How Are Stradivari Violins Different: A Partial Explanation

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Abstract Why old Italian violins – in particular, ones made by Stradivari – sound so good is still largely a mystery. Violinists who play these instruments make the following two observations: (1) usually, the old Italian violins require softer (less strong) touch than modern violins, and still produce, in the concert hall, a louder impression of sound, and (2) in general, if one applies more force to a violin, then a violin leads to a louder impression in the concert hall – until it reaches a certain limit; for an old Italian violin, such a limit starts at a much smaller force. In this paper, we provide a partial explanation for the above empirical observations: namely, we show that, under a reasonable formalization of the first observation, the second observation naturally follows.

1 Formulation of the problem

Empirical fact. It is well known that violins made by old Italians violin makers such as Stradivari sound differently than modern violins. Specifically, the violinists who play these instruments make the following two observations (see, e.g., [1, 2]):

- usually, the old Italian violins require softer (less strong) touch than modern violins, and still produce, in the concert hall, a louder impression of sound, and

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- in general, if one applies more force to a violin, then a violin leads to a louder impression in the concert hall – until it reaches a certain limit; for an old Italian violin, such a limit starts at a much smaller force.

It should be mentioned that these are subjective evaluations, we are talking about louder *impressions* of the sound – since experiments did not observe the difference between the actual loudness of the two types of violins [1, 2].

Problem. As of now, there is no convincing explanation for the above empirical observations.

What we do in this paper. In this paper, we provide a partial explanation for the above empirical observations: namely, we show that, under a reasonable formalization of the first observation, the second observation naturally follows.

In line with this objective, in Section 2, we describe the formalization of the first observation, and in Section 3, we explain how the second observation is derived in this formalization.

2 Let us describe the first observation in precise terms

How can we describe the effect of a violin. Both observations deal with the dependence of the perceived loudness ℓ in the concert hall on the (perceived) strength s applied to the violin, i.e., with the dependence $\ell = \ell(s)$.

Specifically, both observations compare this dependence:

- for modern violins, and
- for the old Italian violins (such as violins produced by Stradivari).

Let us denote:

- the dependence corresponding to the modern violins by $\ell_{(m)}(s)$, and
- the dependence corresponding to old Italian violins by $\ell_{(i)}(s)$.

When the strength s changes a little bit, the perceived loudness ℓ should also change a little bit. In precise terms, this means that both functions $\ell_{(m)}(s)$ and $\ell_{(i)}(s)$ should be continuous.

Of course, when we do not apply any force, i.e., when $s = 0$, the violin does not produce any sound, so $\ell_{(m)}(0) = \ell_{(i)}(0) = 0$. When we apply more force, the perceived loudness increases – or at least stays the same. This means that both functions $\ell(s)$ are (non-strictly) increasing:

$$\text{if } s < s', \text{ then } \ell_{(m)}(s) \le \ell_{(m)}(s') \text{ and } \ell_{(i)}(s) \le \ell_{(i)}(s').$$

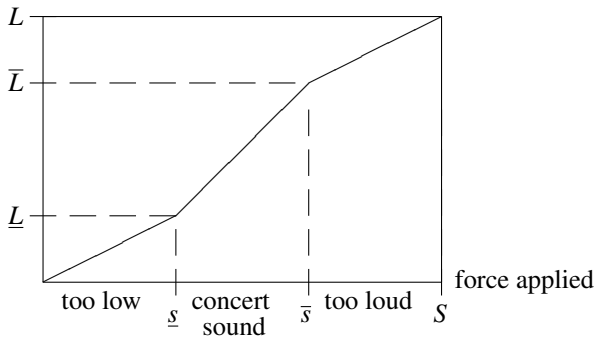
Natural bounds. In general, the possible force s varies from 0 to some limit value S. Similarly, the perceived loudness in the concert hall can vary from 0 to the largest physically possible perceived loudness L that one can extract from an instrument.

Not all the values of perceived loudness from 0 to L make musical sense.

- When the perceived loudness is too small – smaller than some threshold $\underline{L}$ – then the listeners cannot enjoy the music, since they can be barely hear what is being played – if they hear anything at all.
- Similarly, when the music is too loud, it is not possible to enjoy the music. Namely, as we have mentioned, once the perceived loudness reaches some threshold value $\overline{L}$, the listeners practically stop feeling the difference in loudness – and such differences form an important part of a musical experience.

For each violin, let us denote the strength values at which this violin reaches the perceived levels $\underline{L}$ and $\overline{L}$ by, correspondingly, $\underline{s}$ and $\overline{s}$, so that $\ell(\underline{s}_{(i)}) = \ell(\underline{s}_{(m)}) = \underline{L}$ and $\ell(\overline{s}_{(i)}) = \ell(\overline{s}_{(m)}) = \overline{L}$. So, we can divide the range of possible strengths into three zones:

- for small values of force, smaller than the last value $\underline{s}$ for which $\ell(\underline{s}) = \underline{L}$, the perceived loudness is too small, and the listeners cannot enjoy the music;
- for large values of force, larger than the first value $\overline{s}$ for which $\ell(\overline{s}) = \overline{L}$, the listeners cannot perceive the difference in loudness, so the listeners also cannot enjoy the music; and
- for intermediate values of force, values s for which $\underline{s} \le s \le \overline{s}$ and $\underline{L} \le \ell(s) \le \overline{L}$, the listeners enjoy the music.



The first observation says that the old Italian violins sound louder. We are talking perceived loudness. Our perception has limits: we cannot feel the difference in loudness if this difference is too small. There is some threshold ε for which we only feel a difference if it is larger than or equal to this threshold. (Of course, the difference $\overline{L} - \underline{L}$ between the largest and smallest perceived loudness of concert sound is larger than this threshold: $\overline{L} - \underline{L} > \varepsilon$.)

So, the first observation means that not only old Italian violins have larger perceived loudness, but also that the difference between their perceived loudness values is larger than or equal to ε.

In these terms, the first observation takes the following form: for every reasonable strength s of the modern violin for which the perceived loudness is distinguishable from $\overline{L}$, there exists a reasonable strength s' of the old Italian violin that is smaller

than s and that leads to a sound that is distinguishably louder. In precise terms, this observation can be described as follows:

$$\forall s \in [\underline{s}_{(m)}, \overline{s}_{(m)}]\,(\ell_{(m)}(s) \le \overline{L} - \varepsilon \Rightarrow$$

$$\exists s' \in [\underline{s}_{(i)}, \overline{s}_{(i)}]\,(s' < s \,\&\, \ell_{(i)}(s') \ge \ell_{(m)}(s) + \varepsilon)). \tag{1}$$

3 Let us show that the second observation indeed follows from the first one

Let us describe the second observation in precise terms. The second observation means that for the old Italian violins, the maximum strength $\overline{s}_{(i)}$ at which it produces a concert sound is smaller than the similar quantity $\overline{s}_{(m)}$ for modern violins:

$$\overline{s}_{(i)} < \overline{s}_{(m)}. \tag{2}$$

Let us prove that the second observation follows from the first one. Let us show that the inequality (2) follows from the condition (1).

The function $\ell_{(m)}(s)$ is continuous. The values of this function for inputs from the interval $[\underline{s}_{(m)}, \overline{s}_{(m)}]$ change from $\underline{L}$ to $\overline{L}$. Since $\overline{L} - \underline{L} > \varepsilon$, the value $\overline{L} - \varepsilon$ belongs to the interval $[\underline{L}, \overline{L}]$. Thus, by the properties of continuous functions, there exists a point s_0 from the interval $[\underline{s}_{(m)}, \overline{s}_{(m)}]$ for which $\ell_{(m)}(s_0) = \overline{L} - \varepsilon$.

Due to the property (1), there exists a value s' from the interval $[\underline{s}_{(i)}, \overline{s}_{(i)}]$ for which

$$s' < s_0 \tag{3}$$

and

$$\ell_{(i)}(s') \ge \ell_{(m)}(s_0) + \varepsilon. \tag{4}$$

Here, $\ell_{(m)}(s_0) = \overline{L} - \varepsilon$, so the inequality (4) implies that

$$\ell_{(i)}(s') \ge \overline{L}. \tag{5}$$

For all the strength values s from the interval $[\underline{s}_{(i)}, \overline{s}_{(i)}]$ – in particular, for the value s' – the perceived loudness $\ell_{(i)}(s)$ is smaller than or equal to $\overline{L}$; so,

$$\ell_{(i)}(s') \le \overline{L}. \tag{6}$$

From (5) and (6), we conclude that $\ell_{(i)}(s') = \overline{L}$.

We defined $\overline{s}_{(i)}$ as the smallest strength for which the perceived loudness is equal to $\overline{L}$. Thus, we cannot have $s' < \overline{s}_{(i)}$ – otherwise $\overline{s}_{(i)}$ would not be the smallest

strength for which the perceived loudness is equal to $\overline{L}$. Since $s' \le \overline{s}_{(i)}$ and we cannot have $s' < \overline{s}_{(i)}$, we thus conclude that $s' = \overline{s}_{(i)}$.

So, the inequality (3) implies that $\overline{s}_{(i)} < s_0$. The value s_0 belongs to the interval $[\underline{s}_{(m)}, \overline{s}_{(m)}]$, so $s_0 \le \overline{s}_{(m)}$.

From $\overline{s}_{(i)} < s_0$ and $s_0 \le \overline{s}_{(m)}$, we can now conclude that $\overline{s}_{(i)} < \overline{s}_{(m)}$. So, the property (1) indeed implies (2).

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