60/60 = 100/100

CS 3350 Automata, Computability, and Formal Languages
Fall 2016, Test 1

Name: [redacted]

1-2. Use a general algorithm to design a non-deterministic finite automaton recognizing the language $(1 \cup 0)^*1$. After that, use the general algorithm to design a deterministic finite automaton recognizing this same language.

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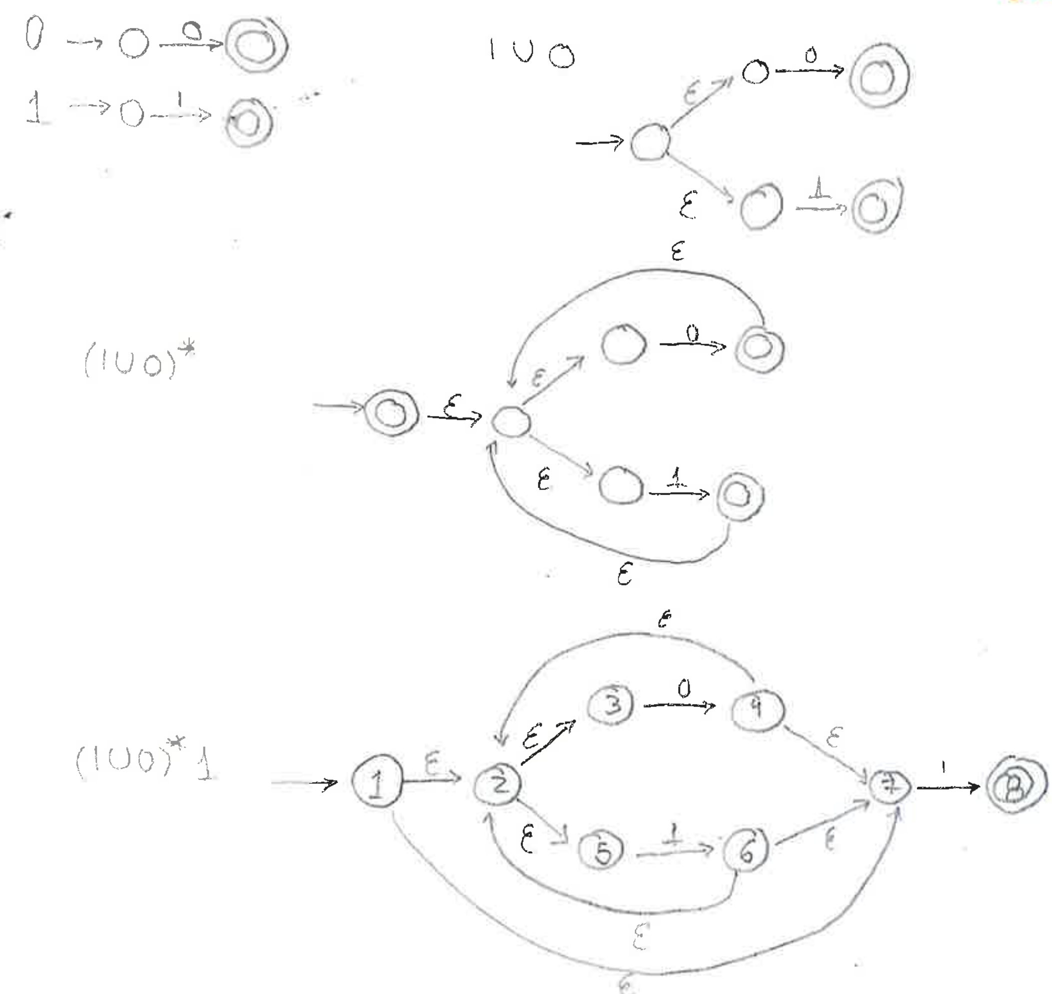

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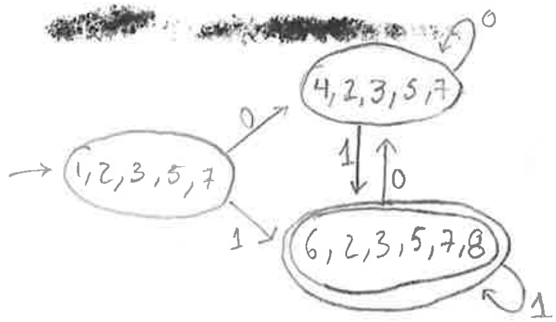

3. Use a general algorithm to transform the following finite automaton into the corresponding regular expression. This automaton has two states: a state q1 which is a start state, and a state q2 which is a final state. In the state q1, 0 leads to q2, and 1 leads to q1. In the state q2, 0 leads to q2, and 1 leads to q1.

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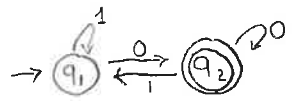
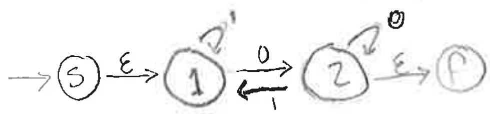
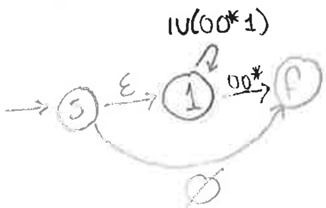




deleting q_2 first



$R'_{s1} = R_{s1} \cup (R_{s2} R_{22}^* R_{21})$
$= \Lambda \cup (\emptyset \ldots) = \Lambda$

$R'_{11} = R_{11} \cup (R_{12} R_{22}^* R_{21})$
$= 1 \cup (00^* 1)$

$R'_{1f} = R_{1f} \cup (R_{12} R_{22}^* R_{2f})$
$\emptyset \cup (00^* \Lambda) = 00^*$

$R'_{sf} = R_{sf} \cup (R_{s2} R_{22}^* R_{2f})$
$\emptyset \cup (\emptyset \ldots) = \emptyset$

$R'_{sf} = R_{sf} \cup (R_{s1} R_{11}^* R_{1f})$
$\emptyset \cup (\Lambda (1\cup(00^*1))^* 00^*) = (1\cup(00^*1))^* 00^*$

4. On the example of the automaton from Problem 3, explain, in detail, how the sequence 001100 will be presented as xyz according to the pumping lemma. For this sequence, check -- by tracing step-by-step -- that the sequence xy^iz for $i = 2$ is indeed accepted by the automaton.

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|0|0|1|1|0|0|

q_1 q_2 q_2 q_1 q_1 q_2 q_2

x y z

X- before 1st repeating state

Y- between 1st and 2nd rep.

z- after 2nd rep

$x=0$ $y=0$ $z=1100$

The first repeating state is q_2

so x is what is before [illegible] 0,

[illegible] in between the rep [illegible] 0)

and z is what is after the rep which is 1100

$xyyz = \underset{x}{0}\underbrace{00}_{yy}\underbrace{1100}_{z}$

|0|0|0|1|1|0|0|

q_1 q_2 q_2 q_2 q_1 q_1 q_2 q_2

xyyz is accepted

5. Use the Pumping Lemma to prove that the language L consisting of all the words of the type *www* is not regular, where *w* can be any word. Here:

- if *w* is an empty string, we get the word ~~*a*~~ Λ,
- if *w* is *ab*, we get *ababab*,

etc.

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L = { www , where w is any word } is not regular

Proof by Contradiction:

Let us asume that L is regular then by pumping lemma $\exists p$ such that every word s from L whose length is $\geq p$ can be represented as $s = xyz$, where $len(y) > 0$, $len(xy) \leq p$ and for every i, $xy^iz \in L$

Lets take $w = a^p b^p$ then $s = a^p b^p a^p b^p a^p b^p$. Here the $len(s) = 6p \geq p$ so by pumping lemma there exist corresponding x, y and z. Since s starts with xy and $len(xy) \leq p$, this means that x and y are in the first set of a's. So when we take xyyz we add a's to the first set of a's but the # of a's in the other sets does not change. So the # of a's in the first w is no longer equal to the # of a's in the remaining w's. So $xyyz \notin L$, but according to pumping lemma $xyyz \in L$, a contradiction. This contradiction shows that our assumption is wrong so L is not regular.

6. Use the general algorithm that we had in class to design a context-free grammar which generates exactly the words accepted by the automaton from Problem 3. Show how the word 001100 will be generated by this grammar.

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$Q_1 \to 0Q_2$

$Q_1 \to 1Q_1$

$Q_2 \to 0Q_2$

$Q_2 \to 1Q_1$

$Q_2 \to \Lambda$