Why burst of physical activity is good for your health: a possible explanation

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Abstract Recent studies have shown that a short burst of intensive physical activity is better for a person's health than the same amount of activity spread over time. In this paper, we provide a theoretical explanation for this empirical fact.

1 Formulation of the problem

Empirical fact. Recent research showed that a short burst of high physical activity is good for your health [1, 2]. Moreover, a short burst has a much better effect than the same amount of activity spread over time.

A natural question. How can explain this phenomenon?

What we do in this paper. In this paper, we provide an explanation for this empirical fact.

2 Analysis of the problem and the resulting explanation

Let us describe the situation in precise terms.

- Let us denote the overall amount of physical activity by D_0.

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- How we spread this activity over time can be characterized by a function $D(t)$, that describes how much activity has been done by time t.
- We start at time $t = 0$, when the activity-so-far is 0: $D(0) = 0$.
- At each moment of time, we can only add more activity. So, this function is (non-strictly) increasing: if $t \le t'$ then $D(t) \le D(t')$.
- At the end of the session, we should have $D(t) = D_0$.
- This amount should remain the same until the next session.
- The time until the next session – e.g., a day – is much larger than the session's duration. Thus, it makes sense, when describing the current session:
  - to ignore this future session – which is too far away from now, and
  - to simply assume that the function $D(t)$ remains equal to D_0 for all non-negative value t.

We need to select a schedule. In these terms, selecting an appropriate schedule means selecting a (non-strictly) increasing function $D(t)$ for which $D(0) = 0$ and

$$\lim_{t\to\infty} D(t) = D_0.$$

There are many such functions, which of them should we choose? Informally, we should select the best of these functions. The question is how we describe "the best" in precise terms.

How to describe "the best" in precise terms: general case. Usually, "the best" means that some objective function attains the largest (or the smallest) possible value. However, this is not the most general way of describing optimality.

For example, if we have two different schedules that have the same health effect, it is reasonable to use this non-uniqueness to optimize something else. In this case, instead of the original single objective function $f(a)$, we have a more complicated scheme, when an alternative a is better than an alternative b, if:

- either $f(a) > f(b)$,
- or $f(a) = f(b)$ and $g(a) < g(b)$ (for some other function $g(a)$).

If this more complex scheme still selects several alternatives, then we can use this non-uniqueness to optimize something else, etc., until we reach the *final* optimality criterion, in which we have only one optimal alternative.

The only thing we can say about such more general optimization settings is that we should be able, for any two alternatives a and b, to decide:

- whether a is better than b (we will denote it by $a > b$),
- or b better than a ($b > a$),
- or a and b have the same quality (we will denote it by $a \sim b$).

These relations $a > b$ and $a \sim b$ should satisfy natural consistency requirements. For example, if a is better than b and b is better than c, then a should be better than c. Thus, we arrive at the following definition.

Definition 1.

- *Let A be a set. Its elements will be called* alternatives.
- *By an* optimality criterion*, we mean a pair of binary relations* $\langle >, \sim \rangle$ *that satisfy the following conditions for all a, b, and c:*
  - *if* $a > b$ *and* $b > c$*, then* $a > c$*;*
  - *if* $a > b$ *and* $b \sim c$*, then* $a > c$*;*
  - *if* $a \sim b$ *and* $b > c$*, then* $a > c$*;*
  - *if* $a \sim b$ *and* $b \sim c$*, then* $a \sim c$*;*
  - *if* $a > b$*, then we cannot have* $a \sim b$*.*
- *We say that an alternative* a_{opt} *is* optimal *with respect to the optimality criterion* $\langle >, \sim \rangle$ *if for every* $a \in A$*, we have either* $a_{\text{opt}} > a$ *or* $a_{\text{opt}} \sim a$*.*
- *We say that the optimality criterion is* final *if there exists exactly one alternative which is optimal with respect to this criterion.*
- *In our case, alternatives are different (non-strictly) increasing functions* $D(t)$ *for which* $D(0) = 0$ *and* $D(t) \to D_0$ *as* $t \to \infty$*. We will call them* D_0-alternatives.

Natural invariance. There is no fixed unit of time relevant for this process. So it makes sense to require that the optimality criterion will not change if we use a different measuring unit to measure time.

If we know the dependence $D(t)$ in the original scale, how will this dependence look like in the new scale? If we replace the original measuring unit by a one which is λ times larger, then a moment t in the new scale corresponds to moment $\lambda \cdot t$ in the original scale. For example, if we replace second with minutes – which are 60 times larger – then 2 minutes in the new scale is equivalent to $2 \cdot 60 = 120$ seconds in the original scale.

In general, the value $D_{\text{new}}(t)$ corresponding to moment t in the new scale is thus equal to the value $D(\lambda \cdot t)$ when time is described in the original scale. Thus, $D_{\text{new}}(t) = D(\lambda \cdot t)$, and we arrive at the following definition.

Definition 2.

- *Let* D_0 *be a real number.*
- *For every* $\lambda > 0$ *and for every* D_0*-alternative* $D(t)$*, by a* λ-rescaling $R_\lambda(D)$*, we mean a* D_0*-alternative* $D_{\text{new}}(t) \stackrel{\text{def}}{=} D(\lambda \cdot t)$*.*
- *We say that the optimality criterion of the set of all* D_0*-alternatives is* scale-invariant *iff for every* $\lambda > 0$ *and for every two* D_0*-alternatives a and b, we have the following:*
  - *if* $a > b$*, then* $R_\lambda(a) > R_\lambda(b)$*, and*
  - *if* $a \sim b$*, then* $R_\lambda(a) \sim R_\lambda(b)$*.*

Main result. Now, we are ready to formulate our main result.

Proposition. *Let* D_0 *be a real number, let* $(<, \sim)$ *be a final scale-invariant optimality criterion on the set of all* D_0*-alternatives. Then, the optimal* D_0*-alternative has the form* $D(t) = D_0$ *for all* $t > 0$*.*

Comments.

- This result explains the empirical fact that the burst of physical activity indeed leads to the best health results.
- The proof of this result is contained in the following Proofs section. It is similar to the proof of a similar health-related result in [3].

3 Proof

Part 1. Let us first prove that for every final scale-invariant optimality criterion on the set of all D_0-alternatives, the optimal D_0-alternative $D_{\rm opt}$ is itself scale-invariant, i.e., $R_\lambda(D_{\rm opt}) = D_{\rm opt}$ for all $\lambda > 0$.

Indeed, by definition, the fact that $D_{\rm opt}$ is optimal means thatfor every D_0-alternative D, we have either $D_{\rm opt} > D$ or $D_{\rm opt} \sim D$.

This is true for every D_0-alternative D. Thus, this property holds for $R_{\lambda^{-1}}(D)$. So, we have either $D_{\rm opt} > R_{\lambda^{-1}}(D)$ or $D_{\rm opt} \sim R_{\lambda^{-1}}(D)$.

Since the optimality criterion is scale-invariant, we can conclude that:

- either $R_\lambda(D_{\rm opt}) > R_\lambda(R_{\lambda^{-1}}(D)) = D$
- or $R_\lambda(D_{\rm opt}) \sim R_\lambda(R_{\lambda^{-1}}(D)) = D$.

This is true for all D_0-alternatives D. Thus, by definition of optimality, this means that the D_0-alternative $R_\lambda(D_{\rm opt})$ is also optimal.

However, we assumed that our optimality criterion is final. This means that there is only one optimal D_0-alternative, and thus, $R_\lambda(D_{\rm opt}) = D_{\rm opt}$.

The statement is proven.

Part 2. Let us now use the result from Part 1 to prove that the optimal D_0-alternative has the desired form.

Indeed, the equality $R_\lambda(D_{\rm opt}) = D_{\rm opt}$ means that the values of these two functions coincide for all t. By definition of λ-rescaling, this means that for every t and every $\lambda > 0$, we have $D_{\rm opt}(\lambda \cdot t) = D_{\rm opt}(t)$.

In particular, by taking $\lambda = s > 0$ and $t = 1$, we conclude that for every $s > 0$, we have $D_{\rm opt}(s) = D_{\rm opt}(1)$. Thus, the function $D_{\rm opt}(s)$ attains the same constant value $D_{\rm opt}(1)$ for all $s > 0$. In particular, for $s \to \infty$, we have $D_{\rm opt}(s) \to D_{\rm opt}(1)$.

By definition of a D_0-alternative, this limit must be equal to D_0. Thus, $D_{\rm opt}(1) = D_0$ and therefore, for all $s > 0$, we have $D_{\rm opt}(s) = D_0$.

The proposition is proven.

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