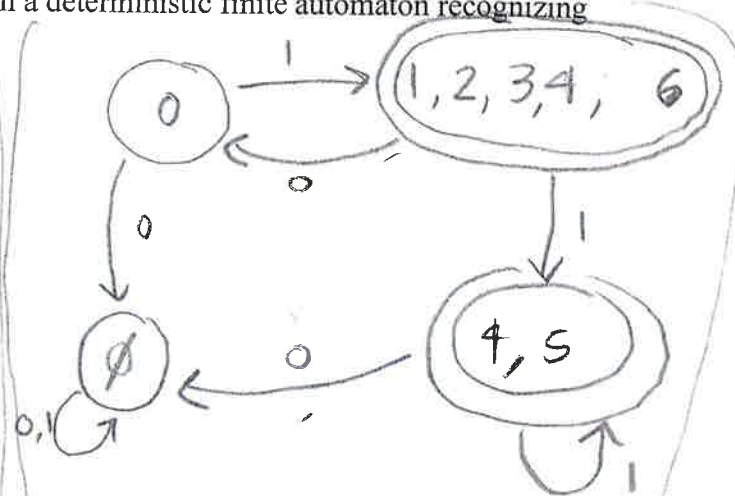


$$\frac{10}{10}$$

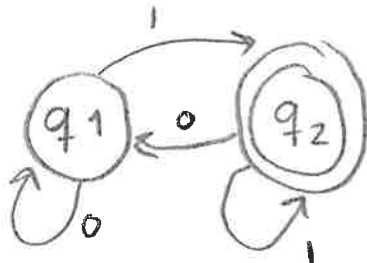
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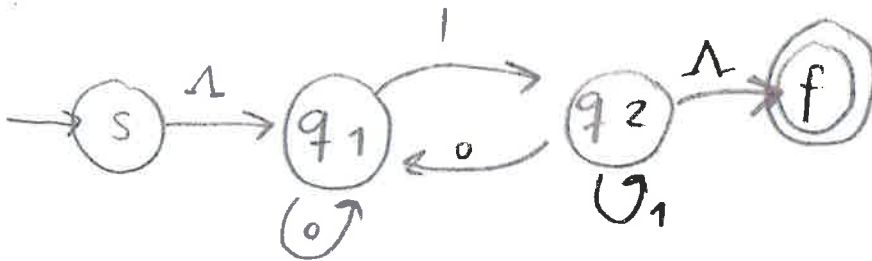
Deterministic finite automaton

3-4. Use a general algorithm to transform the following finite automaton into the corresponding regular expression.

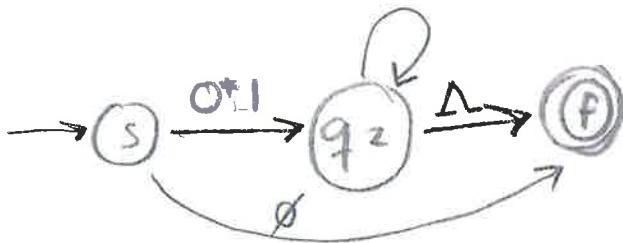
- This automaton has two states: a state q_1 which is a start state, and a state q_2 which is a final state.
- In the state q_1 , 1 leads to q_2 , and 0 leads to q_1 .
- In the state q_2 , 1 leads to q_2 , and 0 leads to q_1 .



1. Add new start and final



2. Delete q_1 , $U(00^*1)$

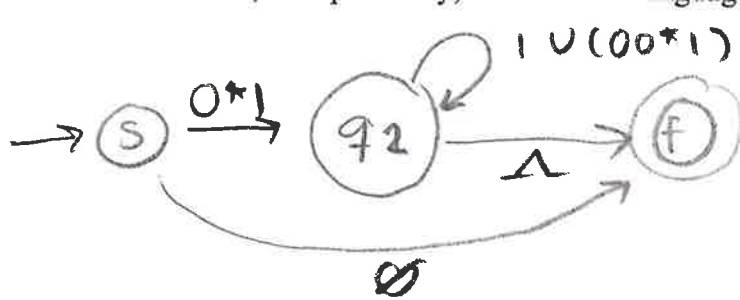


$$\begin{aligned}
 R'_{s2} &= R_{s2} \cup (R_{s1} R_{11}^* R_{12}) \\
 &= \emptyset \cup (\Lambda 0^* 1) \\
 &= 0^* 1
 \end{aligned}$$

$$\begin{aligned}
 R'_{sf} &= R_{sf} \cup (R_{s1} R_{11}^* R_{1f}) \\
 &= \emptyset \cup (\Lambda 0^* \emptyset) = \emptyset
 \end{aligned}$$

$$\begin{aligned}
 R'_{22} &= R_{22} \cup (R_{21} R_{11}^* R_{12}) \\
 &= 1 \cup (0 0^* 1)
 \end{aligned}$$

$$\begin{aligned}
 R'_{2f} &= R_{2f} \cup (R_{21} R_{11}^* R_{1f}) \\
 &= \Lambda \cup (0 0^* \emptyset) = \Lambda
 \end{aligned}$$



3. Delete q_2



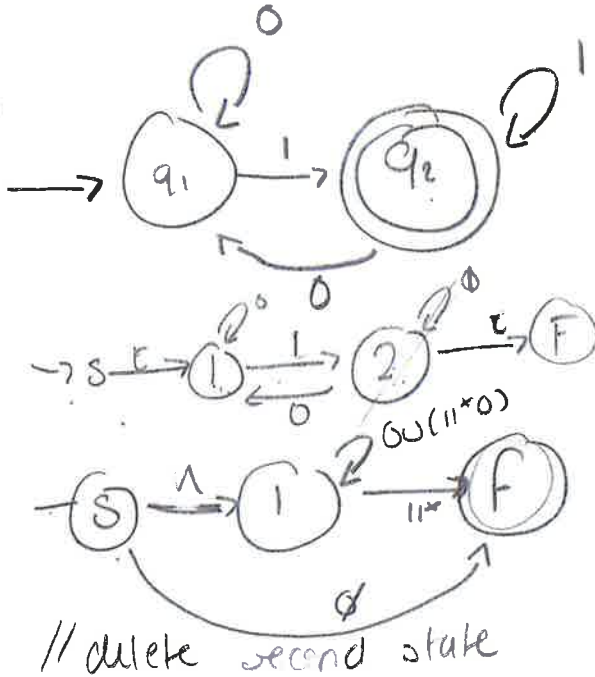
$$R'_{sf} = R_{sf} \cup (R_{s2} R_{22}^* R_{2f})$$

$$= \emptyset \cup (0^*1(1 \cup (00^*1))^* \Lambda)$$

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3-4. Use a general algorithm to transform the following finite automaton into the corresponding regular expression.

- This automaton has two states: a state q_1 which is a start state, and a state q_2 which is a final state.
- In the state q_1 , 1 leads to q_2 , and 0 leads to q_1 .
- In the state q_2 , 1 leads to q_2 , and 0 leads to q_1 .



$$R'_{S1} = R_{S1} \cup (R_{S2} R_{22}^* R_{21})$$

$$\Lambda \cup (\emptyset \dots) = \Lambda$$

$$R'_{11} = R_{11} \cup (R_{12} R_{22}^* R_{21})$$

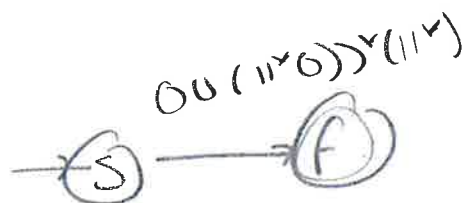
$$0 \cup (1 1^* 0) = 0 \cup (1^* 0)$$

$$R'_{1F} = R_{1F} \cup (R_{12} R_{22}^* R_{2F})$$

$$\emptyset \cup (1 1^* \Lambda) = 1^*$$

$$R'_{SF} = R_{SF} \cup (R_{S2} R_{22}^* R_{2F})$$

$$\emptyset \cup (\emptyset \dots) = \emptyset$$



$$R'_{SF} = R_{SF} \cup (R_{S1} R_{11}^* R_{1F})$$

$$\emptyset \cup (\Lambda (0 \cup (1^* 0))^* 1^*)$$

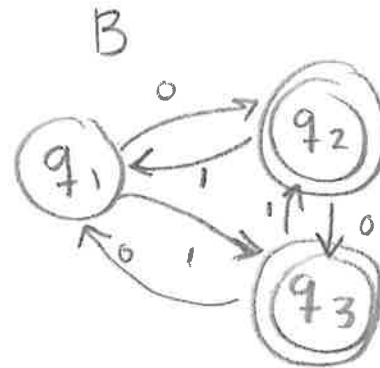
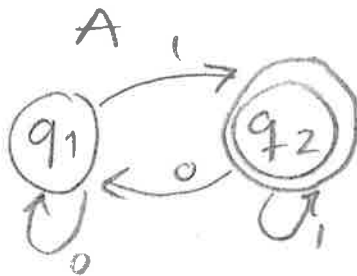
$$\boxed{(0 \cup (1^* 0))^* (1^*)}$$

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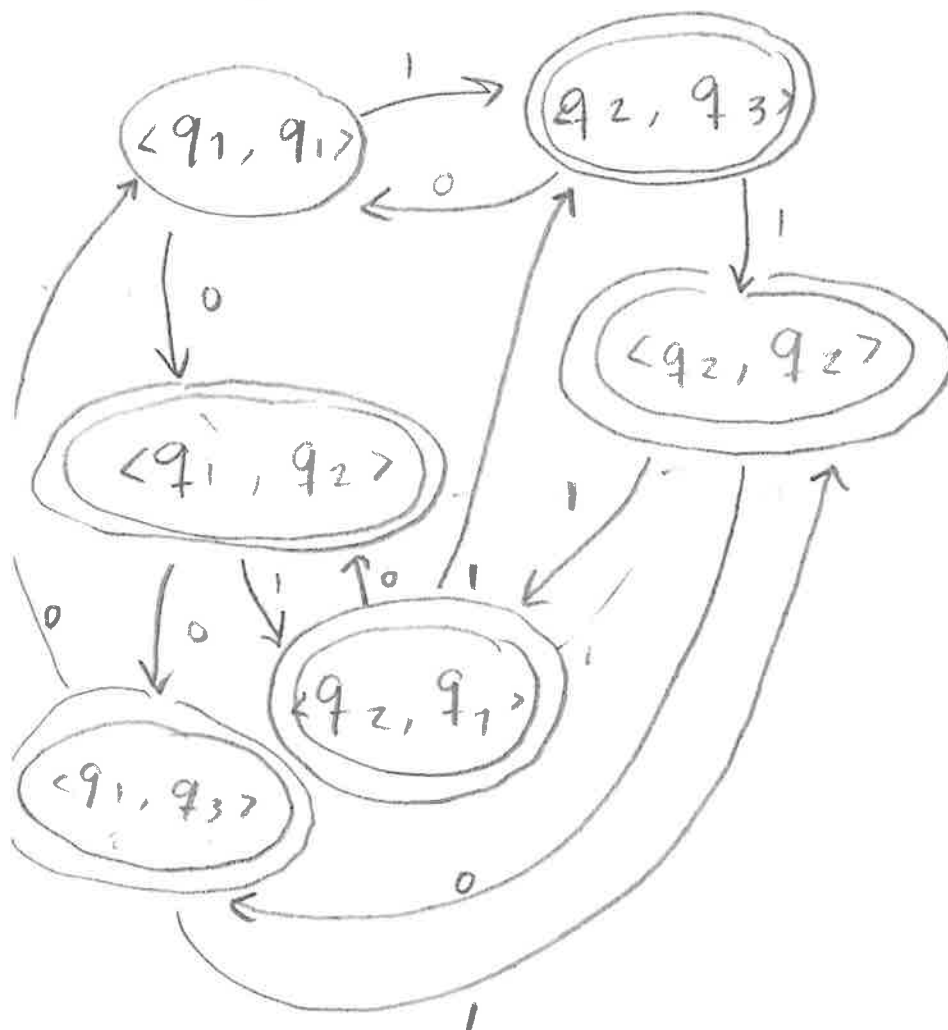
5. Let A be a language recognized by the automaton from Problem 3, and let B be the language corresponding to the following automaton:

- This automaton has 3 states, the starting state q_1 , and the two final states q_2 and q_3 .
- When you see 0, from q_1 you move to q_2 , from q_2 you move to q_3 , and from q_3 you move to q_1 .
- When you see 1, from q_1 you move to q_3 , from q_2 you move to q_1 , and from q_3 you move to q_2 .

Use the algorithm that we learned in class, with pairs of states from two automata as states of the new deterministic automaton, to describe the automata recognizing the union and the intersection of these two languages.



$A \cup B$



$A \cap B$

