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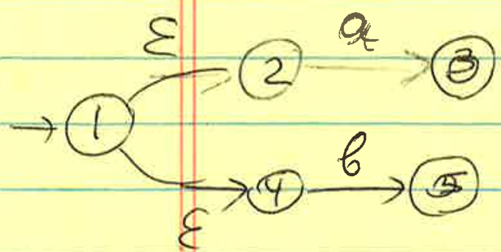
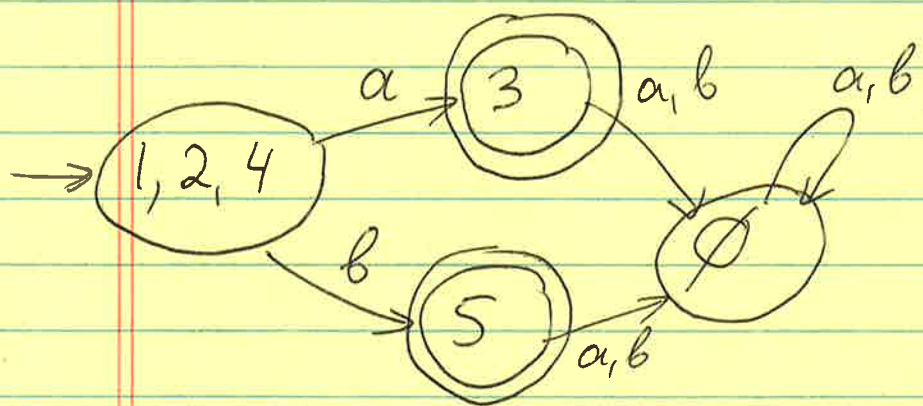
-1-

$a \cup b$



{ }



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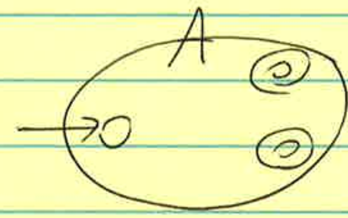
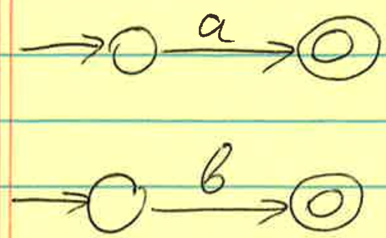
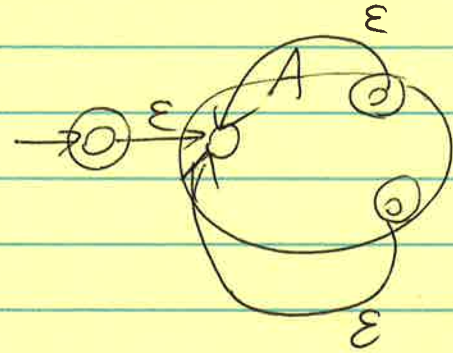
$a \cup b^*$







Kleene star:

- add a new starting state which is also final
- add jumps from new starting state to old starting state
- add jumps from old final states to old starting state

$\{a\}^* = \{\Lambda, a, aa, \ldots\}$

$\{a, b\}^* = \{\Lambda, a, b, aa, ab, ba, bb, \ldots\}$

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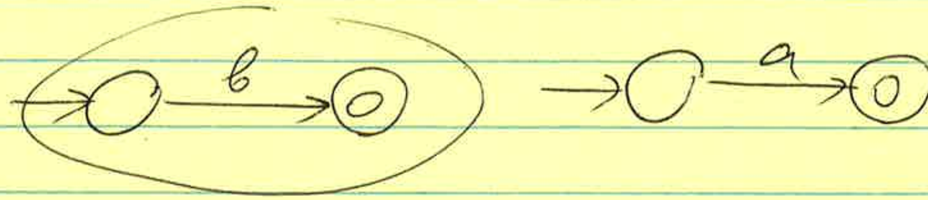
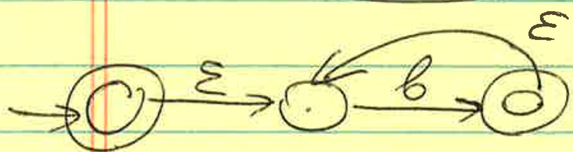
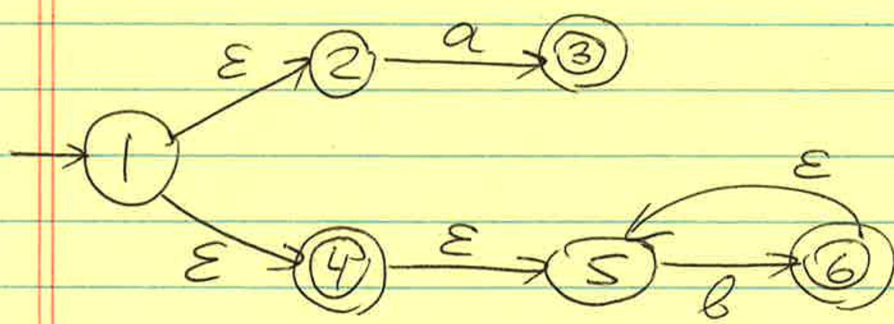
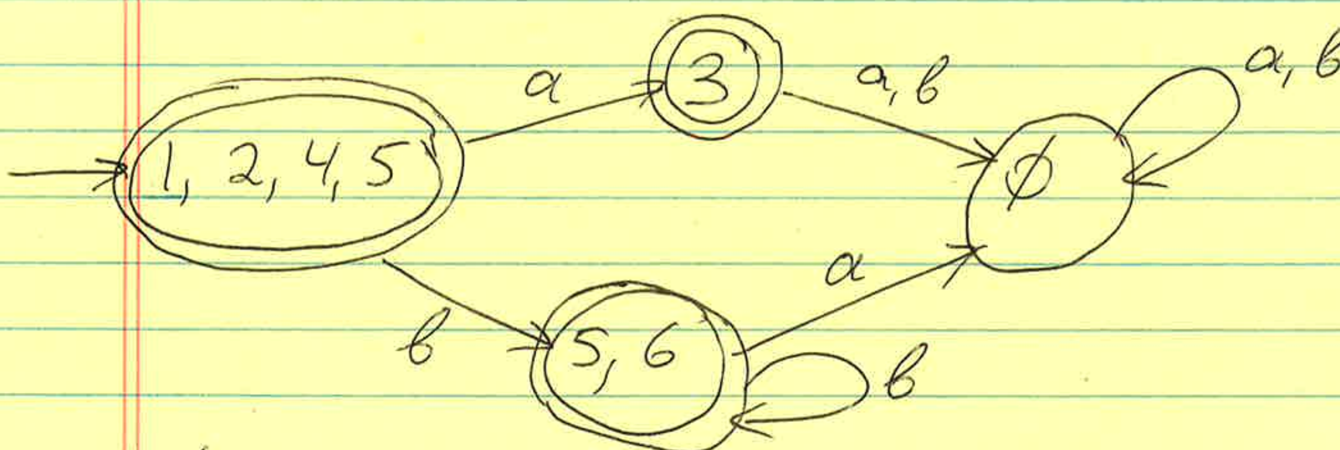
$a \cup b^*$









$L = \{\Lambda, a, b, bb, bbb, \ldots\} =$

$\{a\} \cup \{\Lambda, b, bb, bbb, \ldots\} = a \cup b^*$

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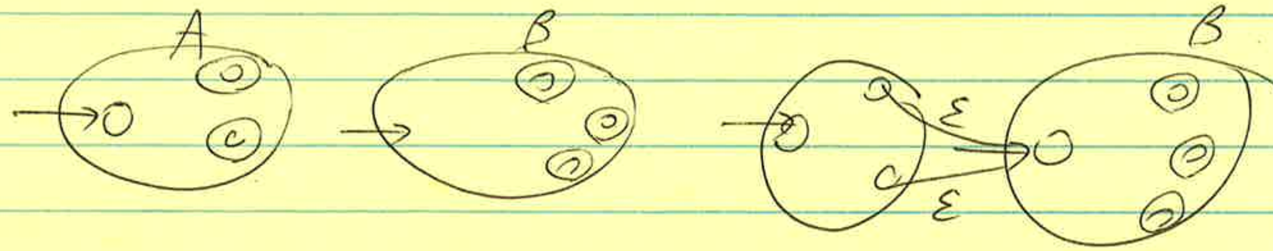
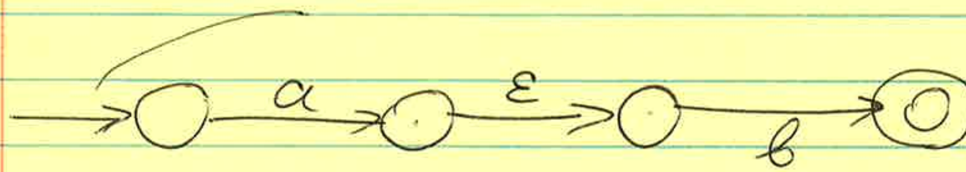
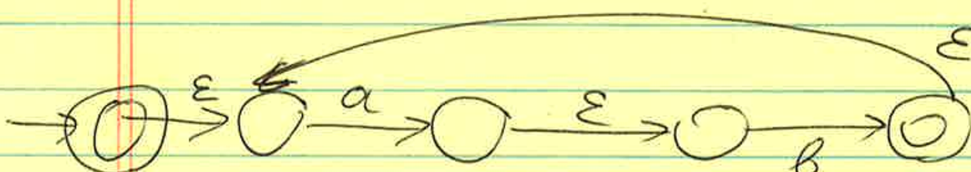
$ab \cup (ab)^*$

$\boxed{ab}$

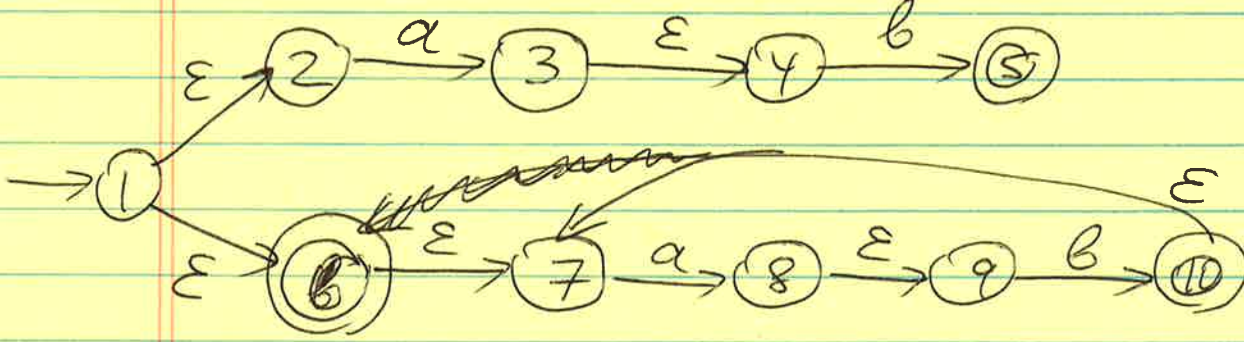




ab



$(ab)^*$



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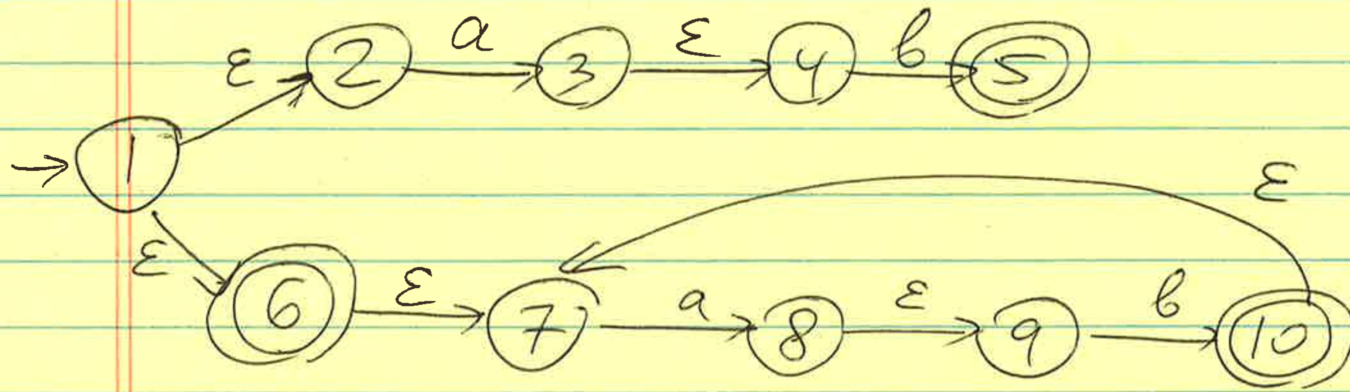
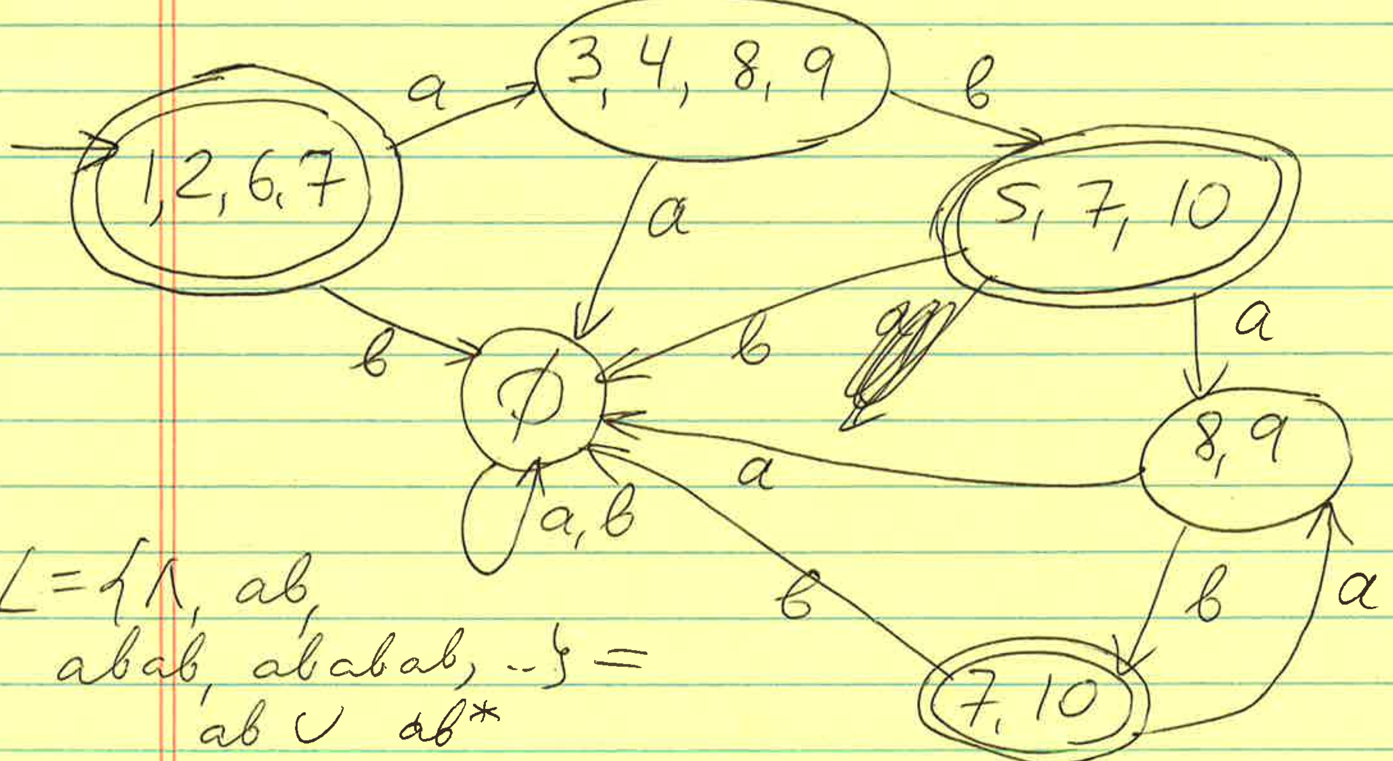
$ab \cup (ab)^* = \{ab, \Lambda, abab, ababab, \ldots\}$





$L = \{\Lambda, ab, abab, ababab, \ldots\} = ab \cup ab^*$

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Regular expression:

$a, b, \ldots$ mean $\{a\}, \{b\}, \ldots$

$L_1 L_2$ means composition

$L_1 \vee L_2$ union

L^* Kleene star

Regular expression $\equiv$ anything obtained from letters by using composition, union, and Kleene star

Regular expression $\longrightarrow$ Finite Automaton

$RL \longrightarrow \underbrace{NDA}_{\text{non deterministic automata}} \longrightarrow DA \longrightarrow$ determ. automata

regular language

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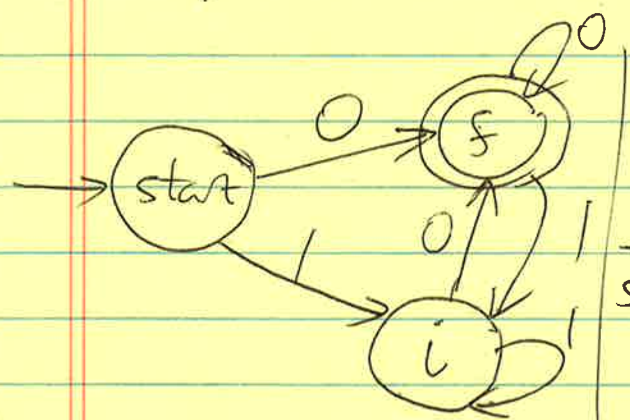
- We have a deterministic finite automaton.
- We want to describe a regular expression that covers exactly all the words accepted by this automaton and only these words



L = even binary integers
binary integers ending in 0.

FA → RL

Step 1: We add new starting state, with jump to old starting state.

step 2: We add new final state, with jumps from old final states

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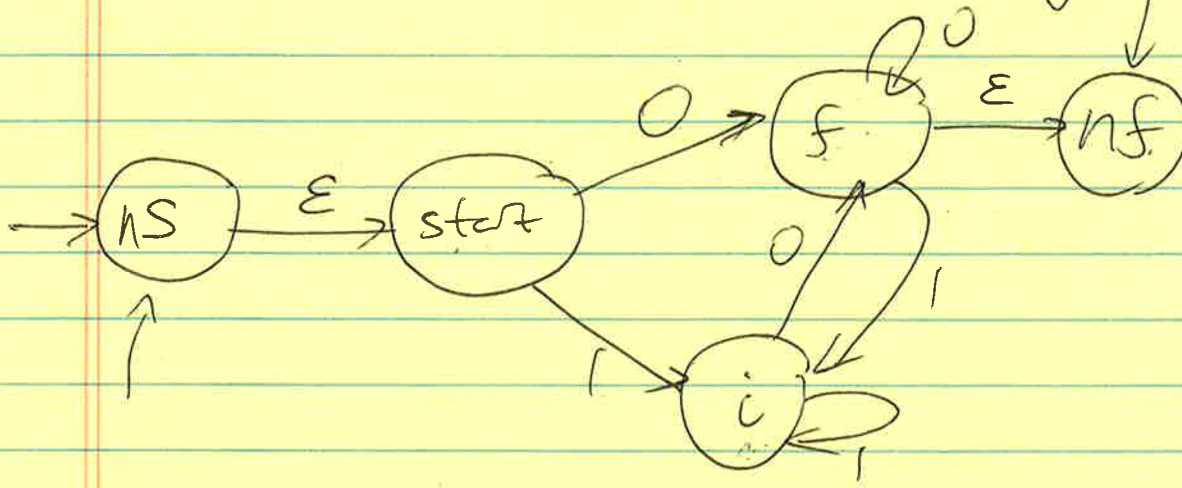
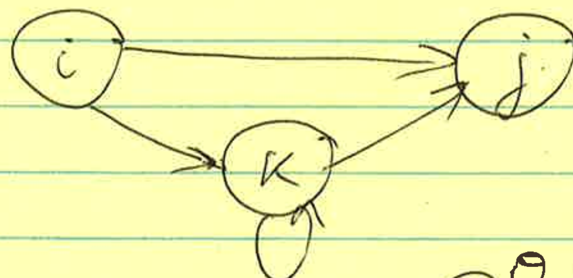
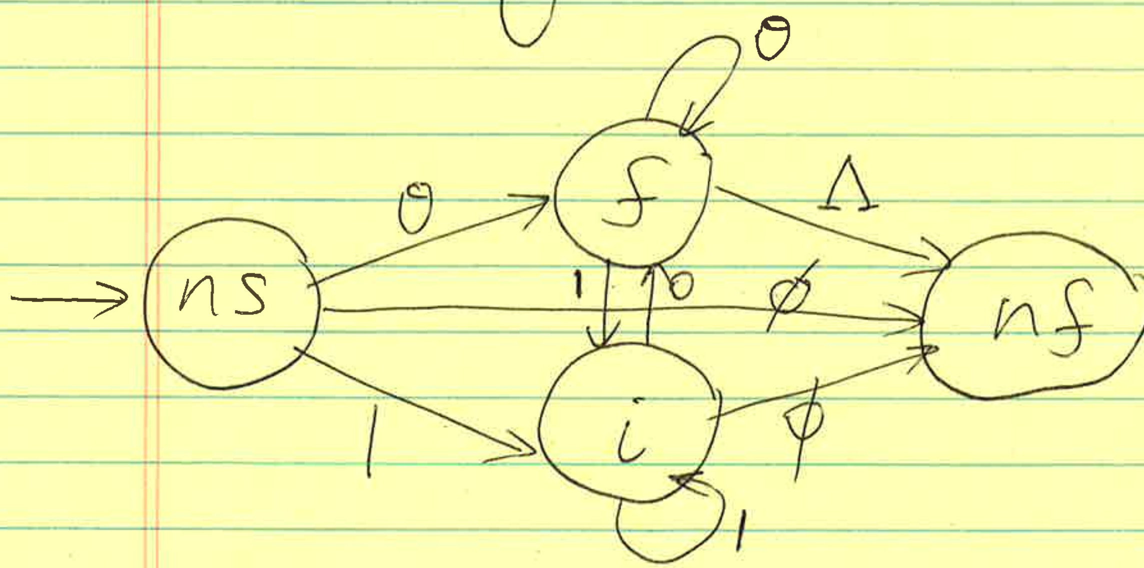
$$R'_{ij} = R_{ij} \cup (R_{ik} R_{kk}^* R_{kj})$$





$\boxed{k = \text{start}}$



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$$R'_{ij} = R_{ij} \cup (R_{ik} R^*_{kk} R_{kj})$$

$$R'_{ns,f} = R_{ns,f} \cup (R_{ns,start} R^*_{start,start} R_{start,f})$$

$i = ns$

$j = f \rightarrow \quad k = start$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$a = 3 \quad (3+4)^2 = 3^2 + 4^2 + 2 \cdot 3 \cdot 4$

$b = 4$

$$\boxed{\emptyset^* = \Lambda}$$

$$R'_{ns,f} = \emptyset \cup (\Lambda \emptyset^* \emptyset)$$

$$\Lambda \Lambda \emptyset = \emptyset$$

$$A^* = \{\Lambda, (a \in A), a_1 a_2, (a_i \in A), a_1 a_2 a_3 (a_i \in A), \ldots\}$$

~~$\emptyset^* = \Lambda$~~ $\boxed{\Lambda A = A}$ $\boxed{A \cup \emptyset = A}$

$$R'_{ns,f} = \emptyset$$

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$$R'_{ij} = R_{ij} \cup (R_{ik} R_{kk}^{*} R_{kj})$$

$i = ns, \quad j = nf, \quad k = start$

$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,start} R_{start,start}^{*} R_{start,nf})$$

$$R'_{ns,nf} = \phi \cup (\Lambda \underset{\Lambda}{\overset{}{\phi^{*}}}\!\!\!\!\!\!\!\!\quad\; \phi) = \phi$$

$$\boxed{\Lambda \phi = \phi}$$

$$R'_{ns,i} = \underset{\phi}{R_{ns,i}} \cup (\underset{\Lambda}{R_{ns,start}} \; \underset{\phi^{*} = \Lambda}{R_{start,start}^{*}} \; \underset{1}{R_{start,i}})$$

$$\phi \cup 1 = 1$$