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SPACE-TIME PERCEPTION: WHY EVOLUTION HAS LED TO LOGARITHMIC (AND POWER LAW) SUBJECTIVE TIME AND WHY WE CANNOT REMEMBER EARLY CHILDHOOD

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Аннотация. It is known that subjective perception of time is biased: the perceived time is, in general, different from the physical time. Empirical studies have shown that in the first approximation, the dependence of subjective time on physical time is described reasonably well by a power law – and even better by a logarithm. In this paper, we show that such a dependence is indeed optimal with respect to any reasonable optimality criterion. Since we humans are a product of billions of years of optimizing evolution, this proven optimality explains why these dependencies reasonably accurately describe our time perception. We also explain why we cannot remember early childhood events.

Ключевые слова: Subjective time, logarithmic dependence, optimality, early childhood memories..

1. Formulation of the problem

Subjective time: empirical observation. It is known that subjective time s – time that we perceive – differs from physical time t. For example, as we age, time seems to go faster; see, e.g., [2, 3, 5].

According to [5], in the first approximation, the dependence of subjective time s on physical time t is well described by a power law $s \sim t^\alpha$. The most accurate description comes when we take α close to 0, when we have

$$t^\alpha = (\exp(\ln(t))^\alpha = \exp(\alpha \cdot \ln(t)) \approx 1 + \alpha \cdot \ln(t),$$

i.e., in effect, when we consider a logarithmic dependence.

A natural question: why? We humans are the result of billions of years of improving evolution. This means that practically every feature of our biology is optimal (or at least close to optimal). If a feature was not optimal, during the evolution process, a better feature would have appeared and replaced it. In particular, power law and logarithmic dependence of subjective time on physical time must be optimal in some reasonable sense. So why are these particular dependencies optimal?

What is known. There exist partial explanations for the difference between subjective and objective time. For example, in [1], this difference is explained by the need to make decisions under uncertainty. However, to the best of our knowledge, there have been no explanations for the emergence of the observed specific dependencies: power law and logarithm.

What we do in this paper. In this paper, we prove that in the first approximation, power law and logarithmic dependencies are indeed optimal – and optimal in the sense of all reasonable optimality criteria. Our result also explains why we cannot remember early childhood.

2. Our explanation

We may have the whole family of optimal functions. At first glance, one may think that we need to select *the* optimal function $s = f(t)$ that describes how the numerical value of the subjective time s depends on the numerical value of the physical time t. However, the numerical value of the physical time depends on the choice of the measuring unit: if instead of hours, we count minutes, then 2 hours becomes 120 minutes. The numbers are different, but the physical situation is the same. In general, in a new unit with is λ times smaller than the original measuring unit, the numerical value of time changes from t to $t' = \lambda \cdot t$. Thus, $t = t'/\lambda$. So, in terms of the new unit, the formula for subjective time has a different form: $s = f(t'\lambda)$. In other words, instead of the original function $f(t)$, we now have a new function $f'(t) \stackrel{\text{def}}{=} f(t/\lambda)$.

The two functions $f(t)$ and $f'(t)$ describe the exact same dependence of subjective time on physical time. Thus, if one of these function is optimal, the other one should be optimal too. Hence, we indeed have the whole family of optimal functions. So, instead of individual functions, we should consider families of functions.

In general, a family of function can be described as $\{f(t, c_1, c_2, \ldots)\}$, for possible parameters $c_1, c_2, \ldots$ The more parameters, the more accurately we can describe the phenomenon of interest – but the more complex the corresponding model becomes. Thus, in the first approximation, it makes sense to consider families $\{f(x, c_1, \ldots, c_k)\}$ with the smallest possible number of parameters k.

What do we mean by "optimal". In principle, we can have different optimality criteria for selecting a family of functions. What they all have in common is that they all enable us, for every two families a and b, to decide whether a is better than b (or of the same quality as b) – according to this criterion; we will denote this by $a \succeq b$. This relation has to be transitive: if a is better than b and b is better than c, then a should be better than c. Also, by the meaning of the relation $\succeq$, we should have $a \succeq a$.

A family a_{opt} is *optimal* if it is better than all other families, i.e., $a_{\text{opt}} \succeq a$ for all families a.

The optimal criterion should be final. If several different families are optimal, then we can use this non-uniqueness to optimize something else. This means that the original optimality criterion was not final: we have a better criterion that takes the additional optimality into account. So, it makes sense to consider only *final* optimality criteria, i.e., criteria for which there is exactly one optimal family.

The optimality criterions should not change if we use different measuring units and/or different starting points. In addition to selecting a different measuring unit, we can select a different starting point. If we select, as a new starting point, a moment of time which is t_0 units earlier than the original one, then for each value t in the original setting, the value in the new setting is equal to $t+t_0$. In general, if we can change both the measuring unit and the starting point, then the new value becomes $t'=\lambda\cdot t+t_0$. Similarly, we change the starting point and the unit for subjective time. The relative quality of two families should not change if we simply change the measuring units and starting points but leave the physics intact.

In the new units, the original function $f(t)$ takes the form $f'(t)\stackrel{\text{def}}{=}A\cdot f(c\cdot t+d)+B$ for some values $A>0$, B, $c>0$, and d. We will say that the original function $f(t)$ and the new function $f'(t)$ are *linearly equivalent*. Thus, in the new units, the original family a becomes $T_{A,B,c,d}(a)\stackrel{\text{def}}{=}\{A\cdot f(c\cdot t+d)+B : f(t)\in a\}$. In these terms, the desired *invariance* means that if $a\succeq b$, then, for each $A>0$, B, $c>0$, and d, we should have $T_{A,B,c,d}(a)\succeq T_{A,B,c,d}(b)$.

We are ready to prove that the selected functions are indeed optimal. This optimality of logarithm and power law dependencies follows from the following result (proven in [4]):

Теорема 1. *The smallest k for which there exists a final invariant optimality criterion is $k=3$. For this k, for each final invariant optimality criterion, each function from the optimal family is linearly equivalent either to* $\log(x)$*, or to* x^α *for some* α*, or to* $\exp(x)$.

But why logarithm? The above result lists three different functions, so why is logarithm better describing the observed phenomenon and not any other function? To answer this question, let recall that subjective time is, by definition, subjective: it only makes sense starting from the moment T at which the corresponding person was born. So, it makes sense to have function $f(t)$ that cannot be defined for $t<T$.

This immediately excludes exponential functions – since they are defined for all possible values t. This also excludes power law functions – since they can be extended to all values t, e.g., as $\text{sign}(t)\cdot|t|^\alpha$. So, the only remaining function is log – and logarithm that is not defined at the birthday T, i.e., logarithm of the form $A\cdot(c\cdot\log(t-T))+B$. Since the logarithm of the product is equal to the sum of the logarithms, we get

$$A\cdot\log(t-T)+(A\cdot\log(c)+B.$$

After an appropriate linear re-scaling of subjective time, we get the function $f(t)=\log(t)$, which is, in the first approximation, exactly what is observed.

Why logarithm works better than power law: an alternative explanation. The above result includes both log and power law as possibly optimal functions, so why is log – which, as we have shown in Section 1, is corresponding to $\alpha\to 0$ – better?

To provide an alternative answer to this question, let us recall that in general, the maximum of a function on a bounded domain is attained either at a local maximum – where all the partial derivatives are 0s – or at a point on a domain's boundary. Often, the domain is small enough so that it does not contain any local maxima. In this case, the maximum is attained at the boundary.

In our case, the domain consists of all non-negative values α. The boundary of this domain is exactly the point $\alpha = 0$. So, it is not surprising that, according to empirical data, the optimum is attained at this boundary point $\alpha = 0$.

So why cannot we remember early childhood? Early childhood are times for which $t \approx T$ and $t - T \approx 0$. For $t = T$, the logarithm $\log(t - T)$ is equal to $-\infty$. Thus, the subjective time that has passed between that time and now, tends to infinity as we get closer and closer to the birth date. So, no matter how far away in subjective time we remember, we cannot remember up to an infinite subjective time interval – there is always a limit to our ability to store information. Whatever happened earlier than this limit, we cannot remember.

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