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$L = \{a^n b^n\} = \{\Lambda, ab, aabb, aaabbb, ..\}$

1) Simplest element of this language:
In terms of the grammar

$S \rightarrow \varepsilon$ This rule means that ε is an element of L

2) if we have a word from L

$\underbrace{a \dots a}_{n} \underbrace{b \dots b}_{\in L} \rightarrow a \underbrace{a .. a b .. b}_{word} b \in L$

$S \rightarrow aSb$

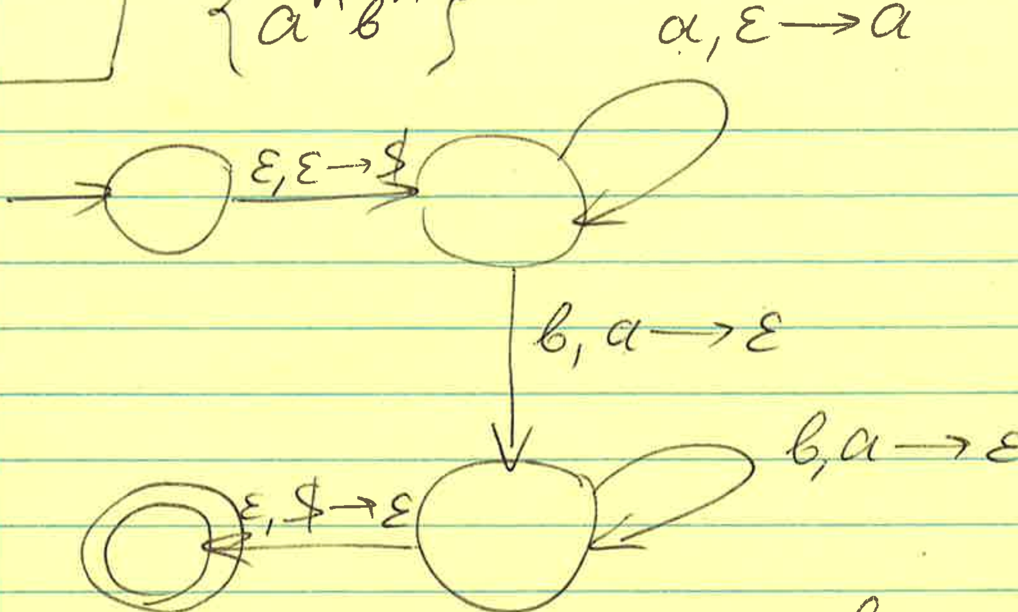
if we have a word w from L, then awb is also in L

$a^n b^{2n} = \{\Lambda, abb, aabbbb, \dots\}$

$S \rightarrow \varepsilon$

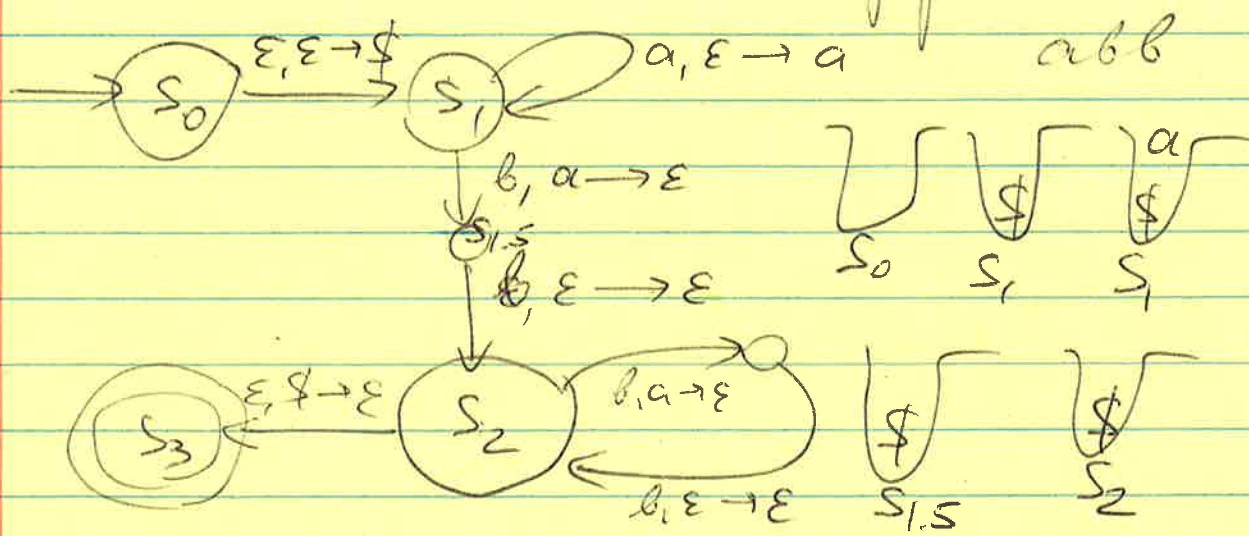
$S \rightarrow aSbb$

$\{a^n b^n\}$



a, b → c
what we see
what we pop
what we push

$\{a^n b^{2n}\}$



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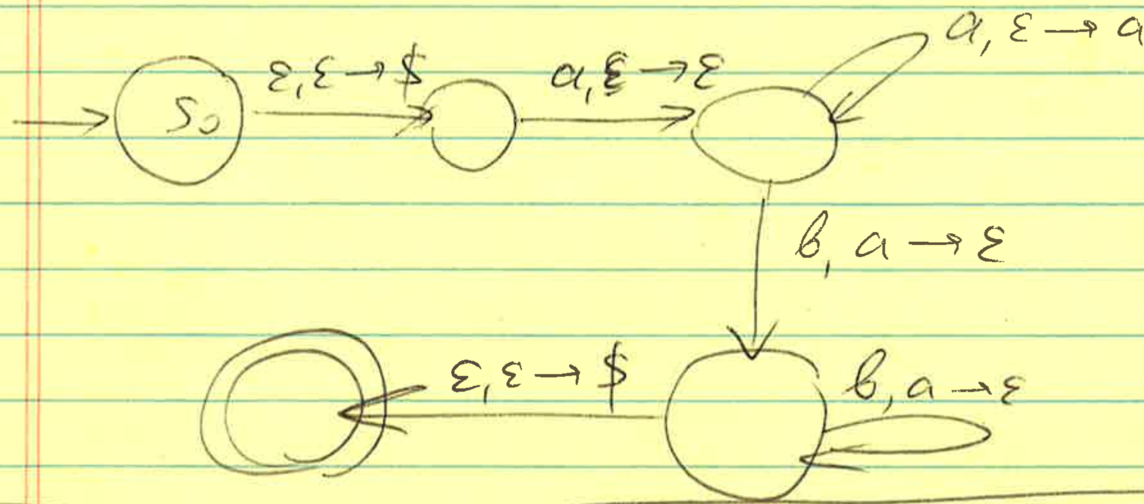
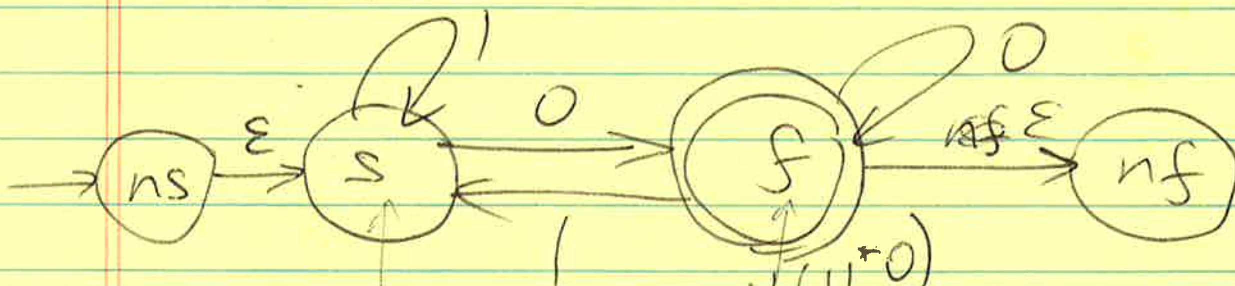
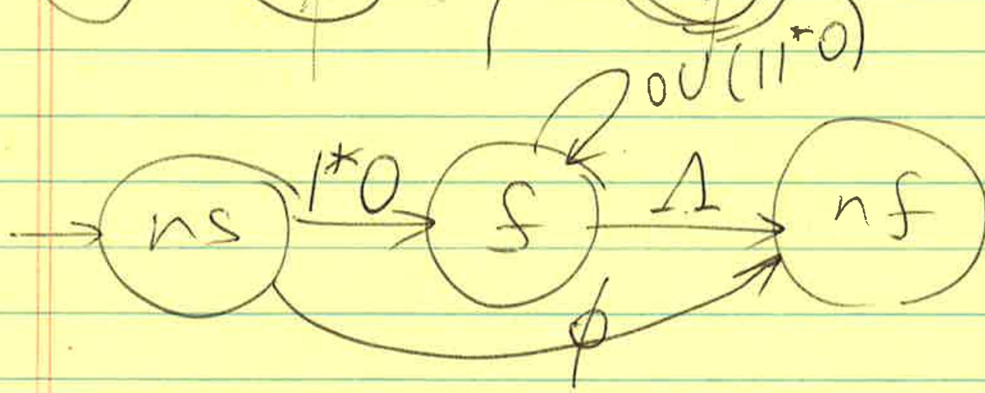
$\{a^{n+1}b^n\} = \{a, aab, aaabb, \ldots\}$

$S \rightarrow a$

$S \rightarrow aSb$







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$$R'_{ij} = R_{ij} \cup (R_{ik} R_{kk}^{*} R_{kj})$$

$$R'_{ns,f} = R_{ns,f} \cup (R_{ns,s} R_{s,s}^{*} R_{sf})$$

$$\emptyset \qquad \Lambda \quad 1^{*} \quad 0$$

$$R'_{ns,nf} \Rightarrow R_{ns,nf} \cup (R_{ns,s} R_{ns,s}^{*} R_{s,nf})$$

$$\emptyset \qquad \underbrace{\Lambda \quad 1^{*} \quad \emptyset}$$

$$R'_{f,f} = R_{f,f} \cup (R_{f,s} R_{s,s}^{*} R_{s,f})$$

$$0 \ \cup \ (1 \quad 1^{*} \quad 0)$$

$$R'_{f,nf} = R_{f,nf} \cup (R_{f,s} R_{s,s}^{*} R_{s,nf})$$

$$\Lambda \qquad \underbrace{1 \quad 1^{*} \quad \emptyset}$$

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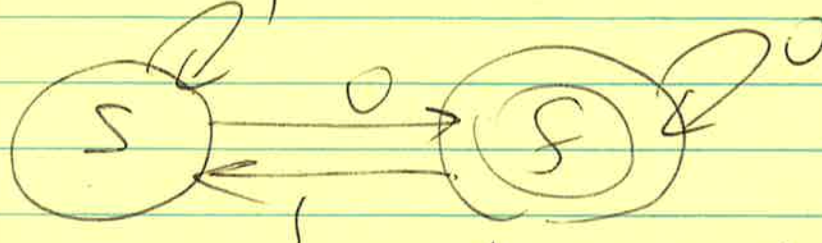
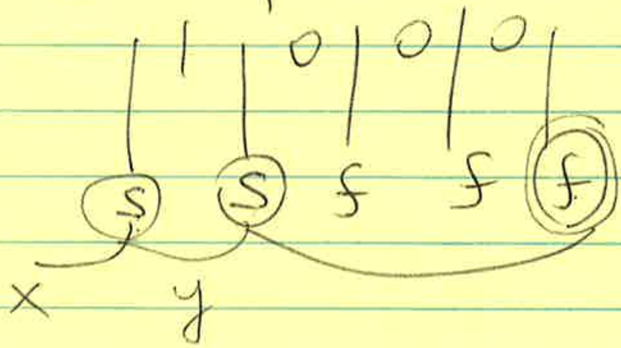
$$R'_{ns,nf} = R_{ns,nf} \cup (R_{ns,f} R_{f,f}^* R_{f,nf})$$

$\cancel{\emptyset}$ $1^*0(0\cup(11^*0))^*\cancel{\Lambda}$

$$\boxed{1^*0(0\cup(11^*0))^*}$$



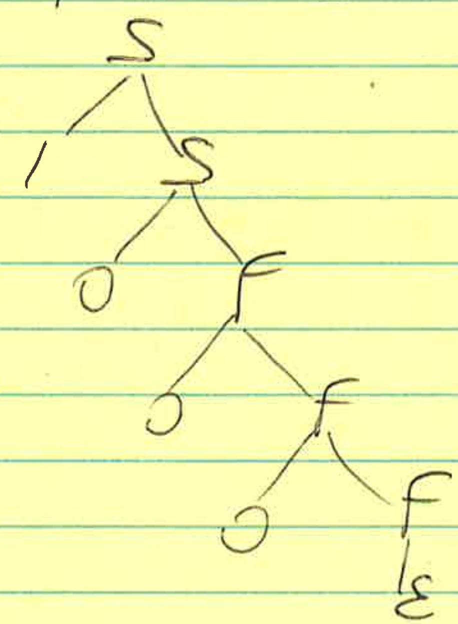
1000



$x = \Lambda$
$y = 1$
$z = 000$

$q_1 \xrightarrow{a} q_2$ $\qquad Q_1 \rightarrow aQ_2$

$S \rightarrow 0F$
$S \rightarrow 1S$
$F \rightarrow 0F$
$F \rightarrow 1S$
$F \rightarrow \varepsilon$



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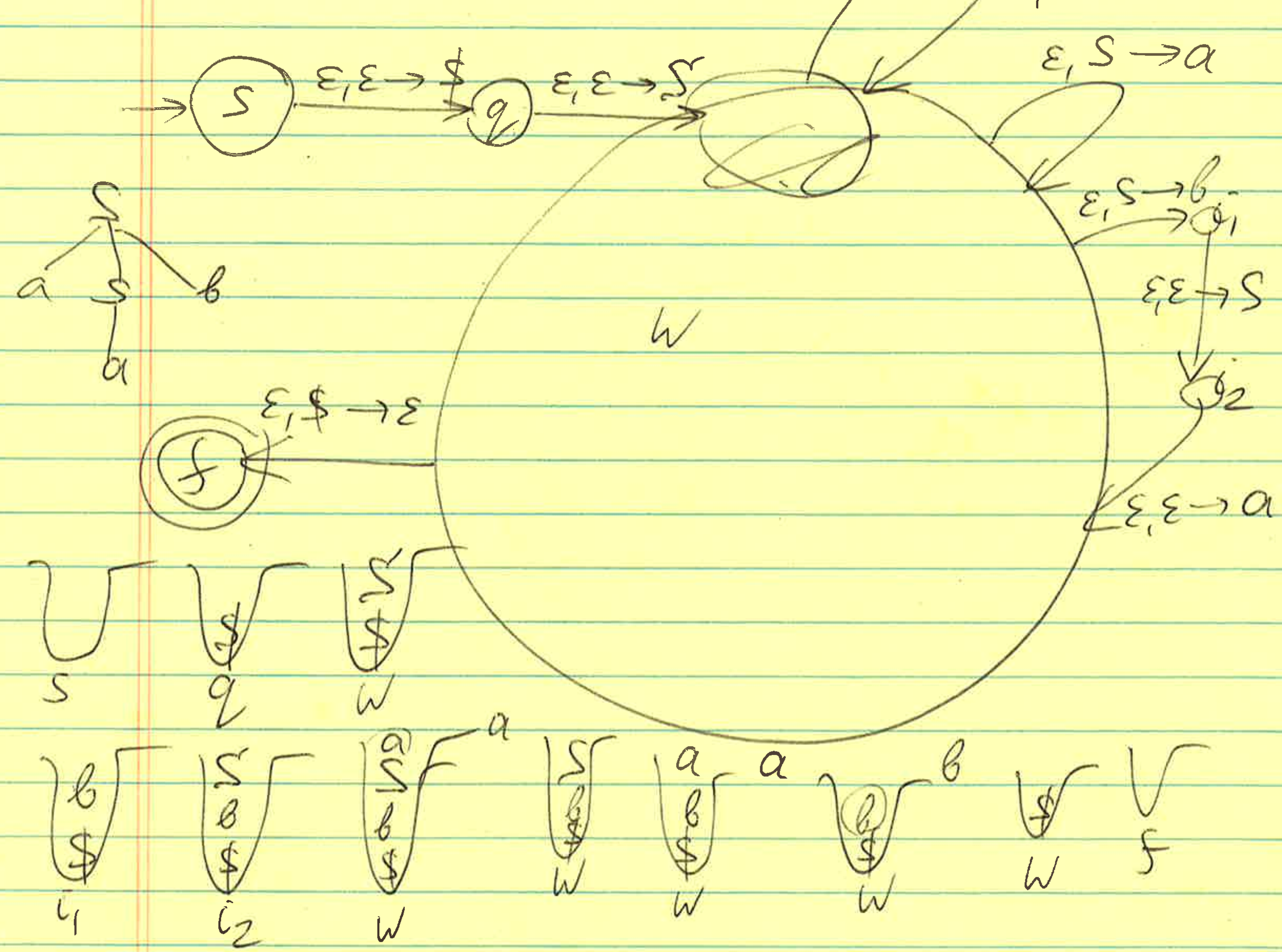
$S \to a$

$S \to aSb$

$\{a^{n+1}b^n\}$



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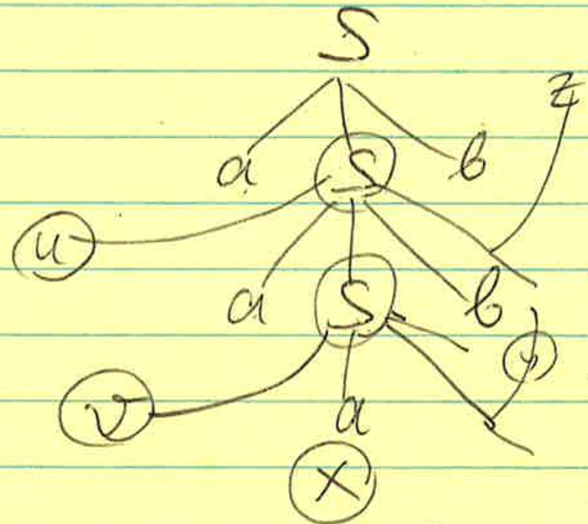
$S \to a$
$S \to aSb$

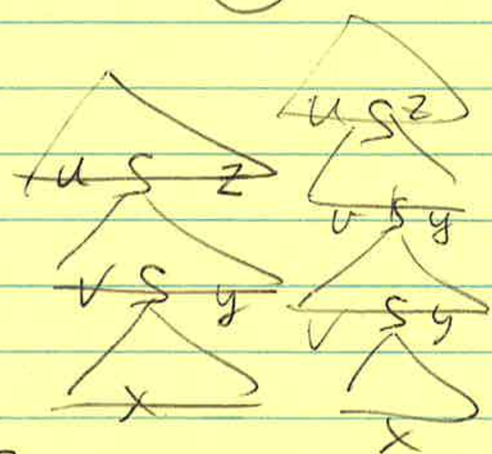
aaabb

~~uvxv~~
uvxyz

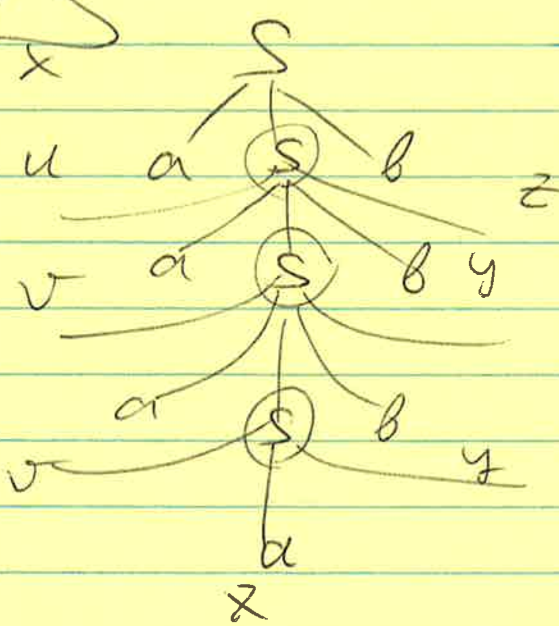


Look for lowest pair of repeating variable on the same branch
u - before 1st rep. to the left
v - between 1st & 2nd to the left
x - under 2nd rep.
y - between 1st & 2nd to the right
z - after 1st rep. to the right

u = a
v = a
x = a
y = b
z = b



uvvxyyz



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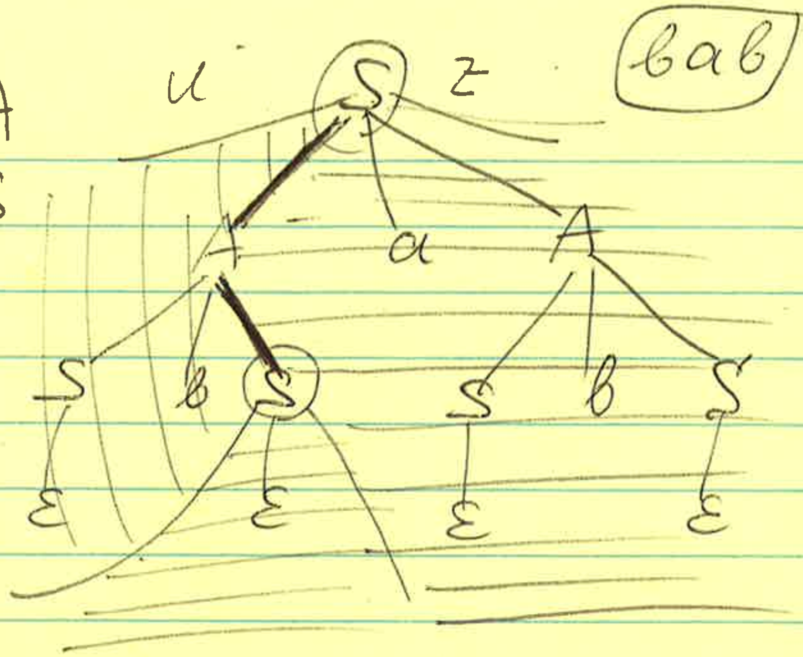
1. $S \to AaA$
2. $A \to SbS$
3. $S \to \varepsilon$



$u = \Lambda$
$v = b$
$x = \Lambda$
$y = ab$
$z = \Lambda$

$\forall CFG\ \exists p\ \forall w \in L\ (len(w) \geq p \to$
$\exists u, v, x, y, z\ (w = uvxyz\ \&\ len(vxy) \leq p$
$\&\ len(vy) > 0\ \&\ \forall i\ (uv^i x y^i z \in L)))$

$L = \{a^n b^n c^n\} = \{\Lambda, abc, aabbcc, aaabbbccc, \ldots\}$
is not CFG.

Proof. by contradiction. Let's assume that L is CFG. Then $\exists p \ldots$ $u\boxed{vxy}z$
Let's take $w = a^p b^p c^p = \underbrace{a \ldots a}_{p} \underbrace{b \ldots b}_{p} \underbrace{c \ldots c}_{p \text{ times}}$
$len(w) = 3p \geq p$
thus $\exists u, v, x, y, z$ s.t. $w = uvxyz\ \&\ len(vxy) \leq p$
$\&\ len(vy) > 0\ \&\ uvvxyyz \in L$.

We cannot have vxy covering a's, ~~b's~~ and c's since then len(vxy) > p.

So: we have one of the cases:

1) vxy is in a's. When we pump we add a's but not b's or c's.
So $uvvxyyz \notin L$

2) vxy is between a's and b's. When we pump we add a's and b's but not c's balance is discrupted, so $uvvxyyz \notin L$

3) vxy is in b's, then we add b's but not a's or c's, so $uvvxyyz \notin L$

4) vxy is between b's and c's, then we add b's and c's but not a's, so $uvvxyyz \notin L$

5) vxy is in c's. Then we add c's but not a's and b's, so $uvvxyyz \notin L$.

In all possible cases, we get a contradiction so L is not CFG.