A Physical Argument in Defense of Free Will

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Abstract According to current physical theories, the future state of the world is uniquely determined by its initial state. This contradicts our intuition, according to which we have free will, our actions are not pre-determined – we can choose different actions. However, for physicists, arguments from intuition are not the most convincing one: for example, many observed quantum phenomena are inconsistent with our intuition. In this paper, we provide a physical argument in favor of free will. Specifically, in physics, it is usually believed that if a property holds for almost all values, it must be true in the real world: this is, e.g., how physicists make conclusions about cosmology. We show that a similar argument leads to the need to consider several different future states corresponding to the same initial state – which leaves room for freedom of will.

1 Physics and Free Will: Formulation of the Problem

There is a serious contradiction between physics and intuition: is there a free will? In Newton's physics, the initial conditions uniquely determine the state of the world at all consecutive moments of time. In particular, whether a person will raise his/her hand at this moment is uniquely determined by the initial state of the world. This conclusion is in clear contradiction with our intuition, according to which we have a free will: in particular, each of us can decide, at any given moment of time, whether he/she raises a hand or not (unless, of course, there is some problem with this action).

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In quantum physics, the state of the world – as described by the so-called wave function – is also uniquely determined by the initial conditions. However, the wave function does not uniquely determine the resulting observation and action, it only determines the probabilities of different actions. This is still in conflict with our intuition: whatever the probability is, one day a person can raise his hand every minute, the other day not at all.

Some people seriously believe that there is no free will, but this leads to weird conclusions. What can we do about this contraction? Physical equations have been supported by millions of experiments, many of them very accurate, so it is difficult to claim that physics is wrong. But it is still very difficult to believe that all our actions are pre-determined. A few people believe that: e.g., Einstein seriously believed in this; see, e.g., [1]. For example, he loved to go yachting along, to the horror of his friends who knew that he could not swim. To them, he replied that since everything is pre-determined by the initial state of the Universe (he did not believe in quantum physics), it is also pre-determined whether he will drown or not, so none of his actions can change that. This is definitely a weird argument. All our civilization is based on the assumption that we are responsible for our actions, that we have an ability not to harm others. Otherwise, why would we punish criminals if their actions have been uniquely pre-determined?

But can we come up with more physical arguments in defense of free will? To many physicists, arguments based on our intuition are not very convincing: after all, many quantum phenomena contradict our intuition, but these phenomena are still observed in real life. To deal with the conflict between physics and free will, we need to come up with more physical arguments in favor of free will.

What we do in this paper. In this paper, we provide exactly this – we provide physical arguments in favor of free will.

2 Our Physical Argument in Favor of Free Will

Motivation for our argument. The main motivation for our argument comes from cosmology. There exist many different solutions to Einstein's equations of General Relativity, equations that determine the geometry of our space-time. Some of these solutions are *generic*, in the sense that a small modification of this solution still keeps it quantitatively the same. Other solutions are *degenerate*, in the sense that no matter how small modifications we allow, there is always a modification that leads to a drastic change of space-time. Physicists believe that the actual solutions are the generic ones; see, e.g., [2, 3].

There are some arguments in favor of this belief. For example, according to quantum physics, there are always quantum fluctuations, they are small, but they are there. So, even if we start with a degenerate solution, such a fluctuation will transform the original degenerate solution into a generic one.

What we will do. We will show that:

- physical theories with no free will – in which the future state is uniquely determined by the initial state of the Universe – are degenerate,
- theories in which, for each initial state, there are several different future states – opening the room for free will – are generic.

3 What is generic: towards a precise definition

Intuitive idea of generic: a reminder. We say that a set G is generic if:

- for each element of this set, all sufficiently close objects are also in S, and
- for each element which is not in this set, no matter how close to this element we look, we will always find an element which is in G.

Let us transform this idea into a precise definition. The usual understanding of two objects x and x' being close is that the distance $d(x,x')$ between these objects (defined in some reasonable way) is small. So, we naturally arrive at the following definition.

Definition 1. *Let A be a set with a metric $d(x,x')$. A subset G of the set A is called* generic *if it satisfies the following two conditions:*

$$\forall x \in G\, \exists \varepsilon > 0\, \forall x'\, (d(x,x') \le \varepsilon \to x' \in G) \text{ and}$$

$$\forall x \notin G\, \forall \varepsilon > 0\, \exists x' \in G\, (d(x,x') \le \varepsilon).$$

Comment. In mathematical terms:

- a set with a metric is called a *metric space*,
- a set that satisfies the first condition is called *open*, and
- a set that satisfies the second condition is called *everywhere dense*.

4 How can we apply this idea to the free will problem?

What is the free will problem? In traditional physics, once we know the initial conditions, we can uniquely determine the future state. In other words, the set x of all possible future states consists of a single state $x = \{s\}$. To provide room for free will, we need to have a theory in which the initial condition does not uniquely determine the future state, i.e., in which the set x of all possible future states has several elements.

In most physical theories, the set of all possible states is continuous. In particular, for each state s and for each $\varepsilon > 0$, there exists a ε-close state $s' \neq s$, i.e., a state for which $d(s,s') \le \varepsilon$. Let us describe this in precise terms.

Definition 2. *We say that a set S is* continuous *if for every $s \in S$ there exists an $s' \neq s$ for which $d(s,s') \leq \varepsilon$.*

How to describe the distance between the sets of states? To apply the generic idea to our problem, we need to describe the distance between the sets of states. There is a known natural way to do it – known as the *Hausdorff metric*:

Definition 3. *Let S be a metric space, and x and x' two subsets of S. The* Hausdorff distance *$d_H(x,x')$ is the smallest ε for which the following two conditions are satisfied:*

- *for each $s \in x$, there exists $s' \in x'$ for which $d(s,s') \leq \varepsilon$, and*
- *for each $s' \in x'$, there exists $s \in x$ for which $d(s,s') \leq \varepsilon$.*

Now, we are ready to formulate and prove our main result.

5 Our main result: formulation and proof

Proposition. *Let S be a continuous metric space, let A be the set of all its non-empty subsets, let n be a positive integer, and let G_n denote the class of all the sets x that have at least n elements. Then, the class G_n is generic.*

Discussion. So, the general physical idea that in the real world, everything is generic, leads us to a conclusion that for each initial condition and for each integer n – no matter how large it is – there should be at least n different future states. In other words, for each initial state, there cannot be finitely many future states, there should be infinitely many future states.

Of course, by itself, this does not prove that there is free will, but it provides the *possibility* of free will – and thus, eliminates the discrepancy between physics and free will.

Proof.

1°. Let us first prove that the set G_n is open.

Let x be any set from G_n. By definition of the class G_n, the set x contains at least n different elements. Let us pick any n different elements from x, we will denote them $s_1, \ldots, s_n$. Since all these elements are different, we have $d(s_i, s_j) > 0$ for all $i \neq j$. Let δ denote the smallest of the positive values $d(s_i, s_j)$ corresponding to all $i \neq j$, and let us take $\varepsilon = \delta/3$. We will show that if for some set x', we have $d_H(x,x') \leq \varepsilon$, then the set x' also belongs to G_n, i.e., that the set x' contains at least n different elements.

Indeed, by the definition of Hausdorff metric, the inequality $d_H(x,x') \leq \varepsilon$ implies that for every $s \in x$ there exists a point $s' \in x'$ for which $d(s,s') \leq \varepsilon$. In particular, for each i from 1 to n, there exists a point $s'_i \in x'$ for which $d(s_i, s'_i) \leq \varepsilon$. Let us prove that all n elements $s'_1, \ldots, s'_n$ are different. Indeed, for each $i \neq j$, the triangle inequality implies that

$$d(s_i,s_j) \le d(s_i,s_i') + d(s_i',s_j') + d(s_j',s_j').$$

If we subtract $d(s_i,s_i')$ and $d(s_j',s_j')$ from both sides of this inequality, we conclude that

$$d(s_i,s_j) - d(s_i,s_i') - d(s_j',s_j') \le d(s_i',s_j').$$

Here, $d(s_i,s_j) \le \delta = 3\varepsilon$, $d(s_i,s_i') \le \varepsilon$, and $d(s_j',s_j') \le \varepsilon$. Thus, we conclude that

$$d(s_i',s_j') \ge 3\varepsilon - \varepsilon - \varepsilon = \varepsilon > 0,$$

so indeed all n points s_i' are different, and $x' \in G_n$.

2°. To complete the proof, let us prove that the set G_n is dense.

Denseness means that for every non-empty set x that is not in G_n and for every $\varepsilon > 0$, there exists a set x' from G_n for which $d(x,x') \le \varepsilon$.

Indeed, let us pick any element s_1 from the given set x. Let us use the fact that the set S is continuous to construct elements $s_2,\ldots,s_n$ that will be all different from each other and for which

$$d(s_1,s_n) < d(s_1,s_{n-1}) < \ldots < d(s_1,s_2) \le \varepsilon.$$

Finding s_2 is easy: since the set S is continuous, there exists a state $s_2 \ne s_1$ for which $d(s_1,s_2) \le \varepsilon$.

Let us now assume that we have already found the elements $s_1,\ldots,s_i$, and let us show how to select the next element s_{i+1}. Indeed, due to continuity of the set S, for $\varepsilon_0 = d(s_1,s_i)/2$, there exists a point $s_{i+1} \in S$ that is different from s_1 and for which $d(s_1,s_{i+1} \le \varepsilon_0$ – and thus, $d(s_1,s_{i+1}) < d(s_1,s_i)$.

Since all the distances $d(s_1,s_i)$ are different, this means that all the elements s_i are different from each other. Thus, the union $x' \stackrel{\text{def}}{=} x \cup \{s_2,\ldots,s_n\}$ has at least n different elements – and therefore, x' belongs to the class G_n.

Let us show that $d_H(x,x') \le \varepsilon$. Indeed:

- Every element s from the set x also belongs to x', so for $s' = s$, we have $d(s,s') = 0 \le \varepsilon$.
- By definition of the set x', every element s from x' either belongs to the set x or is equal to one of the newly constructed elements s_i. In the first case, for $s' = s \in x$, we have $d(s,s') = 0 \le \varepsilon$. In the second case, when $s = s_i$, we can take $s_1 = s' \in x$, then, by the construction of s_i, we have $d(s,s') = d(s_1,s_i) \le \varepsilon$.

The denseness is proven, and so is the proposition.

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References

1. W. Isaacson, *Einstein: His Life and Universe*, Simon & Schuster, New York, 2008.
2. C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*, Princeton University Press, Princeton, New Jersey, 2017.
3. K. S. Thorne and R. D. Blandford, *Modern Classical Physics: Optics, Fluids, Plasmas, Elasticity, Relativity, and Statistical Physics*, Princeton University Press, Princeton, New Jersey, 2021.