CS 3350 Automata, Computability, and Formal Languages
Fall 2017, Test 1

60/60 = 100/100

Name: ______________________

General comments:

- you are allowed up to 5 pages of handwritten notes;
- if you need extra pages, place your name on each extra page;
- the main goal of most questions is to show that you know the corresponding algorithms; so, if you are running of time, just follow the few first steps of the corresponding algorithm;
- each question will be graded on its own merit; so, for example, if on one stage, you got a wrong non-deterministic automaton, but on the second stage, you correctly transform it into a deterministic one, you will get full credit for the second stage.

Good luck!

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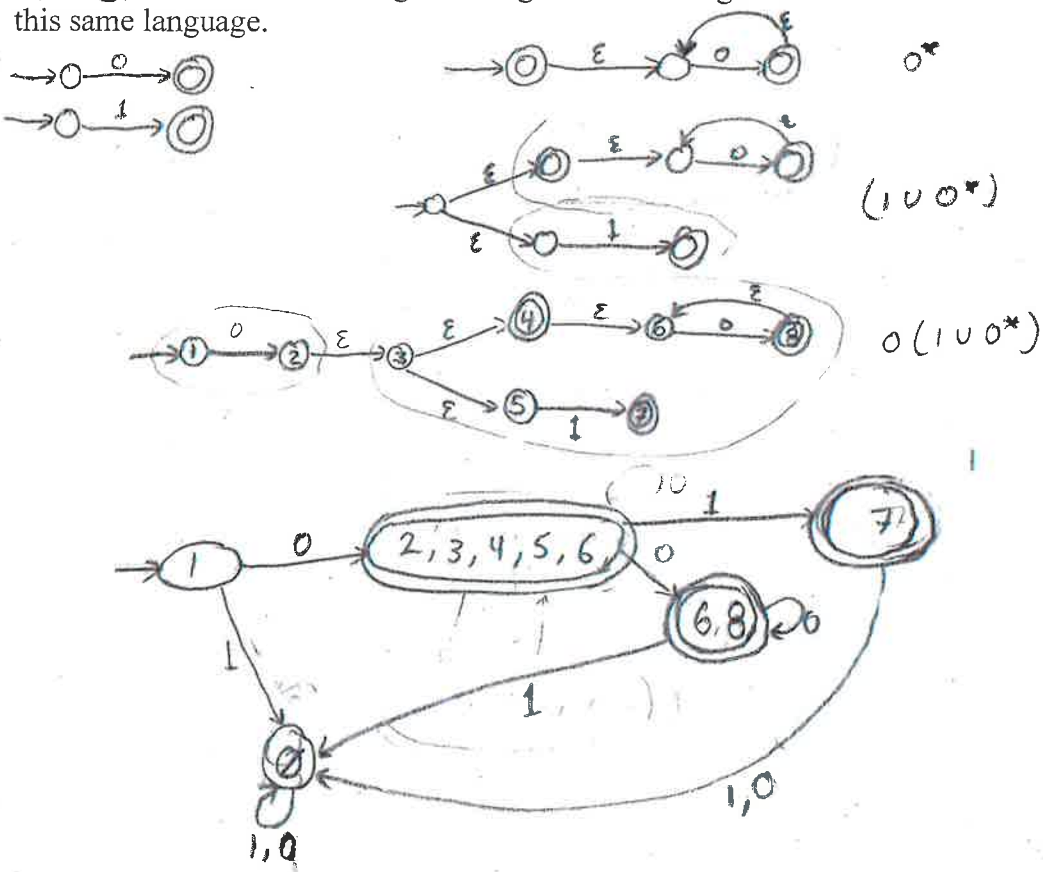
1-2. Use a general algorithm to design a non-deterministic finite automaton recognizing the language $0(1 \cup 0^*)$. After that, use the general algorithm to design a deterministic finite automaton recognizing this same language.



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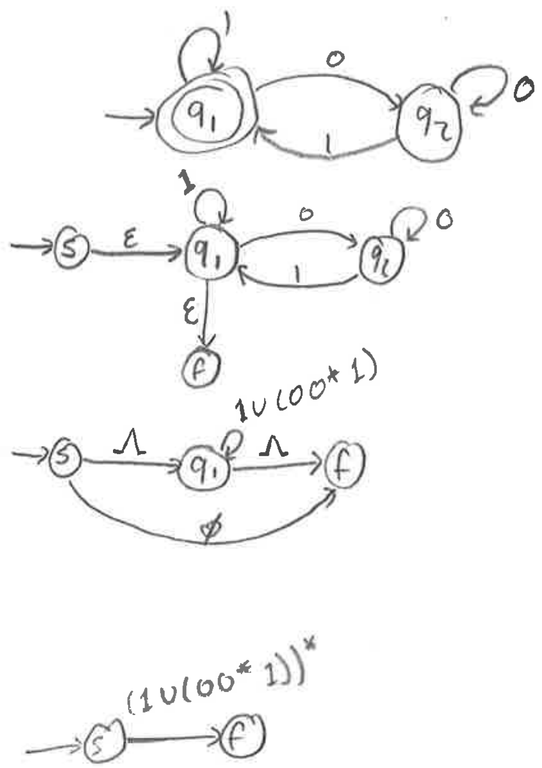
3-4. Use a general algorithm to transform the following finite automaton into the corresponding regular expression.

- This automaton has two states: a state q_1 which is a start state and a final state, and another state a state q_2.
- In the state q_1, 0 leads to q_2, and 1 leads to q_1.
- In the state q_2, 0 leads to q_2, and 1 leads to q_1.



$R'_{ij} = R_{ij} \cup (R_{ik} \; R^*_{kk} \; R_{kj})$

$R'_{sq_1} = R_{sq_1} \cup (R_{sq_2} \; R^*_{q_2q_2} \; R_{q_2q_1})$

$= \Lambda \cup (\emptyset \; 0^* \; 1)$

$= \Lambda$

$R'_{q_1q_1} = R_{q_1q_1} \cup (R_{q_1q_2} \; R^*_{q_2q_2} \; R_{q_2q_1})$

$= 1 \cup (0 \; 0^* \; 1)$

$R'_{q_1f} = R_{q_1f} \cup (R_{q_1q_2} \; R^*_{q_2q_2} \; R_{q_2f})$

$= \Lambda \cup (0 \; 0^* \; \emptyset)$

$= \Lambda$

$R'_{sf} = R_{sf} \cup (R_{sq_2} \; R^*_{q_2q_2} \; R_{q_2f})$

$= \emptyset \cup (\emptyset \; 0^* \; \emptyset)$

$= \emptyset$

$R'_{sf} = R_{sf} \cup (R_{sq_1} \; R^*_{q_1q_1} \; R_{q_1f})$

$= \emptyset \cup (\Lambda \; (1 \cup (00^*1))^* \; \Lambda)$

$= \emptyset \cup (1 \cup (00^*1))^*$

$= (1 \cup (00^*1))^*$

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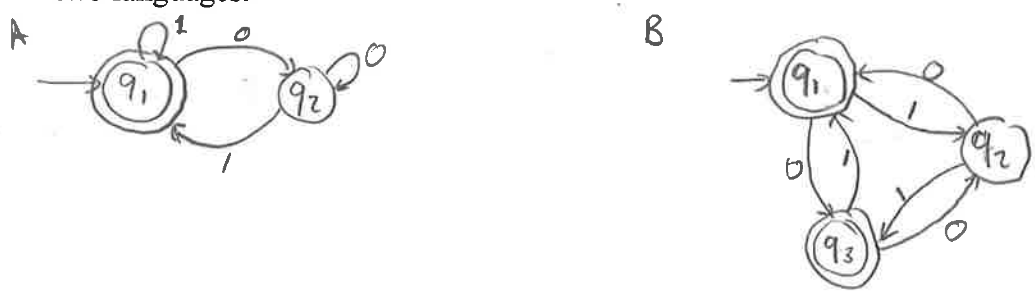
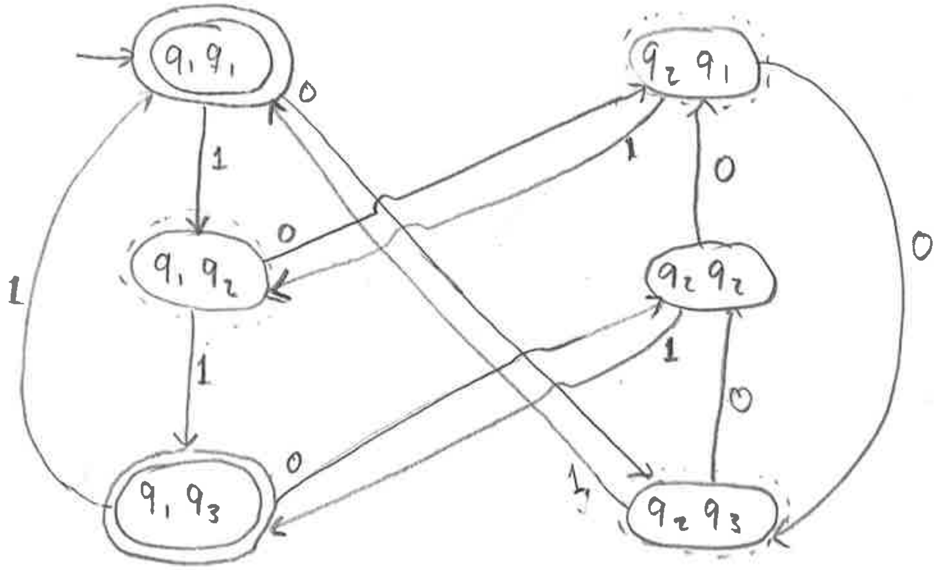
5. Let A be a language recognized by the automaton from Problem 3, and let B be the language corresponding to the following automaton:

- This automaton has 3 states, the starting state q_1 which is also final, a state q_2, and a final state q_3.
- When you see 0, from q_1 you move to q_3, from q_2 you move to q_1, and from q_3 you move to q_2.
- When you see 1, from q_1 you move to q_2, from q_2 you move to q_3, and from q_3 you move to q_1.

Use the algorithm that we learned in class, with pairs of states from two automata as states of the new deterministic automaton, to describe the automata recognizing the union and the intersection of these two languages.





6. Prove that the language $\{a^n b^{n+1}\} = \{\Lambda, abb, aabbb, aaabbbb, ...\}$ is not regular.

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$L = \{a^n, b^{n+1}\}$ is not regular

Proof by contradiction,

Let's assume that L is regular. Then by pumping Lemma there exists p s.t. $\forall w \in L\ (len(w) \geq p \rightarrow \exists\ x y z\ (w = x y z\ \&\ len(y) > 0\ \&\ len(xy) \leq p\ \&\ \forall i\ (xy^iz \in L)$. Let us take $w = a^p b^{p+1}$ and let's get a contradiction

$w = a^p b^{p+1} = a \cdot a \quad b \quad b \quad b$

$len(w) = 2p+1 \geq p$, so

$w = x y z \qquad len(xy) \leq p$

So, for this word, there exist x, y, z as in Pumping Lemma

Since $len(xy) \leq p$, x and y are among the first p symbols of the word w, and these symbols are a's

So y consists only of a's

By pumping lemma $x y y z$ is in L.

But when go from xyz to $xyyz$, we add y, i.e. we add a's and we do not add any b's, So $xyyz$ is not L.