Intuitionistic Fuzzy Techniques, It from Bit, 10- or 11-Dimensional Quantum Field Theory, and Color Optical Computing

Vladik Kreinovich[1[0000−0002−1244−1650]], Olga Kosheleva[2[0000−0003−2587−4209]], Victor Timchenko[3[0000−0003−4259−3919]], and Yuriy Kondratenko[4[0000−0001−7736−883X]]

[1] Department of Computer Science, University of Texas at El Paso
500 W. University, El Paso, Texas 79968, USA, vladik@utep.edu
[2] Department of Teacher Education, University of Texas at El Paso
500 W. University. El Paso, Texas 79968, USA, olgak@utep.edu
[3] Admiral Makarov National University of Shipbuilding, Mykolaiv, Ukraine
vl.timchenko58@gmail.com
[4] Petro Mohyla Black Sea National University, Mykolaiv, Ukraine
y_kondrat2002@yahoo.com

Abstract. It is known that a consistent application of quantization leads to the emergence of all logically possible physical models. This idea – that logic underlies physics – is known as "it from bit". Traditional applications of this idea are based on the 2-valued logic. In this paper, we show that to get a more adequate picture, we need to use ideas behind intuitionistic fuzzy approach. In the first qualitative approximation, this approach leads to a logic that can be naturally interpreted in color terms – and for which related computations can be performed by color optical computing. The simplest quantitative approximation to this approach leads to a possible physically and logically meaningful explanation of the fact that the smallest space-time dimension for which a consistent quantum field theory is possible is 10 or 11.

Keywords: Intuitionistic fuzzy techniques · It from Bit · 10- or 11-Dimensional Quantum Field Theory · Color Optical Computing

1 It from bit – physics is based on logic: a brief reminder

The main idea behind quantum physics. (See, e.g., [3, 28] for details.) In Newtonian physics the future state of the world is uniquely determined by its cirrent state. Once we know how all the forces depend on the location and velocities of all the objects, we can use Newton's equations to predict the future state of the system. For example, once we know the current positions and velocities of all celestial bodies, we can predict their positions hundreds of years ahead – and indeed, we can predict Solar eclipses, comet appearances, etc., many decades and even centuries ahead.

At the end of the 19th century, it turned out the Newtonian predictions were not perfectly accurate: for example, the actual location of Mercury deviated from the predicted one by 43 angular seconds every 100 years. This discrepancy was resolved by Einstein whose General Relativity theory provides perfect predictions. From this viewpoint, the behavior of the macro-world is perfectly deterministic.

However, it turned out that the we cannot make deterministic predictions for objects in the microworld. One of the first examples was radioactivity, when atoms spontaneously emit some particles. Absolutely identical atoms of the same material emit particles at different times – and it is not possible to predict these times. We can, at best, predict the probability that an atom will edit duing a given time interval.

The same impossibility of exact prediction was observed for all other quantities of micro-objects such as molecules, atoms, and elementary particles. These observation lead to one of the main ideas behind quantum physics – that nothing is pre-determined, that we can make, at best, only probabilistic predictions.

Further development of this idea – from primary to secondary quantization. In the first formalizations of this idea, the physicists took into account that particles randomly deviate from their Newtonian trajectories. To compute the underlying Newtonian trajectories, these formalizations used the original Newtonian deterministic formulas describing the dependence of force on the location and velocity of all the objects.

These formalizations provided very accurate description of the micro-world – e.g., the spectrum of light emitted by different materials. However, later, more accurate measurements showed that there is a small discrepancy between the measurement results and the predictions of these formalizations.

To explain these discrepancy, physicists again applied the main idea of quantum physics: that nothing is pre-determined, that we can make, at best, only probabilistic predictions. In the original formalization, particle trajectories were non-deterministic, but the formulas for the foces were still deterministic. When physicists took into account that the forces can also randomly deviate from their Newtonian values – they called this *secondary quantization* – they got a perfect match between theory and experiment.

Need for ternary (and further) quantizations. So far, secondary quantization explains all observed physical phenomena. However, a mathematical analysis of the corresponding equations shows that we may not be able to compute higher-order terms needed for moe accurate predictions: some of these terms have physically meaningless infinite values, and even when these tems remain finite, the series that should lead to a more and more accurate predictions actually diverge; see, e.g., [3, 28].

To solve this problem, several physicists – starting with John Wheeler [17] suggested to again use the main quantum idea – that nothing is pre-determined, that we can make, at best, only probabilistic predictions. Specifically, while in secondary quantization, force is non-deterministic, we can have different force values with different probability – the formulas for describing these probabilities are deterministic. A natural idea is thus to allow that the actual probabilities may differ from these values.

The same argument can be applied to all other physical assumptions – the structure of space-time in which all physically processes take place, the properties of the particles, etc. Once we apply this principle to its logical conclusion, we see that there are no deterministic constaints on a physical model, all logically consistent physical models are possible – with some probability. From this viewpoint, to understand our Universe, we need to consider all possible – i.e., all logically consistent – physical models (and some physics explanations were indeed provided this way). This idea is known as "it from bit" – we can potentially describe all the properties of the physical world ("it") based on logic. And since physicists use binary "true-false" logic, corresponding to 0-1 binary digits (bits), they use "bit" as a catchy shorthand for logical foundations.

2 Logic is important for physics, but what logic should we use? An argument for intuitionistic fuzzy approach

Need for the third option – in addition to "true" and "false". Two-valued logic assumes that every statement is either true or false in the corresponding theory. However, since the famous Gödel's result, it is known for each well-defined theory (that is sufficiently strong to contain arithmetic), some statements are independent from this theory: neither this statement not its negation can be derived from the theory.

How is this related to intuitionistic fuzzy approach. This is in line with the main ideas behind intuitionistic fuzzy logic (see, e.g., [1, 29]). Indeed, in the traditional fuzzy logic (see, e.g., [2, 8, 16, 18, 20, 31]), once we know the degree of confidence μ in a statement, the degree of confidence in its negation is estimated as $1 - \mu$, so that the sum of these two degrees will be exactly 1. This means, in effect, that we only consider two possibilitites: that the statement is true and that the statement is false. In reality, often there is a third possibility: that the expert does not know whether the statement is true or false. To capture this third option, intuitionsitic fuzzy logic uses two degrees: the degree μ_+ to which the expert believes that the statement is true and the degree μ_- to which the expert believes that the statement is false. The sum of these two degee nay be smaller than 1, and the difference $\mu_0 \stackrel{\text{def}}{=} 1 - \mu_+ - \mu_-$ is the degree to which the expert is not sure whether the given statement is true or false. In this case, the sum of the *three* degree is equal to 1: $\mu_+ + \mu_- + \mu_0 = 1$.

Comment. In the following text, we will argue that there is an even deeper relationship between the it-from-bit approach and the intuitionistic fuzzy ideas.

We need to go further than three values. In principle, for each formal theory and for each statement, we can try all possible derivations. If one of these derivations leads to the given statement, this statement is true. If one of these derivations leads to the negation of the given statement, this means that the statement is false. However, this argument implicitly assumes that we have a potentially infinite time. In reality, according to modern physics, our Universe has a finite lifetime, so there may be not enough time to determine whether a statement is true or not.

One may naively think that this is only important for very long and/or very complex statements, but in reality, even for simple propositional formulas, checking whether they are true or not is, in general, NP-hard (see, e.g., [15, 21]) – meaning that unless P = NP (which most computer scientists believe to be impossible) checking the truth of these formulas requires exponential, astronomical time – that for reasonable size formulas exceeds the lifetime of the Universe.

From this viewpoint, even if we spend the whole lifetime of the Universe, we may not be able to check whether a statement is true or not. Thus, in addition to the cases when the is true (T), false (F), or independent (I), we may have several other options:

– we may only know that the statement is either true or independent; we will denote this case by TI;
– we may only know that the statement is either false or independent; we will denote this case by FI;
– we may only know that the statement is either true or false, but it cannot be independent; we will denote this case by TF;
– we may not have any information about the statement, so it may be false, it may be true, and it may be independentS; we will denote this case by TFI.

Comment. These are not just theoretical possibilities, such situations did happen in the past. For example:

– most mathematical open problems are of the type TFI;
– about the Continuum Hypothesis – that there is no cardinality strictly between the cardinalities of the set of natural numbers and of the set of real numbers – at first, it was proven that this hypothesis is consistent with set theory, i.e., that it is of the type TI; it was later shown that it's actually independent;
– similarly, for the negation of the Continuum Hypothesis, it was originally only proven that it is ether false or independent, i.e., it was of the typoe FI;
– finally, for some very large integers, we do not know yet whether they are prime, but we know that the corresponding statement cannot be independent: in principle, we can always check whether a given number is prime by checking that it is not divisible by any smaller number; in this case, we have a statement of type TF.

How this is related to intuituionistic fuzzy ideas. To explain this relation, let us recall the usual fuzzy logic. Its main idea is that, in addition to statements that the expert knows to be true and the statements that the expert knows to be false, there are also statements about which the expert is not sure. In terms of the degree of confidence μ, this means that in addition to statements for which $\mu = 1$ and statements for which $\mu = 0$, there are also statements for which $0 < \mu < 1$. This description can be viewed as the first – qualitative – approximation to the usual fuzzy logic.

Similarly, we can consider the first – qualitative – approximation to the intuitionistic fuzzy logic. In the intuitionistic fuzzy logic, we have three degrees of confidence μ_+, μ_-, and μ_0 instead of a single degree μ. So, it makes sense to consider, for each of these three degrees μ_i, similar three options: $\mu_i = 1$, $\mu_i = 0$, and $0 < \mu_i < 1$. Since the sum of these three degrees is 1, if one of these degrees is equal to 1, then the other two degrees must be equal to 0. With this in mind, we end up with the following possible cases:

- the case when $\mu_+ = 1$ and $\mu_- = \mu_0 = 0$; this case corresponds to T ("true");
- the case when $\mu_- = 1$ and $\mu_+ = \mu_0 = 0$; this case corresponds to F ("false");
- the case when $\mu_0 = 1$ and $\mu_+ = \mu_- = 0$; this case corresponds to I ("independent");
- the case when $\mu_+ > 0$, $\mu_0 > 0$, and $\mu_- = 0$; this case corresponds to TI;
- the case when $\mu_- > 0$, $\mu_0 > 0$, and $\mu_+ = 0$; this case corresponds to FI;
- the case when $\mu_+ > 0$, $\mu_- > 0$, and $\mu_0 = 0$; this case corresponds to TF;
- finally, we can have the case when $\mu_+ > 0$, $\mu_0 > 0$, and $\mu_- > 0$; this case corresponds to TFI.

One can see that we have exactly the same cases as in the previous subsection.

3 How this qualitative application of intuitionistic fuzzy approach is related to color interpretation and color optical computing

Color interpretation. We have three truth values T, F, and I, and we have three basic colors: red (R), green (G), and blue (B). It therefore makes sense to use different basic colors to illustrate different truth values; see, e.g., [24, 26]. Other options like TI can then be represented by the combination of the corresponding colors.

In principle, we can have any mapping between these two triples, but it makes sense to select the following specific mapping:

- The "true" value T gives us the largest amount of information, so it makes sense to associate the truth value T with the color that brings the most information. Light is a periodic process consisting of cycles. In principle, each cycle can carry a number. The more cycles per second, the more numbers – and thus, the more information – we can carry. The number of cycles per second is nothing else but the frequency. Thus, it makes sense to associate T with the basic color that has the largest frequency – i.e., with color blue (B).
- From I value, we can go to T and we can go to F. From this viewpoint, I is close to both T and F, while T and F are not close to each other. Thus, it makes sense to assign the two remaining colors – R and G – so that the

color associated with I be closer to the color B associated with T than the color associated with F. Among the two remaining colors, G is closer to B than R, so it makes sense to associate I with G and F with R.

Comment. The association of "false" (F) with red (R) is also in good accordance with the cultural references:

– in traffic lights, read means prohibited – i.e., false;
– in accounting, read means losing money – which is, of course, negative (= false);
– in pedagogy, red color is usually used to mark mistakes – e.g., false statements.

Color optical computing. Colors not only provide a visual interpretation of truth values, they also enable us to perform computations really fast; see, e.g., [9, 12, 25, 27].

There are several reasons why color optical computing is fast. The first two reasons refer to optical computing in general:

– first, according to relativity theory, the speed of all motions – and, in particular, the speed of all communications – is limited by the speed of light; thus, light is the fastest possible process;
– second, light consists of many particles (photons), so there is a huge possibility to parallelize computing by assigning different computations to different parts of the light beam – and parallelization is a well-known way to speed up computations.

Color computing adds an additional possibility of parallelization (and thus, of a speedup) – we can simultaneously perform three different computations in lights of three different colors.

4 From qualitative to quantitative use of intuitionistic fuzzy approach

Need for a quantitative use of intuitionistic fuzzy approach. Let us illustrate this need on a simple example of, e.g., the case TI. In some cases, we have a lot of arguments in favor of T, but since we do not yet have a proof of the given statement, we assign this statement to the class TI. In other cases, we may have more arguments that this statement is probably independent – e.g., because many similar statements have been proven to be independent. Both these situations are of the same type TI, but they are clearly different. It is desirable to explicitly describe this difference – e.g., by assigning to each of the two options T and I different degrees of confidence.

How can we make the use quantitative. The desired numerical degrees of confidence are used in intuitionsitic fuzzy logic. There – – similarly to the usual fuzzy logic – each degrees of confidence can take any real value from the interval $[0, 1]$.

Of course, using all possible real numbers from this interval is an idealization. No one can meaningfully distinguish, e.g., between degree 0.80 and degree 0.82. We can only meaningfully distinguish between a small finite number of different degrees of confidence. So, to get the quanitative description more adequate, it makes sense to select some small positive integer $n > 1$ and consider degrees 0, $1/n$, $2/n$, ..., $(n-1)/n$, and 1.

What is the simplest quantitative use. The large n, the more complex the corresponding approach. So, to get the simplest possible interpretation, it makes sense to consider the smallest numbe n that allows us to capture all above-listed qualitative cases,

Let us analyze what is this smallest n. The smallest integer $n > 1$ is $n = 2$. In this case, we can only use degrees 0, $1/2$, and 1. In this case, we cannot represent the case of TFI, where all three degrees are positive, since in this case, the smallest sum of positive degrees is $3/2$ – which is larger than 1. Since $n = 2$ does not work, let us consider the next integer $n = 3$. For this n, it is already possible to cover all the qualitative cases. Specifically, we have the following options:

– three cases T, F, and I, for which one of the degrees is 1; in these cases, $\mu = (\mu_+, \mu_-, \mu_0)$ is equal to:

$$(1, 0, 0), \quad (0, 1, 0), \quad (0, 0, 1);$$

– for each of the types TI, FI, and TF, two possible combinations of degrees; in these cases, the triple $\mu = (\mu_+, \mu_-, \mu_0)$ is equal to:

$$\left(\frac{2}{3}, 0, \frac{1}{3}\right), \left(\frac{1}{3}, 0, \frac{2}{3}\right), \left(0, \frac{2}{3}, \frac{1}{3}\right), \left(0, \frac{1}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{1}{3}, 0\right), \left(\frac{1}{3}, \frac{2}{3}, 0\right);$$

– for the type TFI, there is one possible triple:

$$\mu = (\mu_+, \mu_-, \mu_0) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right).$$

Overall, there are 10 possible options.

5 The may explain the appearance of 10- or 11-dimensional space-time in quantum field theory

Fact that needs to be explained. It is known that in our usual 4-dimensional space-time – consisting of 3-dimensional space and 1-dimensional time – it is not possible to have a consistent quantum field theory. The smallest space-time dimension for which a consistent quantum field theory is possible is 10 or 11, which corresponds to 9- or 10-dimensional proper space and 1-dimensional time; see, e.g., [4, 5, 22, 30].

This conclusion is based on complex mathematical analysis whose physical meaning is not very clear. It is therefore desirable to come up with a physically meaningful explanation for this dimension.

The above 10-options results leads to a possible explanation. To provide such an explanation, let us recall some simple facts from quantum physics. Specifically, let us describe a quantum analogue of the classical (non-quantum) system that can be in N possible states $s_1, \ldots, s_N$. In quantum physics, in addition to all these states, the system can also be in a *superposition* state

$$c_1 \cdot |s_1\rangle + \ldots + c_N \cdot |s_N\rangle,$$

where $c_1, \ldots, c_N$ are complex numbers with the only constraint $|c_1|^2 + \ldots + |c_N|^2 = 1$.

This fact is well known and well-used in *quantum computing* (see, e.g., [19]), where the quantum analogue of a bit – a system with two possible states 0 and 1 – is a *qubit*, with possible states $c_0 \cdot |0\rangle + c_1 \cdot |1\rangle$, where c_i are complex numbes for which $|c_0|^2 + |c_1|^2 = 1$. In this case, instead of two possible states, we have a 2-dimensional set of states – to be more precise, the set of all possible states is a unit sphere in the 2-dimensional complex space.

In general, a quantum analogue of an system with N possible states in a system with N-dimensional space. So, for the above system that has at least 10 possible states (it can have more if consider $n > 3$) its quantum analogue is a 10-dimensional space. So, to describe a state of this system at each moment of time, we need a point in a 10-dimensional space – exactly the same dimension as coming from the complex mathematics of quantum field theory.

Comment. This is not the only possible physical explanation of the fact that the dimension of proper space in consistent quantum field theories is 9 or 10; let us mention two other explanations.

- One of the reasons why quantum field theory is inconsistent in 3-dimensional space is because, according to relativity theory, elementary particles are point-wise objects. As a result, when we compute the overall energy of the electric field generated by a charged particles, we get a physically meaningless infinite value. A natural way out is to take into account that, due to the quantum fluctuations, particles are not exactly point-wise, they follow some probability distribution of possible locations. Each particle is affected by many possible quantum fluctuations. It is known that, in general, the joint effect of a large number of small random disturbances is distributed according to the Gaussian (normal) distribution; the corresponding mathematical result – known as the *Central Limit Theorem* (see, e.g., [23]) explains the ubiquity of normal distributions. A normal distribution in an d-dimensional space is uniquely determined by its mean $m = (m_1, \ldots, m_d)$ and its covariance matrix c_{ij}. To describe the components of the mean vector and of the covariance matrix, we need

$$m + \frac{m \cdot (m+1)}{2}$$

parameters. In particular, in the 3-dimensional space, we need $4+6=9$ parameters [10, 11].

– Another explanation is related to another well-known problem: that while it is known that space-time is curved, it is difficult to take this fact into account in a quantum description. One way to take this into account is to use another known fact: that the equation of General Relativity Theory – that describes our curved space-time – can be derived in the flat (not curved) space-time if we simply take into account that the source of the gravitational field is enegy – including the energy of the gravitation field itself; see [6, 7, 13]. From this viewpoint, it makes sense to embed the curved space-time into a flat one with higher dimension, and use the usual quantum field theory in this high-dimensional space-time. Interestingly, the smallest dimension of a flat space-time into which we can embed (at least locally) any curved 4-dimensional space-time is exactly 10 – which provides one more explanation of the use of 10-dimensional space; see [11]. The same dimension appears if we consider global embeddings [14].

Acknowledgments. This work was supported by the AT&T Fellowship in Information Technology, by the Institute for Risk and Reliability, Leibniz Universitaet Hannover, Germany, by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) Focus Program SPP 100+ 2388, Grant Nr. 501624329, by the European Union under the project ROBOPROX (No. CZ.02.01.01/00/22 008/0004590), by the Center of Excellence in Econometrics, Faculty of Economics, Chiang Mai University, Thailand, by the Ho Chi Minh City University of Banking, Vietnam, and by Thang Long University, Hanoi, Vietnam.

Disclosure of Interests. The authors have no competing interests to declare that are relevant to the content of this article.

References

1. Atanassov, K.: Intuitionistic Fuzzy Sets: Theory and Applications. Springer-Verlag, Berlin, Heidelberg (1999)
2. Belohlavek, R., Dauben, J. W., Klir, G. J.: Fuzzy Logic and Mathematics: A Historical Perspective. Oxford University Press, New York (2017).
3. Feynman, R., Leighton, R., Sands, M.: The Feynman Lectures on Physics. Addison Wesley, Boston, Massachusetts (2005)
4. Greene, B.: The Elegant Universe: Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory. W. W. Norton & Company, New York, (2003)
5. Green, M. B., Schwarz, J. H., Witten, E.: Superstring Theory. Cambridge University Press, Cambridge, UK (2012)
6. Gupta, S. N.: Quantization of Einstein's gravitational field: general treatment. Proceedings of Physical Society A **65**, 608–619 (1952)
7. Gupta, S. N.: Einstein's and other theories of gravitation. Reviews of Modern Physics **29**, 334–336 (1957)
8. Klir, G., Yuan, B.: Fuzzy Sets and Fuzzy Logic. Prentice Hall, Upper Saddle River, New Jersey (1995)

9. Kondratenko, Yu. P. (ed.): Research Tendencies and Prospect Domains for AI Development and Implementation. River Publishers, Aalborg, Denmark (2024)
10. Korlyukov, A. B., Kreinovich, V.: Equations of physics become consistent if we take measurement uncertainty into consideration. Reliable Computing, Supplement (Extended Abstracts of APIC'95: International Workshop on Applications of Interval Computations, El Paso, Texas, USA, February 23–25, 1995), 111–112 (1995)
11. Kosheleva, O., Kreinovich, V.: Why fully consistent quantum field theories require that the space-time be at least 10-dimensional: a commonsense field-based explanation. In: Dantsin, E., Kreinovich, V. (eds.), Uncertainty Quantification and Uncertainty Propagation under Traditional and AI-Based Data Processing (and Related Topics): Legacy of Grigory Tseytin. Springer, Cham, Switzerland, pp. 227–241 (2026)
12. Kosheleva, O., Kreinovich, V., Timchenko, V. L., Kondratenko, Yu. P.: From fuzzy to mobile fuzzy. Journal of Mobile Multimedia **20**(3), 651–664 (2024) DOI: 10.13052/jmm1550-4646.2035
13. Kreinovich, V.: Gupta's derivation of Einstein equations. Soviet Physics Doklady **20**(5), 341–342 (1975)
14. Kreinovich, V.: Why 3-D space? Why 10-D space? A possible simple geometric explanation. Mathematical Structures and Modeling **40**, 55–58 (2016)
15. Kreinovich, V., Lakeyev, A., Rohn, J., Kahl, P.: Computational Complexity and Feasibility of Data Processing and Interval Computations. Kluwer, Dordrecht (1998)
16. Mendel, J. M.: Explainable Uncertain Rule-Based Fuzzy Systems. Springer, Cham, Switzerland (2024)
17. Misner, C. W., Thorne, K. S., Wheeler, J. A.: Gravitation. Princeton University Press, Princeton, New Jersey (2017)
18. Nguyen, H. T., Walker, C. L., Walker, E. A.: A First Course in Fuzzy Logic. Chapman and Hall/CRC, Boca Raton, Florida (2019)
19. Nielsen, M. A., Chuang, I. L.: Quantum Computation and Quantum Information. Cambridge University Press, Cambridge, U.K. (2013)
20. Novák, V., Perfilieva, I., Močkoř, J.: Mathematical Principles of Fuzzy Logic. Kluwer, Boston, Dordrecht (1999)
21. Papadimitriou, C.: Computational Complexity. Addison-Wesley, Reading, Massachusetts (1994)
22. Polchinski, J.: String Theory, Vols. 1 and 2. Cambridge University Press, Cambridge, UK (2005)
23. Sheskin, D. J.: Handbook of Parametric and Nonparametric Statistical Procedures. Chapman and Hall/CRC, Boca Raton, Florida (2011)
24. Timchenko, V. L., Kondratenko, Y. P., Kreinovich, V,: Interval-valued and set-valued extensions of discrete fuzzy logics, belnap logic, and color optical computing. In: Massanet, S., Montes, S., Ruiz-Aguilera, D., Gonzalez-Hidalgo, M. (Eds.), Fuzzy Logic and Technology, and Aggregation Operators. Proceedings of the 13th Conference of the European Society for Fuzzy Logic and Technology EUSFLAT 2023 and 12th International Summer School on Aggregation Operators AGOP 2023, Palma de Mallorca, Spain, September 4–8, 2023, Springer Lecture Notes in Computer Science **14069**, 297–303 (2023) doi https://doi.org/10.1007/978-3-031-39965-7_25
25. Timchenko, V., Kondratenko, Yu., Kreinovich, V.: Logical platforms for mobile application in decision support systems based on color information processing. Journal of Mobile Multimedia **20**(3), 679–698 (2024) DOI 10.13052/jmm1550-4646.2037

26. Timchenko, V. L., Kondratenko, Y. P., Kreinovich, V., Kosheleva, O.: Natural Color Interpretation of Interval-Valued Fuzzy Degrees. Proceedings of the 20th World Congress of the International Fuzzy Systems Association IFSA'2023, Daegu, South Korea, August 20–23, pp. 254–258 (2023)
27. Timchenko, V., Kreinovich, V., Kondratenko, Yu. (eds.): Optical Color Computing: Fuzzy Logic and Digital Calculations. River Publishers, Aalborg, Denmark (2026)
28. Thorne, K. S., Blandford, R. D.: Modern Classical Physics: Optics, Fluids, Plasmas, Elasticity, Relativity, and Statistical Physics. Princeton University Press, Princeton, New Jersey (2021).
29. Vassilev, P. M., Atanassov, K. Y.: Modifications and Extensions of Intuitionistic Fuzzy Sets. "Prof. Marin Drinov" Academic Publishing House, Sofia, Bulgaria (2019)
30. Witten, E.: Fivebranes and knots. Quantum Topology **3**(1), 1–137 (2012)
31. Zadeh, L. A.: Fuzzy sets. Information and Control **8**, 338–353 (1965)