Reversible and quantum computing involving random processes: local time naturally appears

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Abstract Computers consume 10% of the world's energy, and this proportion continues to increase. One of the main reasons for this energy consumption is that computations use irreversible processes – e.g., an "and"-gate is irreversible – and, according to thermodynamics, this leads to energy consumption. To decrease the energy use, a natural idea is to use reversible computing – of which quantum computing is an important case. It is known how to make deterministic computations reversible. However, since many real-life processes are random, it is often important to simulate random processes as well. In this paper, we analyze how to make simulations of random processes reversible. It turns out that this can be done by using a known notion of *local time* of a random process.

1 Formulation of the problem

Energy problem of modern computing. One of the big problems of modern computing is that computers use a large amount of energy, up to 10% of the world's consumption. And this portion increases.

Technological and physical reasons for the computing's energy problem. There are technological reasons for this consumption.

However, there is also a fundamental physical reason. Namely, according to thermodynamics (see, e.g., [2, 7]), irreversible processes increase entropy and thus, cause heat release – and computing uses a lot of irreversible processes.

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For example, an "and"-gate transforms two signals a and b into a single signal $c = a \& b$. When $c = 0$, we cannot uniquely reconstruct the inputs (a,b):

- it could be $(0,0)$,
- it could be $(0,1)$, or
- it could be $(1,0)$.

Reversible computing – a natural solution to the computing's energy problem. To decrease computers' energy consumption (and heating the environment), it is desirable to make computations reversible. This is known as *reversible computing.*

Quantum computing as an important case of reversible computing. Limitation to reversible computing is also needed for quantum computing; see, e.g., [5]. Indeed, there computation is done on the level of elementary particles – and on this level, all physical processes are reversible.

Reversible computing: how? A general idea. To make computations reversible, a natural idea is to add auxiliary inputs.

Case of an "and"-gate. For example, we can make an "and"-gate reversible if we:

- add an auxiliary input y and
- transform (a,b,y) into $(a,b,y \oplus (a \& b))$, where $\oplus$ denoted exclusive "or" (i.e., addition modulo 2).

Case of processing real numbers. Similarly:

- to make computations $x \mapsto f(x)$ with real numbers reversible,
- we can add additional input y and transform (x,y) into $(f(x),g(x,y))$ for some $g(x,y)$.

Here, reversible means that the number of output states be equal to the number of input states. If we compute with accuracy ε, then the number of input states is proportional to the area of the (x,y) domain. In these terms, reversible means area-preserving, and this leads to

$$g(x,y) = \frac{y}{f'(x)},$$

where $f'(x)$ means the derivative; see, e.g., [3].

What if we want to simulate a random process: formulation of the problem. Many real-life processes are random. It is therefore important to simulate them.

In these simulations, just like in all other simulations, it is desirable to use reversible and quantum computing. For this purpose, we need to replace possibly irreversible computations $t \mapsto x(t)$ with reversible ones $(t,y) \mapsto (x(t),g(t,y))$ for some function $g(t,y)$.

For smooth processes, as we have mentioned, we should take

$$g(t,y) = \frac{y}{x'(t)}. \tag{1}$$

So, we need to extend this expression to general – not necessarily smooth – processes.

What we do in this paper. In this paper, we describe the desired extension. Interestingly, this extension used a construction that already exists – although it has been previously related to reversible computing: the construction of *local time*; see, e.g., [1, 4, 6].

2 Analysis of the problem and the resulting solution

Analysis of the problem. We want to extend the expression (1) to the non-smooth case. Specifically, we can keep y – but we need to extend the derivative $x'(t)$ to the non-smooth case.

Such an extension already exists. It is known that the ratio

$$\frac{1}{x'(t)} \tag{2}$$

is a smooth case of a quantity known as *local time* $\ell(t)$. This quantity is defined as follows.

- For every x-interval $I = [x, x+\varepsilon]$, we consider the overall duration $d(\varepsilon)$ of all the time intervals at which we had $x(t) \in I$.
- Then, we define $\ell(t)$ as the limit of the ratio

  $$\frac{d(\varepsilon)}{\varepsilon}$$

  when $x = x(t)$.

It is known that in the smooth case,

$$\ell(t) = \frac{1}{x'(t)}.$$

From the smooth case to the general case. It is known in the general case:

- when we have a sequence of smooth processes approximating a given one,
- the resulting values $1/x'(t)$ tend to $\ell(t)$.

Conclusion. Thus, in general, we can use local time to have a reversible or quantum simulation of a random process $x(t)$ as

$$(t, y) \mapsto (x(t), y \cdot \ell(t)).$$

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