Is Earth's tilt a resonance?

Luis J. Franco, Andres Soto, Jose R. Chaidez, and Vladik Kreinovich

Abstract The reason why we have seasons is that the Earth's rotation axis is tilted. An interesting fact that the sine of the tilt is almost exactly 2/5. This fact leads to a natural question: is this an indication of a physical resonance – or is this a random coincidence? In this paper, we show that this is an accidental coincidence.

1 Formulation of the Problem

Earth tilt: its importance and its value. The orbits of all the planets of the Solar system lie in the same plane. The rotation axes of all the planets are approximately orthogonal to this plane. However, each actual rotation axis is somewhat tilted in related to this orthogonal direction. This tilt is the reason why, in most places on Earth, we have four seasons:

- Winter, when this location is tilted away from the Sun and thus, gets the smallest amount of the Solar radiation energy;
- Spring, when the amount of Solar radiation increases;
- Summer, when this located is tilted towards the Sun and thus, gets the largest amount of the Solar radional energy; and
- Fall, when the amount of Solar radiation decreases.

Specifically, the Earth's tilt is 23.4°.

An interesting fact about the Earth's tilt. Sines and cosines of the angles appear in many formulas describing a system's dynamics. Interesting, the sine of the Earth's tilt angle is almost equal to 2/5:

Luis J. Franco, Andres Soto, Jose R. Chaidez, and Vladik Kreinovich
Department of Computer Science, University of Texas at El Paso, 500 W. University
El Paso, Texas 79968, USA, e-mail: ljfranco@miners.utep.edu, asoto47@miners.utep.edu, jrchaidez@miners.utep.edu, vladik@utep.edu

$$\sin(23.4^\circ) = 0.397 \approx 0.4 = \frac{2}{5}.$$

Why this is an interesting fact – and the resulting question. This fact is interesting because usually, unit-less combinations of physical quantities are generic real numbers, and it is very rare that their value is close to a fraction of two small integers. In most cases when we have such an exceptional ratio, this value is caused by a *resonance* (see, e.g., [1, 4]) – a known physical phenomenon according to which two nearby pendulums (or other periodic processes) eventually get synchronized:

- if their frequencies were close to each other, they become equal;
- if one of the frequencies was almost twice larger than the second one, it becomes exactly twice larger, etc.

For example, in the past, the period of the Moon's rotation around the Earth was close to – but different from – the period of its rotation around its axis. However, with time, these periods became the same, so now, from the Earth, we can only see one side of the Moon.

Resonance is not only about lifeless physical bodies, we can experience it ourselves:

- when the music is faster – e.g., when we hear dance music – our hearts start beating faster, and we become more active;
- on the other hand, when the music is slower – e.g., when we hear a lullaby – our hearts start beating slower, and we become less active – and may even fall asleep.

So, the fact that we have a value close to 2/5 leads to a natural question: is this a resonance – or is it a random coincidence?

What we do in this paper. In this paper, we try to answer this question. Our answer is: it is most probably not a resonance.

2 Formal Description of the Problem and the Resulting Solution

Towards a formal description of the problem. In statistics, practitioners usually conclude that an event is not a random coincidence if the probability of this event happening randomly is smaller than a certain threshold. Usually this threshold is 0.05 but sometimes, a smaller threshold is selected; see, e.g., [3].

To use this criterion, we need:

- to formally describe the corresponding random situation – i.e., to come up with the appropriate probability distribution on the set of possible outcomes, and
- to formally describe what we mean by the event.

Let us do it.

Let us formally describe the random situation. The outcome is the sine of the tilt angle. In principle, this angle can take any value between 0 and 90 degrees. So, the resulting sine can, in principle, take any value between 0 and 1.

We have no a priori reason to think that some values from the interval $[0,1]$ are more probable than others. It is therefore reasonable to assume that all the values from the interval $[0,1]$ are equally probable, i.e., that we have a uniform distribution on this interval, in which the probability for a random value to belong to some subset – e.g., the union of disjoint intervals – is equal to the overall length of this subset (i.e., to the sum of the lengths of its component intervals).

Comment. The argument about having equal probabilities – if there is no reason to believe that one of the probabilities is larger – goes back to Laplace, one of the founders of probability theory. It is known as *Laplace Indeterminacy Principle*; see, e.g., [2].

Let us formally describe the event. The event is that the sine of the tilt is 0.003-close to a ratio of two small natural numbers – namely, of natural numbers not exceeding 5.

So, what is the probability of this event? Let us list all the ratios of two natural numbers each of which is smaller than or equal to 5 for which the ratio is located between 0 and 1:

- with the denominator 1, we have 0/1 = 0 and 1/1 = 1;
- with the denominator 2, we have 0/2 = 0, 1/2, and 2/2 =1;
- with the denominator 3, we have 0/3 = 0, 1/3, 2/3, and 3/3 = 1;
- with the denominator 4, we have 0/4 = 0, 1/4, 2/4 = 1/2, 3/4, and 4/4 = 1;
- with the denominator 5, we have 0/5 = 0, 1/5, 2/5, 3/5, 4/5, and 5/5 =1.

If we sort all these ratios in increasing order, we get the following list:

$$0 < 1/5 = 0.2 < 1/4 = 0.25 < 1/3 = 0.333\ldots < 2/5 = 0.4 < 1/2 =$$

$$0.5 < 3/5 = 0.5 < 2/3 = 0.888\ldots < 3/4 = 0.75 < 4/5 = 0.8 < 1.$$

For each of the ratios 0 and 1, all possible 0.003-close real numbers form an interval of width 0.003. These intervals are, correspondingly, $[0,0.003]$ and $[1-0.003,1] = [0.997,1]$. For each of the other ratios r, 0.003-close values form an interval $[r-0.003, r+0.003]$ of width 0.006.

It is easy to see that the difference between every two consecutive numbers in the above list is larger than $2\cdot 0.003 = 0.006$. Thus, all the corresponding intervals

$$[0,0.003], [r-0.003, r+0.003], \text{ and } [1-0.003,1] = [0.997,1]$$

are disjoint: the intersection of any two of them is empty. Thus, the overall width of the union of these intervals is simply the sum of the widths of these intervals. We have nine intervals of width 0.006 each and two intervals of width 0.003 each. Thus, the overall width of the union of all these intervals is equal to $9\cdot 0.006 + 2\cdot 0.003 = 0.06$.

So, for the uniform distribution of the interval [0,1], the probability that a random value is within one of these intervals is 0.06.

Conclusion: this is not a resonance, it is an accidental coincidence. The probability that a random value is 0.003-close to one of the ratios of small natural numbers is equal to 0.06 – and is, thus, larger than the usual threshold of 0.05, the threshold that is normally used to reject the assumption that the value is random. And the value 0.06 is definitely larger than all possible smaller-than-0.05 thresholds that are sometimes used.

Thus, we cannot conclude that the Earth's tilt is a resonance.

Comment. The probability becomes the Earth's tilt is a random coincidence becomes even larger if we take into account that the Earth is just one of the eight planets, and that for no other planets, the sine of the tilt is 0.003-close to a ratio of two small natural numbers. As we have mentioned, for each planet, the probability that the random value is not that close is equal to $1-0.06=0.94$. Thus, the probability that the sine is not that close for all 8 planets is equal to $0.94^8 \approx 0.61$. So, the probability that for at least one planet the ratio is that close is equal to $1-0.61=0.39$ which is much larger that the 0.05 threshold.

Acknowledgments

This work was supported by the AT&T Fellowship in Information Technology, by the Institute for Risk and Reliability, Leibniz Universitaet Hannover, Germany, by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) Focus Program SPP 100+ 2388, Grant Nr. 501624329, by the European Union under the project ROBOPROX (No. CZ.02.01.01/00/22 008/0004590), by the Center of Excellence in Econometrics, Faculty of Economics, Chiang Mai University, Thailand, by the Ho Chi Minh City University of Banking, Vietnam, and by Thang Long University, Hanoi, Vietnam.

References

1. R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Addison Wesley, Boston, Massachusetts, 2005.
2. E. T. Jaynes and G. L. Bretthorst, *Probability Theory: The Logic of Science*, Cambridge University Press, Cambridge, UK, 2003.
3. D. J. Sheskin, *Handbook of Parametric and Nonparametric Statistical Procedures*, Chapman and Hall/CRC, Boca Raton, Florida, 2011.
4. K. S. Thorne and R. D. Blandford, *Modern Classical Physics: Optics, Fluids, Plasmas, Elasticity, Relativity, and Statistical Physics*, Princeton University Press, Princeton, New Jersey, 2021.