60/60 = 100/100

CS 3350 Automata, Computability, and Formal Languages
Fall 2016, Test 1

Name: [redacted]

1-2. Use a general algorithm to design a non-deterministic finite automaton recognizing the language $(1 \cup 0)^{*}1$. After that, use the general algorithm to design a deterministic finite automaton recognizing this same language.

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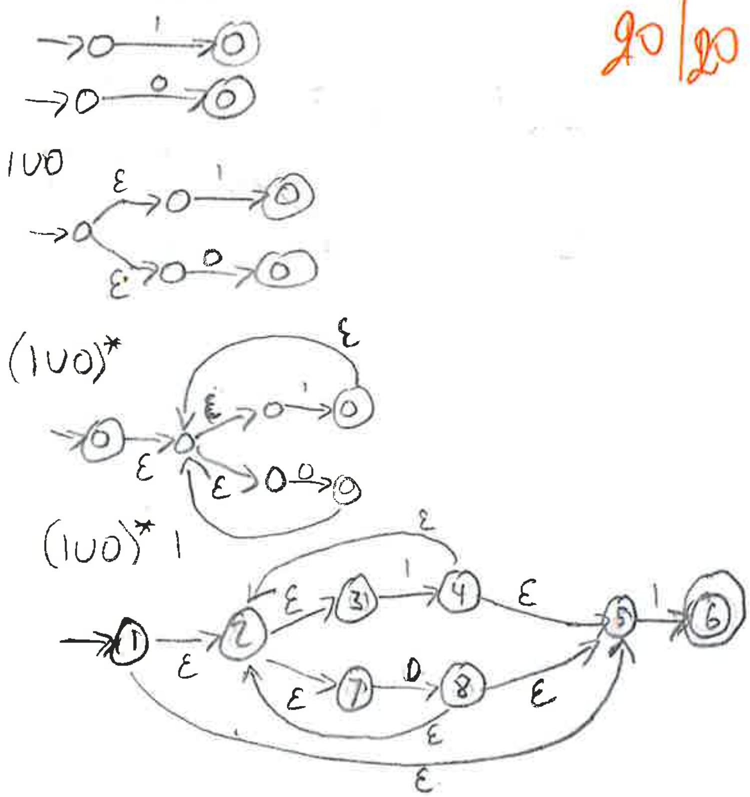
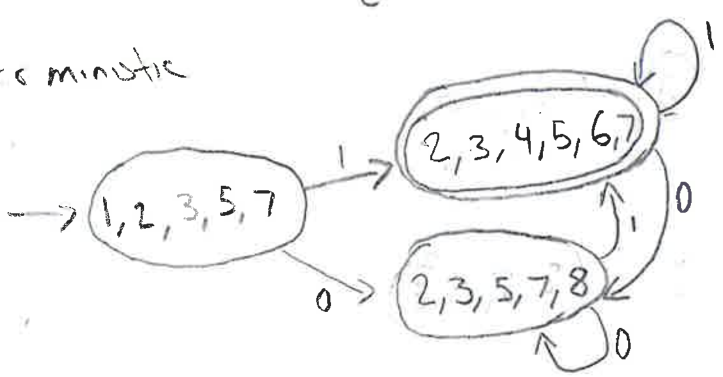


Deterministic



3. Use a general algorithm to transform the following finite automaton into the corresponding regular expression. This automaton has two states: a state q1 which is a start state, and a state q2 which is a final state. In the state q1, 0 leads to q2, and 1 leads to q1. In the state q2, 0 leads to q2, and 1 leads to q1.

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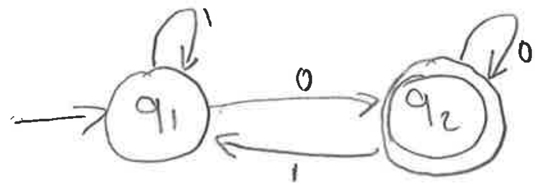
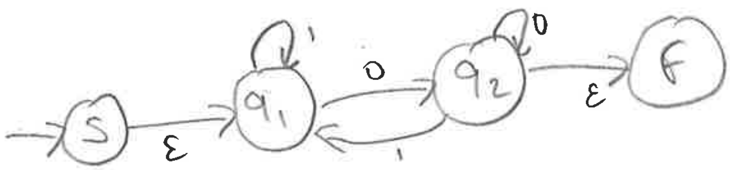
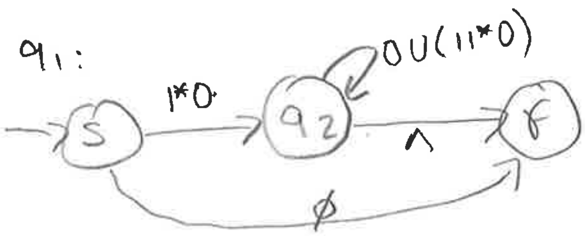




Eliminate q_1:



$$R'_{ij} = R_{ij} \cup (R_{ik} R_{kk}^* R_{kj})$$

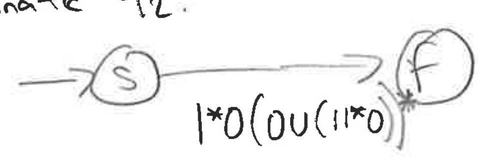
$$R'_{sF} = R_{sF} \cup (R_{sq_1} R_{q_1q_1}^* R_{q_1F}) = \emptyset \cup (\Lambda 1^* \emptyset) = \emptyset.$$

$$R'_{q_2F} = R'_{q_2F} \cup (R_{q_2q_1} R_{q_1q_1}^* R_{q_1F}) = \Lambda \cup (1 1^* \emptyset) = \Lambda$$

$$R'_{sq_2} = R_{sq_2} \cup (R_{sq_1} R_{q_1q_1}^* R_{q_1q_2}) = \emptyset \cup (\Lambda 1^* 0) = 1^*0$$

$$R'_{q_2q_2} = R_{q_2q_2} \cup (R_{q_2q_1} R_{q_1q_1}^* R_{q_1q_2}) = 0 \cup (11^*0)$$

Eliminate q_2:



$$R'_{sF} = R_{sF} \cup (R_{sq_2} R_{q_2q_2}^* R_{q_2F}) = \emptyset \cup (1^*0\,(0\cup(11^*0))^* \Lambda) = 1^*0(0\cup(11^*0))^*$$

4. On the example of the automaton from Problem 3, explain, in detail, how the sequence 001100 will be presented as xyz according to the pumping lemma. For this sequence, check -- by tracing step-by-step -- that the sequence xy^iz for i = 2 is indeed accepted by the automaton.

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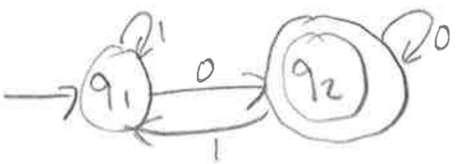
By pumping lemma, there exists a number p such that every word s from L whose length is $\geq p$ can be represented as $s = xyz$, where $len(y) > 0$, $len(xy) \leq p$ and $xy^iz \in L$ for every $i = 0, 1, 2, \ldots$

$L = 1^*0(0 \cup (11^*0))^*$



$s = |0|0|1|1|0|0| = xyz$

$q_1 \; q_2 \; q_2 \; q_1 \; q_1 \; q_2 \; q_2$

$\underbrace{0}_{x}\,\underbrace{0}_{y}\,\underbrace{1100}_{z}$

$p = 2$ states

$len(y) = 1$

$len(xy) = 2 \leq p$

Sequence $xy^2z = xyyz$

$|0|0|0|1|1|0|0|$

$q_1 \; q_2 \; q_2 \; q_2 \; q_1 \; q_1 \; q_2 \; q_2$

$\underbrace{0}_{x}\,\underbrace{0}_{y}\,\underbrace{0}_{y}\,\underbrace{1100}_{z}$

Sequence xyyz ends in final state q_2 it is accepted by the automaton.

5. Use the Pumping Lemma to prove that the language L consisting of all the words of the type *www* is not regular, where *w* can be any word. Here:

- if *w* is an empty string, we get the word Λ,
- if *w* is *ab*, we get *ababab*,

etc.

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$L = www$ {w is any word} is not regular

Proof by contradiction. Let's assume L is regular, then by pumping lemma, there exists a number p such that every word s from L whose length is $\geq p$ can be represented as $s = xyz$, where $len(y) > 0$, $len(xy) \leq p$, and $xy^iz \in L$ for every $i = 0, 1, 2, \ldots$

Let's take $s = 0^p1^p\,0^p1^p\,0^p1^p$. Here, $Len(s) = 6p \geq p$, so, by pumping lemma, there exists corresponding x, y, and z, Since S starts with xy, and $len(y) \leq p$, this means that x and y are in 0's, so when we take $xyyz$, we add 0's, but the number of 0's in the other 2 words does not change, so # of 0's is no longer equal on every word:

$$S = \underbrace{0^{>p}1^p}_{x}\ \underbrace{0^p1^p}_{w}\ \underbrace{0^p1^p}_{w}$$ does not belong to L

So, $xyyz \notin L$. According to pumping lemma $xyyz \in L$ - a contradiction.

This contradiction shows that our assumption is wrong, so L is not regular. The proposition is proven.

6. Use the general algorithm that we had in class to design a context-free grammar which generates exactly the words accepted by the automaton from Problem 3. Show how the word 001100 will be generated by this grammar.

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$q_1 \to 1q_1$

$q_1 \to 0q_2$

$q_2 \to 1q_1$

$q_2 \to 0q_2$

$q_2 \to \Lambda$

0 0 1 1 0 0