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1. Formulate Church-Turing thesis. Is it a mathematical theorem? Is it a statement about the physical world?

Thesis: Anything that can be computed on any physical device can also be computed on a turing machine.

- It is not a mathematical theorem, but a statement about the physical world.

2. Prove that the halting problem is not algorithmically solvable. $\frac{10}{10}$

No algorithm is possible that given a problem p and data d checks if p halts on d.
Prove that a halt checker is impossible.

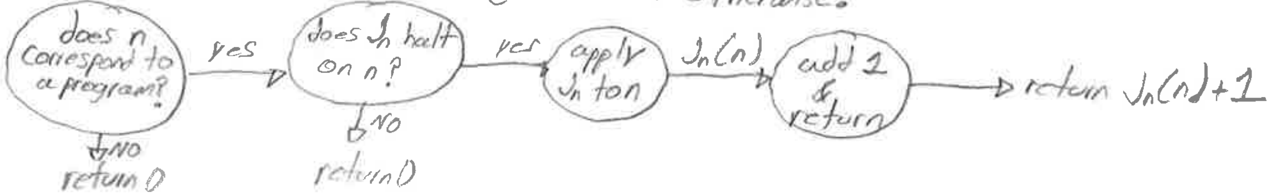
$p, d \to \boxed{} \to \begin{cases} \text{true if } p \text{ halts on } d. \\ \text{False otherwise.} \end{cases}$

A program is a series of ASCII symbols which are used to turn every symbol into 0's & 1's, that put together form a long sequence of 0's & 1's. This sequence is what is stored in the pc when typing a program. Different binary strings may lead to the same number. To avoid that we append a 1 to the begining of the string and interpret it as a binary number.

$001_2 = 1_2$ $\quad 1001_2 = 9$ $\quad 11_2 = 3$

There is an algorithm that given a number n, checks if n corresponds to a program.
If n is valid, a compiler generates an executable file denoted by J_n.

By contradiction
Assume a halt checker exist.
Deffine Function $F(n) = \begin{cases} J_n(n)+1 \to \text{If } n \text{ corresponds to a program \& halts on } n. \\ 0 \longrightarrow \text{otherwise.} \end{cases}$



Let n_0 denote the number corresponding to the program that computes $F(n)$.
For every n, $J_{n_0}(n) = F(n)$

For $n = n_0$
$J_{n_0}(n_0) = F(n_0)$

By definition: n_0 is a number corresponding to a program.
J_0 is F, it always halts, so it halts on n_0 too.
Therefore, $J_0(n_0) = J_{n_0}(n_0) + 1$
$0 \neq 1$
This is a contradiction, thus a haltchecker does not exist.

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3. In the proof that the halting problem is not algorithmically solvable, we use a diagonal function f(n) which was defined as follows:

- $f(n) = j_n(n) + 1$ if n is a valid code of a Java program and j_n halts on n;
- $f(n) = 0$ otherwise.

Write down the first 4 values f(0), f(1), f(2), and f(3) of the diagonal function f(n) for the following case:

- $j_0(n) = 3$;
- $j_1(n) = n$;
- 2 is not a valid numerical code of a program; and
- $j_3(n) = n^2$.

$j_i \backslash n$	0	1	2	3
0	3	3	3	3
1	0	1	2	3
2	—	—	—	—
3	0	1	4	9
F(n)	4	2	0	10

$F(0) = J_0(0)+1 = 3+1 = 4$
$F(1) = J_1(1)+1 = 1+1 = 2$
$F(3) = J_3(3)+1 = 9+1 = 10$

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4. Not all algorithms are feasible, but, unfortunately, we do not have a perfect definition is feasibility. Give two examples:

- an example of an algorithm which is feasible according to the current definition but not practically feasible, and
- an example of an algorithm which is practically feasible but not feasible according to the current definition.

1) $t_n(x) = 10^{150} \cdot len(x)$ → Feasible by definition, but not in practice.

2) $t(x) = \exp(10^{-10} len(x))$ → Not feasible by definition, but feasible in practice.

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5. Briefly describe what is P, what is NP, and what is NP-hard. Give an example of an NP-hard problem. What do we gain and what do we lose when we prove that some problem is NP-hard?

P: Problems that can be solved in feasible time.

NP: Once there is a candidate for a solution, we can check, in feasible time, whether it is indeed a solution.

NP-hard: Every problem from the class NP can be reduced to this problem. This is, problems harder than those of NP.

Example: SAT

What we gain:

- Any method for solving a problem automatically leads to methods for solving all other problems.

What we lose:

- We cannot have an efficient algorithm that solves all problems feasibly.