What can mobile devices learn from bird flocks

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Abstract

To arrange a proper coordination between several mobile devices flying together, it makes sense to analyze how such a coordination is arranged in nature, e.g., in a bird flock. In this paper, we analyze to what extent recent discoveries about the bird flocks can be applied to mobile devices. For this purpose, it is necessary: (1) to make conclusions from these discoveries more convincing, and also (2) to explain why these conclusions can be applied to general flying devices, not only to birds. This is what we do in this paper: (1) we show that additional information from these discoveries provides an additional confirmation that birds in a flock only take into account location of nearest neighbors, and (2) we also explain why the discoveries-based conclusions are applicable not only to birds, but to mobile devices as well. We also explain the relation between these discoveries, seven plus minus two law from psychology, and color optical computing.

Keywords: Mobile devices, coordination, bird flocks, Kepler's conjecture, seven plus minus two law, color optical computing.

1 How to coordinate mobile devices: what is the problem and how a study of bird flocks can help

Need for spatial coordination between mobile devices. The main advantage of 3-D mobile devices is that they are mobile, that they can easily relocate. Sometimes, this ability is used to spread several devices across a certain area – e.g., during search and rescue operations. In other cases, we need to move the whole bunch of devices to a different location. In such cases, we need to maintain spatial coordination between the devices:

- to make sure that they do not collide (and do not get dangerously close to each other), and
- to make sure that none of them moves too far away from others, so that they all stay close to the desired trajectory.

Important challenge. The main challenge in arranging such a coordination is that a mobile device can only rely on the energy that it carries with it, so there are severe restrictions on the amount of computations that a device can do. Also, a mobile device has a limited carrying capacity, so it can only carry a few sensors. All this imposes severe limitations on how mobile devices can coordinate their movements.

How can we solve this problem? A natural approach to solve a practical problem is to look for situations in which similar problems have been solved.

- For example, when people wanted to build a flying machine, they looked at objects that fly – namely, birds. They borrowed some techniques – like having wings and tails – from the bird structure.
- When people thought of the best way to make computers perform intelligent tasks, they started looking into how our brains perform these tasks – and borrowed the neural network structure, which is now the main direction in Artificial Intelligence (AI); see, e.g., [7].

Biological situations are very good for this purpose: not only they show how to solve the corresponding problem, but they also give us a solution that is the result of billions of years of improving evolution – and is therefore most probably optimal (or at least close to optimal).

Comment. From this viewpoint, when there are several biological examples, it makes sense to focus of the examples that correspond to later stages of biological evolution.

How a study of bird flocks can help. In view of the above, to help coordinate the joint movement of mobile devices, it makes sense to look at similar situations where a group of objects – with limited ability to sense and to process information – coordinates their motion. In nature, there are many examples of creatures that fly together: bird flocks, swarms of insects. For all these creatures, the need to keep them flying limits their carrying ability and their ability to process information. It therefore makes sense to use the analysis of bird flocks and insect swarms to come up with techniques for coordinating the joint flight of mobile devices.

Between these two examples, birds correspond to later evolutionary stage. So, in this paper, we will mostly focus on bird flocks.

Need to revisit this idea. Using the experience of bird flocks is a natural idea. This idea has been used in the past when designing coordination techniques for mobile devices; see, e.g., [27]. So why do we need to revisit this idea?

The need to revisit comes from the fact that recently, new discoveries were made about bird flocks [1, 6]. So, it is desirable to analyze how the new discoveries about bird coordination can help to coordinate mobile devices.

What we do in this paper. In this paper, we provide such an analysis.

2 What are the new discoveries and how can we apply them to mobile devices

What was known before. It has been already known, for some time, that coordination between birds is organized by birds taking into account the location of their nearest neighbors; see, e.g., [2, 5, 15, 21, 27].

What is new. One of the main conclusions from the new research is that, on average, a bird in a flock adjust its location based only on the locations of at most about 12 neighbors – and usually fewer than that.

Comment. There are many other conclusions: e.g., that the correlation between birds in a flock linearly decreases with distance between them.

Shall we immediately adopt these recommendations for mobile devices? Based on the new paper's results, shall we immediately recommend that a mobile devise adjusts its position based on at most 12 neighbors? Probably not immediately, there are two problems to take into account.

- The first problem is that we do not directly observe birds' brain processes, we cannot look into their brains, and even when we can, we cannot fully understand what is going on there. As a result, the number of neighbors whose position is taken into account is determined very indirectly, based on the observed trajectories of individual birds in a flock – and this extraction comes with uncertainty.
- Also, it is not immediately clear to what extent recommendations based on the bird flocks are appropriate for mobile devices. In general, not all biological features are useful. For example, while airplanes follow many features of the bird structure in their design – e.g., they have wings and tails just like birds do – airplanes do not follow all the details of the bird structure. For example, birds flap their wings, while airplanes do not.

How can we increase our confidence that these recommendations will work for mobile devices? In view of the above, to make recommendations based on bird flock observations more convincing, we need to do two things:

- we need to gain more confidence that birds only adjust their position based on the positions of their nearest neighbors, and
- we need to gain more confidence that the bird-flock features are of generic nature, that they are applicable to all systems of flying objects, not just to birds.

In this paper, we show how, in both cases, we can indeed increase our confidence – and thus, that we can more confidently apply the bird flock recommendations to mobile devices.

3 How to gain more confidence that birds only adjust their position based on their nearest neighbors

What we plan to do in this section. As we have mentioned earlier, one of the additional pieces of information that the article [1] had is that the correlation between the trajectories of different birds decreases linearly with the distance. Let us show that we can use this fact to provide an additional confirmation that birds only take into account the trajectories of their nearest neighbors.

Analysis of the situation. Let us consider n birds $A_1, \ldots, A_n$ along approximately the same line, so that each bird A_i is a direct neighbor to A_{i+1}. Let $a_1, \ldots, a_n$, denote the deviations of some location characteristic – e.g., coordinate or orientation – from with the ideal trajectory. These deviations are random, so their expected values are 0s: $E[a_1] = \ldots = E[a_n] = 0$. It is also reasonable to assume that for all three deviations, the standard deviations are the same: $\sigma[a_1] = \ldots = \sigma[a_n] = \sigma$ for some $\sigma > 0$. Since $E[a_i] = 0$ for all i, we have $E[a_i^2] = \sigma^2[a_i] = \sigma^2$.

Since each bird takes into account the location of its nearest neighbors, the deviations a_i and a_{i+1} are correlated. Let us assume that the corresponding correlation coefficient r_1 is the same for all i. Since $E[a_i] = 0$ and $\sigma[a_i] = \sigma$ for all i, the correlation coefficient has the following form:

$$r_1 = \frac{E[a_i \cdot a_{i+1}]}{\sigma^2}.$$

From this, we can conclude that $E[a_i \cdot a_{i+1}] = r \cdot \sigma^2$ and thus, that for

$$(a_i - a_{i+1})^2 = a_i^2 + a_{i+1}^2 - 2a_i \cdot a_{i+1},$$

we have

$$E[(a_i - a_{i+1})^2] = 2\sigma^2 - 2r_1 \cdot \sigma^2.$$

Since $E(a_i) = 0$ for all i, the left-hand side of this formula is also equal to the variance of the difference $a_i - a_{i+1}$:

$$V[a_i - a_{i+1}] = 2\sigma^2 - 2r_1 \cdot \sigma^2. \tag{1}$$

Let us see what this implies for the correlation

$$r_k = \frac{E[a_i \cdot a_{i+k}]}{\sigma^2}$$

for which, similarly,

$$V[a_i - a_{i+k}] = 2\sigma^2 - 2r_k \cdot \sigma^2. \tag{2}$$

In situations when a bird does not take into account the locations of birds that are not nearest neighbors, it is reasonable to expect that the differences $a_i - a_{i+1}$ and $a_j - a_{j+1}$ corresponding to different i and j are independent. In this case, the difference $a_i - a_{i+k}$ is the sum of k independent terms:

$$a_i - a_{i+k} = (a_i - a_{i+1}) + \ldots + (a_{i+k-1} - a_{i+k}).$$

Thus, according to the usual properties of the probabilities (see, e.g., [31]), the variance of the difference $a_i - a_{i+k}$ is equal to the sum of the corresponding variances:

$$V[a_i - a_{i+k}] = V[a_i - a_{i+1}] + \ldots + V[a_{i+k-1} - a_{i+k}].$$

According to formula (1), all the variances in the right-hand side are the same, so $V[a_i - a_{i+k}] = k \cdot V[a_i - a_{i+1}]$. Thus the formula (2) takes the form

$$k \cdot V[a_i - a_{i+1}] = 2\sigma^2 - 2r_k \cdot \sigma^2.$$

Substituting the expression (1) for $V[a_i - a_{i+1}]$ into this equality, we conclude that

$$2k \cdot \sigma^2 - 2k \cdot r_1 \cdot \sigma^2 = 2\sigma^2 - 2r_k \cdot \sigma^2.$$

If we divide both sides by $2\sigma^2$, we get

$$k - k \cdot r_1 = 1 - r_k,$$

so

$$r_k = k \cdot r_1 - k + 1.$$

So, in this case indeed, the correlation r_k indeed linearly depends on the distance k between the two birds – exactly as observed in the paper [1].

Comment. Of course, the fact that in the only-interacting-with-nearest-neighbor case, we observe the desired linear dependence, does not fully prove that necessarily birds only take into account the location of nearest neighbors. However, this fact provides an additional argument in favor of the only-interacting-with-nearest-neighbor model.

4 How to gain more confidence that the bird-flock features are of generic nature

What is the optimal configuration of a 3D flock: let us formulate this problem in precise terms. Birds (or other flying objects) should not be too close to each other – if they are too close, there is a risk of collisions. So, the distance between any two objects should be larger than or equal to some threshold d_0.

Under this condition, it is desirable to make sure that the flock takes as little volume as possible. Indeed, if the size of the flock is too big, then:

- it will be more difficult to fly in narrow spaces,
- it will be more difficult to defend each other, and
- it will be more difficult to use aerodynamic effects that help nearest neighbors of a flying bird to fly.

So, we need to find the most compact configuration in which every two objects A and B are at a distance of at least d_0 from each other: $d(A, B) \geq d_0$, for some threshold value d_0.

This problem is equivalent to a known problem formulated by Kepler. From the mathematical viewpoint, placing objects at a distance of at least d_0 from each other is equivalent to placing nonintersecting spheres of radius $d_0/2$. Indeed:

- If we have a configuration of objects for which always $d(A, B) \geq d_0$, then the spheres of radius $d_0/2$ centered in two objects A and B do not intersect, they can only touch. Indeed, if they intersected, then for each intersection point C, we will have $d(A, C) < d_0/2$ and $d(C, B) < d_0/2$. Then, by the trinagle inequality, we would have

  $$d(A, B) \leq d(A, C) + d(C, B) < d_0/2 + d_0/2 = d_0,$$

  but we assumed that $d(A, B) \geq d_0$.
- Vice versa, if have a packing of spheres of radius $d_0/2$, then we can place objects in the centers of these spheres and – for the same reason as in the previous bullet – conclude that always $d(A, B) \geq d_0$, i.e., that this a proper configuration of objects,

From this viewpoint, the problem of forming the most compact flock is equivalent to the problem of the most compact packing of the sphere of a given radius. The sphere packing problem was first formulated by the famous mathematician and physicist Johannes Kepler in 1611 [16].

Kepler's conjecture. Kepler formulated a conjecture that the most compact packing has the following form:

- Let us start with a 1-D configuration, e.g., along the x-axis. In this configuration, sphere centers follow one another at the distance d_0 from each other.
- To combine these 1-D configurations into a 2-D configuration, we place centers of the next row exactly in the middle between the locations of the points on the previous row.
- Finally, to combine the resulting 2-D configurations into a 3-D configuration, we place each center from the next 2-D plane exactly at the center between neighboring centers in the previous row.

In this configuration, each sphere is a neighbor to 6 other spheres in the same plane, to 3 spheres above, and to 3 spheres below – a total of 12 neighbors. In terms of centers, for each center there are exactly 12 directly neighboring centers, i.e., centers located exactly at the minimal distance d_0 from the given center.

Comment. This arrangement may sound complicated, but this is exactly how, e.g., oranges are usually packed when they are transported and when they are sold.

Kepler's conjecture was solved only very recently. For almost four centuries, Kepler's conjecture remained an open problem, until it was proven, in our 21st century, by Thomas C. Hales; see [8, 9, 10, 11, 12, 13].

Comment. Our ultimate goal is to decrease uncertainty caused by the limitations of mobile devices. From this viewpoint, it is worth mentioning that the original proof of Kepler's conjecture (see [9]) was obtained by using *interval computations* – a technique specifically designed for processing uncertainty; see, e.g., [14, 20, 23, 26].

What are consequences for mobile devices. The main consequence is that a limitation to 12 neighbors is not a specific feature of the birds, it comes from a general geometric analysis of the problem. This limitation is, therefore, applicable to all groups of flying objects – in particular, to the groups of flying mobile devices.

But why do birds usually adjust their positions based on the positions of fewer than 12 neighbors? In the ideal flock configuration, a bird has 12 nearest neighbors, but it usually takes into account only some of them. Why not with all of them – they are all at the same distance, there seems to be no reason to prefer some of them.

In our opinion, the problem comes from the fact that birds evolved from already well-evolved creatures who did not fly. For these creatures, the only similar group-traveling problem that they had to solve is how to most compactly travel on a surface. In this case, for the most compact arrangement – which is exactly the 2-D stage of the most compact 3-D arrangement – every creature has exactly 6 neighbors. So, to correctly navigate, the creature had to have in mind information about 7 objects: the creature itself and its 6 neighbors. So, the smallest number of pieces of informaton that a land-based creature should have is seven. Naturally, the brains evolved in such a way that for these 7 pieces of information, the processing is as fast as possible.

This explains why in human data processing we divide all objects into about 7 different categories – this is the famous "seven plus minus two law" (see, e.g., [25, 30]). This law not only follows from the need for a group of creatures to travel compactly – similar estimates come from the need for a sole creature be aware of the environment around it [4]:

- If we only consider information about what is in front, what is at the back, what is to the left, and what is to the right, then, with the information about the object itself, we need 5 pieces of information. The number 5 is exactly the lower endpoint for the 7 ± 2 range $[7-2, 7+2] = [5, 9]$.

- If we also consider diagonal directions such as to the right in front, etc., then we have 8 such directions and thus, we need to process $8+1=9$ pieces of information. The number 9 is exactly the upper endpoint of the 7 ± 2 range.

Comment. The seven plus minus two law is indeed ubiquitous in human reasoning. It explains, for example, why we usually divide all possible colors of rainbow – i.e., all possible frequencies of the visible light – into 7 colors. This also explains why, when we think of words that describe the degree to which a certain imprecise property likes "tall", we usually find about 7 such words. This feature of human reasoning is actively used in fuzzy techniques (see, e.g., [3, 17, 24, 28, 29, 34]).

The fact that we have the same number (about 7) in both cases explains the efficiency of the color optical computing approach to fuzzy computations – when we use 7 colors to represent 7 different degrees of certainty (see, e.g., [18, 19, 22, 32, 33]). Indeed, because of the above equality, by representing degrees as colors:

- on the one hand, we do not lose any information, and,
- on the other hand, we do not introduce additional information (that would have slowed down computations).

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References

[1] I. Ahmed, Md. S. Islam, and I. A. Faruque, “From scattered to focused: task-development connectivity in honey bees, with midge swarms and bird flocks”, *Physics Review E*, 2026, Vol. 113, Paper 044403.

[2] M. Ballerini, N. Cabibbo, R. Candelier, A. Cavagna, E. Cisbani, I. Giardina, V. Lecomte, A. Orlandi, G. Parisi, A. Procaccini, M. Viale, and V. Zdravkovic, “Interaction ruling animal collective behavior depends on topological rather than metric distance: Evidence from a field study”, *Proceedins of the National Academy of Sciences of the USA*, 2008, Vol. 105, pp. 1232–1237, https://doi.org/10.1073/pnas.0711437105

[3] R. Belohlavek, J. W. Dauben, and G. J. Klir, *Fuzzy Logic and Mathematics: A Historical Perspective*, Oxford University Press, New York, 2017.

[4] L. Bokati, V. Kreinovich, and J. Katz, “Why 7 Plus minus 2? a possible geometric explanation”, *Geombinatorics*, 2021, Vol. 30, No. 1, pp. 109–112.

[5] A. Cavagna, A. Cimarelli, I. Giardina, G. Parisi, R. Santagati, F. Stefanini, and R. Tavarone, “From empirical data to inter-individualinteractions: Unveiling the rules of collective animal behavior”, *Mathematical Models and Methods in Applied Sciences*, 2010, Vol. 20, Supplement 01, pp. 1491–1510, DOI: 10.1142/S0218202510004660

[6] S. Chen, “What network structures reveal about the birds and the bees”, *APS Physics*, 2026, Vol. 19, Paper 45, https://physics.aps.org/articles/v19/s45

[7] I. Goodfellow, Y. Bengio, and A. Courville, *Deep Learning*, MIT Press, Cambridge, Massachusetts, 2016.

[8] Th. C. Hales, “Cannonballs and honeycombs”, *Notices of the American Mathematical Society*, 2000, Vol. 47, No. 4, pp. 440–449, ISSN 0002-9920.

[9] Th. C. Hales, “A proof of the Kepler conjecture”, *Annals of Mathematics, Second Series*, 2005, Vol. 162, No. 3, pp. 1065–1185, arXiv:math/9811078, doi:10.4007/annals.2005.162.1065, ISSN 0003-486X

[10] Th. C. Hales, “Historical overview of the Kepler conjecture”, *Discrete & Computational Geometry*, 2006, Vol. 36, No. 1, pp. 5–20, doi:10.1007/s00454-005-1210-2, ISSN 0179-5376

[11] Th. C. Hales, *Dense Sphere Packings: A Blueprint for Formal Proofs*, London Mathematical Society Lecture Note Series, Vol. 400, Cambridge University Press, Cambridge, UK, 2012, doi:10.1017/CBO9781139193894, ISBN 978-0-521-61770-3.

[12] Th. C. Hales and S. P. Ferguson, “A formulation of the Kepler conjecture”, *Discrete & Computational Geometry*, 2026. Vol. 36, No. 1, pp. 21–69, arXiv:math/9811078, doi:10.1007/s00454-005-1211-1, ISSN 0179-5376.

[13] Th. C. Hales and S. P. Ferguson, *The Kepler Conjecture: The Hales-Ferguson Proof*, Springer, New York, USA, 2011, ISBN 978-1-4614-1128-4.

[14] L. Jaulin, M. Kiefer, O. Didrit, and E. Walter, *Applied Interval Analysis, with Examples in Parameter and State Estimation, Robust Control, and Robotics*, Springer, London, 2012.

[15] J. W. Jolles, A. J. King, A. Manica, and A. Thornton, "Heterogeneous structure in mixed-species corvid flocks in flight", *Animal Behavior*, 2013, Vol. 85, pp. 743–750, DOI:10.1016/j.anbehav.2013.01.015

[16] J. Kepler, *Strena seu de nive sexangula*, Frankfurt, 1611 (in Latin), available at https://www.thelatinlibrary.com/kepler/strena.html; English translation: J. Kepler, *The Six-Cornered Snowflake*, Oxford University Press, Oxford, UK, 2014, ISBN 978-0198712497

[17] G. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic*, Prentice Hall, Upper Saddle River, New Jersey, 1995.

[18] Yu. P. Kondratenko (ed.), *Research Tendencies and Prospect Domains for AI Development and Implementation*, River Publishers, Aalborg, Denmark, 2024.

[19] O. Kosheleva, V. Kreinovich, V. L. Timchenko, and Yu. P. Kondratenko, "From fuzzy to mobile fuzzy", *Journal of Mobile Multimedia*, 2024, Vol. 20, No. 3, pp. 651–664, DOI: 10.13052/jmm1550-4646.2035

[20] B. J. Kubica, *Interval Methods for Solving Nonlinear Constraint Satisfaction, Optimization, and Similar Problems: from Inequalities Systems to Game Solutions*, Springer, Cham, Switzerland, 2019.

[21] H. Ling, G. E. Mclvor, K. van der Vaart, R. T. Vaughan, A. Thornton, and N. T. Ouellette, "Costs and benefits of social relationships in the collective motion of bird flocks", *Natute Ecology and Evolution*, 2019, Vol. 3, pp. 943–948, doi: 10.1038/s41559-019-0891-5

[22] A. Lupo, O. Kosheleva, V. Kreinovich, V. Timchenko, and Yu. Kondratenko, "There is still plenty of room at the bottom: Feynman's vision of quantum computing 65 years later", In: Yu. P. Kondratenko (ed.), *Research Tendencies and Prospect Domains for AI Development and Implementation*, River Publishers, Aalborg, Denmark, 2024, pp. 77–86.

[23] G. Mayer, *Interval Analysis and Automatic Result Verification*, de Gruyter, Berlin, 2017.

[24] J. M. Mendel, *Explainable Uncertain Rule-Based Fuzzy Systems*, Springer, Cham, Switzerland, 2024.

[25] G. A. Miller, "The magical number seven plus or minus two: some limits on our capacity for processing information", *Psychological Review*, 1956, Vol. 63, No. 2, pp. 81–97.

[26] R. E. Moore, R. B. Kearfott, and M. J. Cloud, *Introduction to Interval Analysis*, SIAM, Philadelphia, 2009.

[27] G. Nagy, A. Thornton, H. Ling, G. McIvor, N. T. Ouellette, and R. Vaughan, "Computational and structural advantages of pairwise flocking", *Proceedings of the IEEE International Symposium on Multi-Robot and Multi-Agent Systems MRS 2019*, New Brunswick, New Jersey, USA, August 22–23, 2019, pp. 133–135.

[28] H. T. Nguyen, C. L. Walker, and E. A. Walker, *A First Course in Fuzzy Logic*, Chapman and Hall/CRC, Boca Raton, Florida, 2019.

[29] V. Novák, I. Perfilieva, and J. Močkoř, *Mathematical Principles of Fuzzy Logic*, Kluwer, Boston, Dordrecht, 1999.

[30] S. K. Reed, *Cognition: Theories and Application*, SAGE Publications, Thousand Oaks, California, 2022.

[31] D. J. Sheskin, *Handbook of Parametric and Nonparametric Statistical Procedures*, Chapman and Hall/CRC, Boca Raton, Florida, 2011.

[32] V. Timchenko, Yu. Kondratenko, and V. Kreinovich, "Logical platforms for mobile application in decision support systems based on color information processing", *Journal of Mobile Multimedia*, 2024, Vol. 20, No. 3, pp. 679–698, DOI 10.13052/jmm1550-4646.2037

[33] V. Timchenko, V. Kreinovich, and Yu. Kondratenko (eds.), *Optical Color Computing: Fuzzy Logic and Digital Calculations*, River Publishers, Aalborg, Denmark, 2026.

[34] L. A. Zadeh, "Fuzzy sets", *Information and Control*, 1965, Vol. 8, pp. 338–353.