a good approximation, we can take the sum of the first few terms of the Taylor expansion.

The more terms we take into account, the more accurate is our description. As the first approximation, it makes sense to take the smallest number of terms for which we can have a function that is consistent with our knowledge about the desired dependence.

Let us apply this strategy to our case. We are interested in describing how the success y depends on some characteristic x: be it sense of belonging, or effort, or degree of motivation. What do we know about this dependence?

– First, we know that in all these three cases, at first, the success level grows when the value of the corresponding characteristic increases.
– Second, we know that the success level is limited: no matter how much sense of belonging, motivation, or effort we exhibit, there are physical and biological limits on how much we can learn in the course or how much we can do.

Let us see what is the smallest number of terms in the Taylor expansion

$$y = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \ldots \quad (1)$$

for which the resulting dependence is consistent with these two facts.

– If we only take into account the constant term – i.e., if we use $y = a_0$ as a model – then the dependence y of x is not increasing, which contradicts to the first fact. This means that this approximation is not enough, we need to take more terms into account.
– If we take into account the first two terms in the Taylor expansion, i.e., if we use $y = a_0 + a_1 \cdot x$ as a model, then for $a_1 > 0$ we get an increasing dependence, but the resulting function is not bounded – which contradicts to the second fact. This means that this approximation is also not enough, we need to take more terms into account.

So, we need to take into account quadratic terms as well, i.e., we need consider the following model:

$$y = a_0 + a_1 \cdot x + a_2 \cdot x^2. \quad (2)$$

Here, $a_1 > 0$, and to make sure that the dependence is bounded, we need to take $a_2 < 0$.

This explains, at least on the qualitative level, the inverted U-shaped dependence. One can easily check that the function (2) with $a_2 < 0$ indeed grows until some value and then starts decreasing – this is exactly what the inverted U-shaped dependence is about. In other words, at least on the qualitative level, we get an explanation of the ubiquity of the inverted U-shaped dependence.

4 Limitations of the first quantitative model and a more adequate fuzzy-based quantitative model

Limitations of the first quantitative model. In the above model, it makes sense to consider only the values x for which the success level y is non-negative. One can check that for the quadratic dependence (2), the range of all such values is divided into two subranges of equal width:

– the subrange at which the function is increasing, and
– the subrange at which the function is decreasing.

Indeed, the desired range is the range $[x_-, x_+]$ between the two roots x_- and x_+ of the quadratic equation $y = 0$, for which

$$x_\pm = \frac{-a_1 \pm \sqrt{a_1^2 - 2a_0 \cdot a_2}}{2a_2}. \tag{3}$$

The midpoint $\widetilde{x}$ between these two roots is

$$\widetilde{x} = \frac{x_- + x_+}{2} = -\frac{a_1}{2a_2}. \tag{4}$$

The derivative of the expression (2) – whose sign determines whether the function is increasing or decreasing – has the form

$$y' = a_1 + 2a_2 \cdot x = 2a_2 \cdot \left(x + \frac{a_1}{2a_2}\right) = 2a_2 \cdot (x - \widetilde{x}). \tag{5}$$

Since $a_2 < 0$:

– for $x < \widetilde{x}$, we have $y' > 0$, so the function $y(x)$ is increasing on the first half-range $[x_-, \widetilde{x}]$; and
– for $x > \widetilde{x}$, we have $y' < 0$, so the function $y(x)$ is decreasing on the second half-range $[\widetilde{x}, x_+]$.

The fact that these subranges have *the same width* contradicts to common sense, according to which on a *larger* part of the range, the deoendence is increasing, and it is only decreasing on a *smaller* part of the range.

How can we improve this situation? The above-described usual strategy for designing models only takes into account well-defined ("crisp") parts of our knowledge – e.g., that the function is increasing on some subrange or that it is bounded. However, it is well known that a significant part of our knowledge is formulated by using imprecise ("fuzzy") words from natural language – such as "small", "very", etc.

To describe this knowledge in precise computer-understandable terms, Lotfi Zadeh designed special techniques that he called *fuzzy*; see e.g., [1, 8, 10–12, 15]. Let us therefore use fuzzy techniques to design the corresponding model.

Let us use fuzzy techniques to design a new model. To apply fuzzy techniques, we need to start with a description of our knowledge is commonsense terms. In these terms, our knowledge can be described as follows: *the success comes when the sense of belonging is high but not too high.*

In fuzzy techniques, the degrees are usually limited to the interval $[0, 1]$. So, let us assume that both the sense of belonging x and the resulting level of success y are characterized by numbers from the interval $[0, 1]$.

– The simplest way to describe the degree to which x is high is to have

$$m(x) = x.$$

- The simplest way to describe the hedge "too" (which is, in this case, equivalent to "very") is to use the square function: if the degree to which we have a property is m, then the degree to which this property is very much satisfied is m^2. So, the degree to which a level x is too high is m_2.
- The usual way to describe negation is to use $1 - m$: if the degree to which a property is satisfied is m, then the degree to which this property is not satisfied is equal to $1 - m$. Thus, the degree to which the sense of belonging is not too high is $1 - x^2$.
- How can we interpret "but"? From the logical viewpoint, "but" here simply means "and", and the reason why we use "but" and not "and" is that the two statements combined by this connective are somewhat inconsistent with each other. So, we can use the usual way to describe "and" in fuzzy techniques. One of the simplest ways to describe "and" – and the simplest for which we have a smooth dependence, in accordance with the previous section – is to use the usual ("algebraic") product. In other words, if our degrees of confidence in two statements S_1 and S_2 are m_1 and m_2, then we estimate our degree of confidence in the statement $S_1 \,\&\, S_2$ as $m_1 \cdot m_2$. For our commonsense statement, $m_1 = x$ and $m_2 = 1 - x^2$, so we arrive at the following model:

$$y = x \cdot (1 - x^2). \tag{6}$$

This is our second quantitative model for the dependence of success on sense of belonging.

Our fuzzy-based model is more adequate than our first quantitative model. As we have mentioned earlier, the main limitation of our first quantitative model was that:

- for this model, the subranges at which the dependence $y(x)$ is, correspondingly, increasing and decreasing have the same width – while,
- according to our understanding, the increasing subrange should be wider than the decreasing one.

Let us show that for the fuzzy-based model (6) for which $y = x - x^3$, the increasing subrange is indeed wider. Indeed, for the model (6), increasing means that $y' \geq 0$, i.e., that $1 - 3x^2 \geq 0$. This inequality is equivalent to $3x^2 \leq 1$, i.e., $x_2 \leq 1/3$ and $x \leq \sqrt{1/3} \approx 0.58$.

Thus, for our fuzzy-based quantitative model, the width 0.58 of the increasing subrange is indeed larger than the width $1 - 0.58 = 0.42$ of the decreasing subrange.

Comment. The arguments that we used to come up with our fuzzy-based model are similar to the arguments used in [9] to explain the golden proportion.

In that paper, the author was looking for the value x that best satisfies the property that this value is large but not too large. The arguments used in that paper are largely similar to what we used in this section, the only difference is that that paper used minimum to describe "and".

So, in that paper, the degree y to which the statement is satisfied is described by a slightly different formula $y = \min(x, 1 - x^2)$. It turns out that the value x for which degree is the largest possible is exactly the golden proportion

$$x = \frac{\sqrt{5} - 1}{2} \approx 0.62.$$

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