Why Dolphins Age Slower in Small Social Groups and Age Faster in Larger Groups: A Possible Explanation Based on Decision Theory

Dang Pham, Javier Molina, Olga Kosheleva, and Vladik Kreinovich

Abstract A recent paper has shown that dolphins living in small social groups age slower than solitary dolphins, but dolphins living in larger social groups age faster than solitary dolphins. That paper provided explanations based, to some extent, on the specifics of the social life, with its mutual help and – at the same time – stressful conflicts. The current paper intends to provide a more general explanation of the newly observed phenomena, an explanation based on the ideas of the general decision theory.

1 Formulation of the problem

A recent paper [8] has shown that:

- dolphins living in small social groups age slower than solitary dolphins, but
- dolphins living in larger social groups age faster than solitary dolphins.

That paper provided explanations based, to some extent, on the specifics of the social life, with its mutual help and – at the same time – stressful conflicts. The current paper intends to provide a more general explanation of the newly observed phenomena, an explanation based on the ideas of the general decision theory.

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2 Why decision theory?

In general, aging is related to the quality of life: people and animals living in better conditions usually age slower, while people and animals living in worse conditions age faster.

How can we gauge the quality of conditions? The need to gauge – and compare – different alternatives comes from the need to help decision makers decide which options are better. For this purpose, a special *decision theory* was designed; see, e,g., [5, 6, 10, 11, 12, 13, 14]. In this theory, decision maker's preferences are described by a special function called *utility*: the larger the utility, the more preferable the alternative.

In decision-theory terms, the observations from [8] can be described as follows:

- the utility of persons living in small social groups is larger than the utility of solitary persons, while
- the utility of persons living in larger social groups is smaller than the utility of solitary persons.

To explain these observations, let us recall what utility theory says about the effect of social groups on utility.

3 What decision theory says about the effect of social groups on utility

The main idea of this effect is that the utility of each person is determined not only by the objective circumstances of this person, but also by utilities of others from his/her social group. This dependence may be complicated – as most dependencies in the world.

It is reasonable to assume that this dependence is smooth – in the sense that a small change of one person's utility will lead to a similarly small (and even smaller) change in utilities of others. Smooth dependencies can be, in general, expanded in Taylor series, and to get a reasonable approximate description, we can limit ourselves to the first few terms in this expansion. This is a usual way to approach complex phenomena in physics and other disciplines. In physics, often, we get a reasonable approximation already when we consider the very first – linear – terms in the Taylor expansion, i.e., when we consider a linear formula for the desired dependence; see, e.g., [4, 17].

This idea leads to the following linear formula for the dependence of the utility u_i of the i-th person on the utilities of others:

$$u_i = u_i^{(0)} + \sum_{j \neq i} \alpha_{i,j} \cdot u_j, \tag{1}$$

for some coefficients $\alpha_{i,j}$, where $u_i^{(0)}$ is the utility caused only by external circumstances (without the effect of the social group).

4 Specifics of volunteer social groups

In general, the coefficients $\alpha_{i,j}$ can be both:

- positive – when the i-th person attitude to the j-th person is positive, and
- negative – when the i-th person attitude to the j-th person is negative.

In general, in a human society, we have both positive attitudes and negative attitudes – hatred, jealousy, even wars are still in existence.

However, in the volunteer social group – of which the dolphins' social group is a good example – the attitude is positive – or at largely positive: if for some individual, this attitude was largely negative, why would this individual join this group?

5 What happens in such a volunteer group: a first-approximation description

In the first approximation, let us replace individual values $u_i^{(0)}$, u_i, and $\alpha_{i,j}$ with their average values $u^{(0)}$, u, and α. In this case, the formula (1) takes the following form:

$$u = u^{(0)} + \sum_{j \neq i} \alpha \cdot u, \tag{2}$$

i.e., equivalently, the form

$$u = u^{(0)} + (n-1) \cdot \alpha \cdot u, \tag{3}$$

where n is the group size – i.e., the number of individuals in the corresponding social group.

6 In this first-approximation model, how does utility depend on the group size?

To separate the dependence on the group size from everything else, we need:

- to fix the objective circumstances – i.e., the value $u^{(0)}$, and
- to fix the coefficient α that describes the participants' attitude towards each other.

Under these conditions, to find how the resulting utility u depends on the groups size n, we can move all the terms in the formula (3) containing the unknown u into the left-hand side and all the other terms into the right-hand side. This way, we get the following formula:

$$u - (n-1) \cdot \alpha \cdot u = u^{(0)}, \quad (4)$$

i.e., equivalently,

$$u \cdot (1 - (n-1) \cdot \alpha) = u^{(0)}. \quad (5)$$

Dividing both sides by the coefficient at u, we can find the desired dependence of the utility u on the group size n:

$$u = \frac{u^{(0)}}{1 - (n-1) \cdot \alpha}. \quad (6)$$

7 Analysis of the resulting formula helps explain the dolphin observations

Let us analyze the resulting formula (6).

What about a group of two. Let us start with the simplest case, when we have a group of two: $n = 2$. In this case, the formula (6) takes the form

$$u = \frac{u^{(0)}}{1 - \alpha}. \quad (7)$$

In general, we may be glad about our friends – but clearly less glad than if we ourselves have the same luck, so we have $0 < \alpha < 1$. Thus, $0 < 1 - \alpha < 1$, so $u > u^{(0)}$. In other words, participation in a group of two makes a person happier.

What about small groups. The same increase of utility is observed while $1 - (n - 1) \cdot \alpha > 0$, i.e., equivalently, while $(n-1) \cdot \alpha < 1$, $n - 1 < 1/\alpha$ and

$$n < 1 + \frac{1}{\alpha}. \quad (8)$$

In order words, participation in social groups whose size is smaller than the threshold

$$n_0 \stackrel{\text{def}}{=} 1 + \frac{1}{\alpha} \quad (9)$$

make the person happier.

What about larger groups. But what if the group size exceeds the threshold (9)? In this case, the denominator $1 - (n-1) \cdot \alpha$ of the expression (6) for utility becomes negative, and thus, even under favorable objective circumstances (when $u^{(0)} > 0$), the resulting utility becomes negative. This explains why participation in a larger group can make a person less happy – which is exactly what was observed for the dolphins.

Comment. This negativity may sound counter-intuitive, but it is a well-known phenomenon in decision theory; see, e.g., [1, 2, 3, 7, 9, 12, 15, 16, 18].

Additional explanation. The above analysis also explains why, according to [8], increasing the strength of social relationships speeds up ageing. Indeed, in our model, this strength is described by the parameter α. The larger α, the smaller the threshold n_0 after which the effect of the social group membership becomes negative.

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