When Is a Safe Freeway Lane Change Possible? Relativity Principle Explains an Empirical Formula

Ehsan Yahyazadeh Rineh, Ruey Long Cheu,
Olga Kosheleva[0000−0003−2587−4209], and Vladik Kreinovich[0000−0002−1244−1650]

University of Texas at El Paso, 500 W. University, El Paso, TX 79968, USA
eyahyazadeh@miners.utep.edu, {rcheu,olgak,vladik}@utep.edu

Abstract. Lane change is the most safety-critical part of driving on a freeway. Before we start the lane changing maneuver, we need to make sure that on the neighboring lane, there is a sufficient gap between the two cars, so that we can change safely. There exist algorithm for checking for this, including fuzzy-based ones. Most of these algorithm are based on the same empirical formula, according to which the smallest size of the safe gap exponentially depends on the vehicles' speed. To guarantee safety, it is desirable to make sure that the related algorithms are theoretically justified. For lane change techniques, this means, in particular, that all empirical formulas used in this algorithm's design have a theoretical justification. In this paper, we show that the relativity principle from physics indeed provides such a theoretical justification for the empirical exponential dependence of the smallest size of the safe gap on the vehicles' speed.

Keywords: Relativity principle · Freeway lane change · Self-driving cars

1 Formulation of the problem

Practical problem. Lane change is the most safety-critical part of freeway driving – whether we deal with human driving or with self-driving cars. Before we start a lane change, it is important to make sure that this change is, in the current driving situation, indeed possible, i.e., whether a gap between the cars of the next lane is sufficient for a safe lane change maneuver.

How this is done. Several techniques have been proposed for this determination. At present, the most effective is a fuzzy-based system described in [9, 10] – that improves upon the previously proposed fuzzy-based system [2].

All current systems use the same empirical formula [3, 4, 6, 7], according to which the smallest size f of the safe gap exponentially depends on the freeway speed v:

$$d = a \cdot \exp(b \cdot v), \tag{1}$$

for some coefficients a and b that, in general, depend on the specific freeway and specific road conditions.

This formula is applicable in usual situations, when all the cars on all the lanes move at the same speed – the maximum speed allowed on this road segment. There are also versions of this formula that can be used for more complex situations, when some of the lanes are conjested and thus slower than others; these version also use exponential dependence.

Remaining problem. Because of the safety-critical aspect of this practical problem, it is desirable to make sure that all the foundations for the used algorithms are well-justified – and not just come from a purely computational extrapolation of a few empirical data. Since one of the main foundations for the existing lane-change techniques is the above exponential fomula (1), it is therefore desirable to provide, if possible, a theoretical foundation for this formula – or, if such a foundation is not possible, to provide a alternative formula that does have a theoretical foundation.

What we do in this paper. In this paper, we provide a theoretical foundation for the empirical formula (1). Somewhat unexpectedly, this foundation is based on the relativity principle – something that is, at first glance, as far away from our practical problem as possible.

2 Analysis of the problem and the resulting explanation

Relativity principle: a brief reminder. To most readers, the relativity principle is associated with Einstein's Relativity Theory, a theory that is important for describing objects moving with the speeds close to the speed of light. At first glance, it has nothing to do with a car on a freeway whose speed – around 100 km/h – is much much smaller than the speed of light, which is 300,000 km/sec.

However, the relativity principle is not limited to close-to-speed-of-light motions. This principle was known many centuries before Einstein. It has been first formulated by Galileo who noticed that for a passenger in a windowless ship cabin, when the sea is smooth, it is not possible to tell whether the ship is moving or not – all the physical processes remain the same; see, e.g., [5, 8].

This principle is perfectly valid for most processes described by Newton's mechanics. In Newton's mechanics, the only process that violated this principle is light. For all other processes, when we replace the stationary observer with a moving one, the observed speeds change. For example, when we are on a platform at a train station, we can see that the train is moving, but when we are inside the train, we do not feel this motion. In contrast to that, the famous Michelson-Morley experiments showed that the speed of light remains the same, whether we measure it from the viewpoint of a stationary observer or from the viewpoint of a moving observer – which clearly showed that for speeds close to the speed of light, Newtonian physics is not applicable. To come up with a reasonable adjustment of Newton's physics, Einstein decided to keep as many fundamental properties as possible – including relativity principle, and the resulting theory indeed turned out to be a great success.

Comment. Of course, in our practical situation, we are interested not in close-to-speed-of-light effects of relativity principle, but in its low-speed applications.

How is relativity principle relevant for freeway driving. The relation between the freeway driving and relativity principle is straightforward. When you drive on a street, you realize that you are driving, because your position with respect to the houses changes. However, when you drive on a reasonably straight segment of a multi-lane freeway when it is separated by a high barrier from the surrounding city – to decrease noise – and from the cars driving in the opposite direction – to avoid collisions, the picture that you see remains the same, nothing changes. All the cars travel with the same speed – the maximum speed allowed on this road segment. So, when you glance to the left and to the right, all the cars in your lane and on the neighboring lanes seem to freeze in the same position: nothing moves. Of course, your engine hums, and the speedometer shows that you are going at high speed – but without that, you would have had a full illusion that no one actually moves. This is not just a hypothetical illusion: in the movies, scenes that are supposed to be in a moving car are often shot when the car is actually standing still, but in the resulting movie, you get a full impression of a moving car.

So, whether all the cars stay still or all of them drive with the same speed v – without irrelevant factors like noise or speedometer, we would not have noticed the difference, the physics is the same.

Let us use this relation to analyze how d depends on v. With respect to our practical problem, the relativity principle means, in particular, that the desired relation $d = f(v)$ should not change if instead of the stationary coordinate system, we use a coordinate system that moves with some velocity v_0. When we change to such a system, all velocity values change from v to $v' = v + v_0$. In these terms, invariance means that if we have $d = f(v)$ then for $v' = v + v_0$ we should have $d' = f(v')$.

Notice that here we used d' and not the original value d, since what remains the same is physics, and not necessarily the numerical values of the corresponding physical quantities. For example, the whole idea of a wind tunnel – in which smaller-size models of airplanes were tested – is that many aerodynamic processes are the same at different scales – but on small scale, not only the sizes change, we also need to corresponding re-scale time intervals, wind speeds, etc.

So, in general, instead of the original formula $d = f(v)$, we should have a formula $d' = f(v')$ in which d' may be a re-scaled length, i.e., length described in different units. If we change the unit for measuring length from the original one to the one that is c times smaller, then instead of the original numerical value d we get the new numerical values $d' = c \cdot d$. For example, when we replace meters with centimeters, then 1.7 m becomes $1.7 \cdot 100 = 170$ cm. So, we arrive at the following particular case of the relativity principle:

$$\text{for each } v_0\text{, for all } v \text{ and } d\text{, if } d = f(v)\text{, then } d' = f(v'),$$
$$\text{where } v' = v + v_0 \text{ and } d' = c \cdot d\text{, for some } c \text{ depending on } v_0.$$

This property explains the exponential dependence. Substituting $d' = c(v_0) \cdot d$ and $v' = v + v_0$ into the formula $d' = f(v')$, we conclude that $f(v+v_0) = c(v_0) \cdot d$. Since $d = f(v)$, we thus conclude that

$$f(v + v_0) = c(v_0) \cdot f(v). \tag{2}$$

In principle, there are some weird discontinuous functions $f(v)$ that satisfy this equation for some $c(v_0)$. However, in our practical case, it makes sense to assume that small changes in the velocity v should lead to small changes in the safe gap size d, i.e., in mathematical terms, that the function $f(v)$ is continuous. In this case, all the solutions to the functional equation (2) are known: they are exactly the exponential dependencies (1); see, e.g., [1].

So, we have indeed arrived at a theoretical explanation for the empirical formula (1).

Comment. The derivation of (1) from (2) is easy if we also assume that the dependence $f(v)$ is smooth (differentiable). Indeed, in this case, the function $c(v_0) = f(v + v_0)/f(v)$ is also differentiable, as the ratio of two differentiable functions. Thus, we can differentiate both sides of the formula (2) with respect to v_0, and then take $c_0 = 0$. This way, we get $f'(v) = c_0 \cdot f(v)$, where $f'(v)$ means derivative and $c_0 \stackrel{\text{def}}{=} c'(0)$. So, $df/dv = c_0 \cdot f$. If we divide both sides by f and multiply both sides by dv, we get an equivalent equality in which variables are separated: $df/f = c_0 \cdot dv$. Integrating both sides, we get $\ln(f) = c_0 \cdot v + c_1$, where c_1 is the integration constant. Applying $\exp(z)$ to both sides, we get the desired expression

$$f(v) = \exp(c_0 \cdot v + c_1) = \exp(a_0 \cdot v) \cdot \exp(c_1) = a \cdot \exp(b \cdot v),$$

where $a \stackrel{\text{def}}{=} \exp(c_1)$ and $b \stackrel{\text{def}}{=} c_0$.

Acknowledgments. This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), HRD-1834620 and HRD-2034030 (CAHSI Includes), EAR-2225395 (Center for Collective Impact in Earthquake Science C-CIES), and by the AT&T Fellowship in Information Technology.

It was also supported by a grant from the Hungarian National Research, Development and Innovation Office (NRDI), by the Institute for Risk and Reliability, Leibniz Universitaet Hannover, Germany, and by the European Union under the project ROBOPROX (No. CZ.02.01.01/00/22 008/0004590).

The authors are thankful to the anonymous referees for valuable suggestions.

Disclosure of Interests. The authors have no competing interests to declare that are relevant to the content of this article.

References

1. Aczél, Dhombres, J.: Functional Equations in Several Variables, Cambridge University Press (2008)

2. Balal, E., Cheu, R.L.: Modeling lane changing decisions: comparative evaluation of fuzzy inference system, support vector machine and multilayer feed-forward neural network. Neural Network World **28**(3), 361–378 (2018)
3. Caliper: TransModeler User's Guide. Version 2.6, Caliper Corp., Newton, Massachusetts (2011)
4. Choudhury, C.: Modeling Driving Decisions with Latent Plans. Ph.D. Thesis, Massachusetts Institute of Technology (2007)
5. Feynman, R., Leighton, R., Sands, M.: The Feynman Lectures on Physics, Addison Wesley, Boston, Massachusetts (2005)
6. Gipps, P. A.: A model for the structure of lane-changing decisions. Transportation Research **20B**, 403–414 (1986)
7. Nagel, K., Schrekenberg, M.: A cellular automaton model for freeway traffic. Journal de Physique I **2**(12), 2221–2229 (1992)
8. Thorne, K. S., Blandford, R. D.: Modern Classical Physics: Optics, Fluids, Plasmas, Elasticity, Relativity, and Statistical Physics, Princeton University Press, Princeton, New Jersey (2021)
9. Yahyazadeh Rineh, E.: Enhancing Discretionary Lane Changing Models Using Fuzzy Inference Systems, Ph.D. Dissertation, Department of Civil Engineering, University of Texas at El Paso (2025)
10. Yahyazadeh Rineh, E., Cheu, R. L.: Fuzzy inference systems for dscretionary lane changing decisions: model improvement and research challenges. International Journal of Transportation Science and Technology **17**, 312–327 (2025) https://doi.org/10.1016/j.ijtst.2024.05.001