From Discrete (Crisp) to Continuous (Fuzzy) to a Combination of Discrete and Continuous: Case of Partial Credit

Christian Servin, Olga Kosheleva, and Vladik Kreinovich

Abstract There is a problem with the way partial credit is currently assigned. For example, when in a test with 10 questions, a student is almost correct on each of them – getting 9 out of 10 points for each problem – the student's overall grade for the test is 90 out of 100 – which usually corresponds to A ("excellent"). This is not right: a student is classified as excellent, but in all 10 problems, the student's answers are wrong! We show that the solution to this problem is to combine the usual continuous ("fuzzy") approach to partial credit with the more traditional discrete (correct-wrong) grading – by providing, in effect, a bonus for the correct answer. A similar solution is proposed for the forgetting problem, when a student who showed perfect knowledge on all intermediate tests – but did not do so well on the final exam – may still get an A.

1 Formulation of the problem

Traditional discrete (crisp) way of grading in elementary school. When we were studying mathematics in elementary school, there was no such thing as partial credit, grading was very discrete (crisp):

- either your answer is correct, then you get a credit for this particular problem,
- or your answer is wrong, then you do not get any credit for this problem.

Christian Servin
Information Technology Systems Department, El Paso Community College (EPCC)
919 Hunter Dr., El Paso, TX 79915-1908, USA, e-mail: cservin1@epcc.edu

Olga Kosheleva
Department of Teacher Education, University of Texas at El Paso, 500 W. University
El Paso, Texas 79968, USA, e-mail: olgak@utep.edu

Vladik Kreinovich
Department of Computer Science, University of Texas at El Paso, 500 W. University
El Paso, Texas 79968, USA, e-mail: vladik@utep.edu

There was even a joke explaining this approach. A little big-city kid comes home and happily reports to his parents that he got a good grade for biology. When the teacher asked how many legs a chicken has, he said: “Three”. “But a chicken has only has to legs” – wondered his parents. “Yes”, the kid replied, “but everyone else said four.”

Partial credit – at higher educational levels. Later on, in high school and at the university level, usually, when the answer is wrong but the student performed several solution steps correctly, this student no longer gets 0 point, he/she gets a partial credit. In other words, at this educational level, grading is done by using a continuous scale – the grade can go from 0 all the way to 1 (or to 10, or to 100) depending on how close the solution strategy is to the correct one. In effect, this is a fuzzy scale; see, e.g., [1, 2, 3, 5, 6, 8].

Usually, unless some solution stages are more important, all these stages are valued the same. Thus, a student who correctly performed m out of n solution stages, gets partial credit m/n.

There is a problem with the usual way of assigning partial credit. Here is a realistic (and sometimes real) problem with the usual approach to partial credit. Suppose that students take a test that consist of 10 equally important and equally complex problems. Each problem is worth 10 points, so overall, a student can get up to 100 points. As usual, students who get 90 or more points get an A (“excellent”), students who get between 80 and 89 points get a B (“good”), students who get between 70 and 79 get a C (“satisfactory”), students who get between 60 and 69 get a D, and students who get smaller than 60 points get a failing grade F for this test.

Suppose that in each of these ten problem, the student has to follow 10 steps to reach a solution. Since each problem is worth 10 points, each step is worth 1 point. Suppose that in each of the ten problems, the student made one small mistake, i.e., performed one of the ten steps incorrectly. Then, according to the above-described usual arrangement, this student will get 9 out of 10 points for each of the ten problems. So, this student’s overall grade for the test will be 90/100, and this students gets an A for this test.

It may sound reasonable and fair, but let us look at it from the viewpoint of correct answers: all 10 answers are wrong, and still a student get an A grade – indicating excellence. Something is not right here. If this was happening in real life, and a person who needed to perform 10 computations gets all 10 wrong, this person will be probably fired – and he/she will definitely *not* be called an excellent worker.

This is a general problem of continuous grading. One may think that this problem can be corrected by assigning different numbers of points. Yes, changing the grading scheme may help to eliminate the problem in this particular case. However, one can see that a similar problem appears in any continuous grading scheme: if we assign the A letter grade to any numerical grade larger then or equal to some threshold $g_0 < 1$, then a minor mistake in each of N problems – that makes it worth g_0/N points – will lead to an overall A, while all N answers will still be wrong.

What we do in this paper. In this paper, we show how to avoid such a situation.

2 How to solve this problem

Our main idea. Since a fully continuous grading scheme – where a grade for each problem can, in principle, take any value between 0 and 1 – leads to an undesirable situation, a natural idea is to add some discreteness to grading. Specifically, since the problem is caused by the fact that grades for an incorrect (but "almost" correct) answer can be close to 1, a natural idea is to only provide full grade of 1 when the solution is correct – and when even a small mistake is made, assign a value smaller than or equal to some smaller threshold $t_0 < 1$.

With this arrangement, we get a grading scheme that combines discrete and continuous. Namely, possible grades for each problem can be:

- either any number from 0 to t_0 – which corresponds to the continuous part,
- or the value 1 – which corresponds to the discrete part.

But is this fair? At first glance, this may not sound fair to a student – a tiny mistake immediately drops his/her grade from 1 to t_0. However, as we have mentioned, it makes perfect sense in real life. For example, when a construction company promises to build a building by a certain date, there is often a bonus for doing it on time – a bonus that will *not* be paid if the construction is only a few days late.

Maybe this is a way to make it sound fair to students: instead of calling it a penalty, call the difference $1-t_0$ a *bonus* for the correct answer.

Comment. Such a bonus-type description of grading is not unheard of. For example, when one of us (VK) was a student in St. Petersburg University, we had oral final exams. A student would randomly select a card with three questions. If the student answers all three questions correctly, he/she get a guaranteed B (to be more precise, a grade of 4, a Russian equivalent of B) – and of course, if the student fails to answer some of these three questions, the grade is lower. To raise the B grade to an A, the student has to answer all additional questions that the instructors ask. If the student does not answer these additional questions, the B grade remains. This way, B is not viewed as a penalty. Instead, A is viewed as a bonus for perfect knowledge.

What threshold should we choose. We do not want to give a very low grade to a student whose knowledge of the material is almost perfect. On the other hand, if we make the threshold too close to 1, students will not notice the difference and will not strive for having correct answers – which is the main point of the proposed change in the grading policy. So, it is reasonable to select a threshold t_0 for which the difference $1-t_0$ is the smallest still noticeable by an average student.

To find such a difference, we can use the well known seven-plus-minus two law; see, e.g., [4, 7]. According to this law, people usually classify objects into 7 ± 2 categories – between 5 and 9, on average 7. From this viewpoint, for an average person, the smallest noticeable difference is 1/7 – which is between 14 and 15%. So, it makes sense to select $1-t_0 = 0.15$, i.e., $t_0 = 0.85$.

Thus, we arrive at the following recommendation.

Resulting recommendation. For a problem worth 10 points, we should give all 10 points only when the resulting answer is absolutely correct. If there is at least a minor mistake, then immediately the grade goes down to 8.5 points – minus whatever points should be deducted for the student's mistakes.

This grading strategy is equivalent to making each problem worth 8.5 points – with the promise of additional 1.5 bonus points for an absolutely correct answer.

This will avoid undesirable situations like we have described above, when a student gets a A for the test in which all answers are incorrect – almost correct but still incorrect. Indeed, in this case, when all the answers are wrong, in the new grading system, the student will get at most 85 points for the test – which corresponds to B, not to A.

3 Additional problem and related idea

Additional problem. An additional problem with the usual grading scheme is related to students' forgetting. Ideally, the overall grade for the class should reflect the level of the student's knowledge at the end of the class – and further in the future. However, in reality, students forget. As a result of this forgetting, some students who got a perfect A grade on all three midterm exams perform much worse on the final exam – sometimes even on a C level. However, in situations when the final exam is worth only 30 points, losing 25 points – by getting 75 on the final exam – simply means losing $0.3 \cdot 25 = 7.5$ points towards the class grade. So, when all the previous grades were perfect As, the student still gets 92.5 points for the class and thus, an A grade for the class. This is not just a hypothetical situation, it actually happened with one of our students.

This is not right. By the end of the class, the student only shows a satisfactory level of knowledge, but by giving this student an A grade we falsely certify that this student's level of knowledge is excellent. How can we avoid this problem?

Our proposal. To avoid this problem, we need to make sure that the student's grade for the class cannot be significantly higher that this student's grade on the final exam – the grade that reflects the student's level of knowledge at the end of the course. Since, as we have argued, the significant difference means at least 14 points, the proposal is as follows: if grade on the final exam is at least 15 points lower than the overall grade computed the usual way, then the grade for the final exam become an official grade for the class.

If we use the proposed scheme in the above case, when the difference $92.5 - 75 = 17.5$ is larger than 15, the student would get, for the class, not an A, but a C – the letter grade corresponding to this student's 75 points on the final exam.

Acknowledgments

This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), HRD-1834620 and HRD-2034030 (CAHSI Includes), EAR-2225395 (Center for Collective Impact in Earthquake Science C-CIES), and by the AT&T Fellowship in Information Technology.

It was also supported by a grant from the Hungarian National Research, Development and Innovation Office (NRDI).

References

1. R. Belohlavek, J. W. Dauben, and G. J. Klir, *Fuzzy Logic and Mathematics: A Historical Perspective*, Oxford University Press, New York, 2017.
2. G. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic*, Prentice Hall, Upper Saddle River, New Jersey, 1995.
3. J. M. Mendel, *Explainable Uncertain Rule-Based Fuzzy Systems*, Springer, Cham, Switzerland, 2024.
4. G. A. Miller, “The magical number seven plus or minus two: some limits on our capacity for processing information”, *Psychological Review*, 1956, Vol. 63, No. 2, pp. 81–97.
5. H. T. Nguyen, C. L. Walker, and E. A. Walker, *A First Course in Fuzzy Logic*, Chapman and Hall/CRC, Boca Raton, Florida, 2019.
6. V. Novák, I. Perfilieva, and J. Močkoř, *Mathematical Principles of Fuzzy Logic*, Kluwer, Boston, Dordrecht, 1999.
7. S. K. Reed, *Cognition: Theories and Application*, SAGE Publications, Thousand Oaks, California, 2022.
8. L. A. Zadeh, “Fuzzy sets”, *Information and Control*, 1965, Vol. 8, pp. 338–353.