Why Signed Letter Grades Are Mostly Used for Graduate Classes

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Abstract Student's performance is described by a grade. Grades have a large degree of subjectivity. From this viewpoint, they are closer to fuzzy expert estimates than to well-calibrated measurement results. In the US educational system, there are two versions of grading. The most widely used is a 5-level scheme, with letter grades A (Excellent), B (Good), C (Satisfactory), D (Below Satisfactory), and F (Failing). Some places also provide signed letter grade like A− or B+, to provide a more nuanced grading: minus means that the level of knowledge is slightly worse than the average level of knowledge corresponding to this letter grade, and plus means slightly better. Statistics shows that signed letter grades are more typical in graduate classes and less typical in undergraduate classes.In this paper, we explain what this is so.

1 Formulation of the problem

Letter grades and signed letter grades: a brief reminder. In the US education system, the overall grade for a class – usually, from 0 to 100 – is formed by grades for several assignments and tests. Based on this numerical grade, the student gets a letter grade, and this letter grade is the only thing that is recorded in the student's transcript. There are 5 letter grades:

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- A ("excellent") that usually corresponds to numerical grades between 90 and 100;
- B ("good") that usually corresponds to numerical grades between 80 and 89;
- C ("satisfactory") that usually corresponds to numerical grades between 70 and 79;
- D ("below satisfactory", passing grade for some classes) that usually corresponds to numerical grades between 60 and 69; and
- F ("failing") that usually corresponds to numerical grades below 60.

This leads to the following ordering of grades, from the worst to the best:

$$\mathrm{F} < \mathrm{D} < \mathrm{C} < \mathrm{B} < \mathrm{A}.$$

In addition, some universities also use *signed letter grades*, i.e., letters grades with added plus or minus signs. In this arrangement, the range corresponding to a letter grade is divided into three parts:

- the upper part – describing slightly better knowledge level – corresponds to the plus grade,
- the middle part is reserved for original unsigned grade, and
- the lower part – describing slightly worse knowledge level – corresponds to the minus grade.

For example, among students whose knowledge corresponds to the A level, A+ is the best option, unsigned A is next best, and A$-$ is the worst of the three A-level options:

$$\mathrm{A}- < \mathrm{A} < \mathrm{A}+.$$

In general, if we add signed versions of all letter grades, we get the following ordering of possible signed letter grades:

$$\mathrm{F}- < \mathrm{F} < \mathrm{F}+ < \mathrm{D}- < \mathrm{D} < \mathrm{D}+ < \mathrm{C}- < \mathrm{C} < \mathrm{C}+ <$$

$$\mathrm{B}- < \mathrm{B} < \mathrm{B}+ < \mathrm{A}- < \mathrm{A} < \mathrm{A}+.$$

Usually, not all possible signed letter grades are used: e.g., a failure is a failure, so F+ makes not much sense.

Comments. US is not the only country that uses signed grades. For example, in Russia, where OK and VK are from, we had a smilar (but not exactly) system of grades that went into a transcript:

- 5 meaning "excellent",
- 4 meaning "good",
- 3 meaning "satisfactory",
- 2 meaning "failing", and
- 1 meaning "really bad".

In addition, professors could add signs whenever they wanted; some professors did this sometimes, some never. There was even a possibility to give two signs. For example, 5++ would mean exceptionally excellent, while $3--$ would mean the

very low level of satisfactory: one more minor mistake and it would be failing. Not all signed grades were used: e.g., some professors would add minus or minuses to a failing grade (1 or 2), but it made no sense to add plus signs to the failing grade.

How this is all related to fuzzy. Grades are always strongly subjective. How many points to give to a partially correct problem? Do we consider a small mistake a typo – when a student shows, in other cases, that he/she knows this material – or do we count it as a true mistake? All this has a lot of built-in subjectivity. As a result, these grades are not so much an objective measurement results, but, to a large extent, an expert estimate, i.e., in terms of fuzzy techniques (see, e.g., [1, 2, 3, 5, 6, 8]) largely a fuzzy degree.

Related problem: shall we use signed grades or not? At the University of Texas at El Paso (UTEP) where two of us (Olga Kosheleva and Vladik Kreinovich) work and where Chistian Servin studied – and got his degrees – we normally do not use signed letter grades. Several faculty members – mostly those who graduated from schools that use letter grades – proposed to introduce letter grades. At that time, we had two different elected faculty bodies that decided on academic policies:

- the Faculty Senate that decided on academic policies related to undergraduate education, and
- the Graduate Council that decided on academic policies related to graduate programs.

Each of these two bodies had a discussion and a vote about the issue of signed letter grades. The resulting vote was split:

- in the Faculty Senate, most elected representatives voted against using signed letter grades in undergraduate classes, while
- in the Graduate Council, most elected representatives voted for using signed letter grades in graduate classes.

It turned out later on that this was not a peculiar feature of our university: many more schools use signed letter grade in graduate classes than in undergraduate classes. A natural question is: why?

What we do in this paper. In this paper, we provide a possible explanation for this phenomenon.

2 Our explanation

What is the purpose of the letter grades? In order to provide an explanation, let us recall what is the purpose of the grades in the first place.

This question is easy to answer for numerical grades: the numerical grade, in spite of all its level of subjectivity, reflects the level of student knowledge. After receiving this grade – and these grades are officially announced to students – each

student knows where he/she stands. This does not change whether we use unsigned or signed letter grades.

In the US system, these numerical grades are only shown to the student. They are *not* shared with anyone, they are not kept as records anywhere – sharing these numerical grades with someone different from this particular student is prohibited by law.

The only thing that becomes different when signed letter grades are introduced is what goes into the transcript – and the transcript is used when a student applies for a job, when the student applies for further studies, etc. From this viewpoint, the only objective of introducing letter grades is to provide future employers a better understanding of the student's level of knowledge in this class.

This, in turn, encourages students to study better:

- if a student is currently at a lower B level, with expected 80 points out of 100, then this student needs a lot of effort to make it an A, and if he/she does not have time for such a big effort, this student will stay at the same B level;
- on the other hand, if we have a signed letter grade system, the student with expected 80 points (that correspond to B$-$) has a chance to get a better grade (e.g., unsigned B) with a much more reasonable effort, so more students will select to study more – and thus, the average level of knowledge will increase.

Adding signed letter grades provides a higher level of granularity. So, the question of whether to use signed letter grades or not become a question of how much granularity is needed. If we decide to use too few levels, we will not give a good picture. However, if we provide too many details, too many levels, they may not be noticeable neither by a student, so why add them? Thus, to proceed further, we need to analyze which level of granulation should we use.

Which level of granularity should we use? According to the well-known seven-plus-minus-two law (see, e.g., [4, 7]), a person meaningfully distinguishes between from 5 to 9 levels, with 7 levels for most. Thus, to be as detailed as possible – and at the same time to make all these levels clearly distinguishable for the majority – it makes sense to provide 7 levels of granularity.

Let us apply all this to the question about letter grades. In the undergraduate level, if we only use letter grades, we have 5 options: A, B, C, D, and F, which is within the range of optimality. This number has an additional advantage that it leads to distinguishable grades for *all* the students – even for those who usually consider 5 levels. If we add all possible signed grades, we will get too many levels, which will make the difference between the neighboring levels indistinguishable both for the students and the employers – and this defeats the purpose of assigning letter grades. This explains why signed letter grades are rarely used for undergraduate classes.

What is different in the graduate school is that for a graduate class, already C is a failing grade – by definition, graduate students should have good, above-average knowledge. From this viewpoint, it does not make sense to consider signed versions of the C grade – and probably does not even makes sense to consider yet other failing grades D and F. So, if we only use unsigned letter grades, we get only three

levels: A, B, and C. Clearly, we can provide a more detailed granulation that will still make all the levels distinguishable. If we add signs for positive grades A and B, we will have exactly 7 grades:

$$\mathrm{C} < \mathrm{B}- < \mathrm{B} < \mathrm{B}+ < \mathrm{A}- < \mathrm{A} < \mathrm{A}+,$$

i.e., the optimal number of grades. This explains why signed letter grades are often used for graduate classes.

Comment. These two conclusions do not mean that signed letter grades should never be used in undergraduate classes. For example, in highly selective prestigious schools, students are expected to have only As and Bs. For such schools, C for a class is a shame – practically a failure. In this case, it also makes sense to add signed letter grades to supplement the three real grades A, B, and C.

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