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Quiz solution:

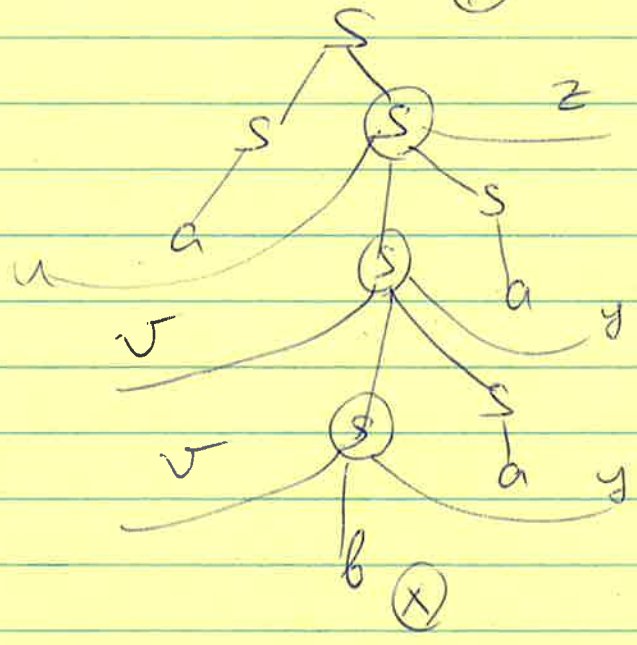
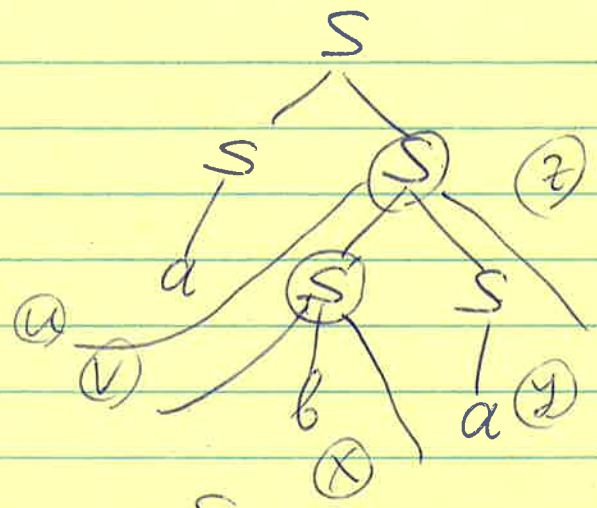
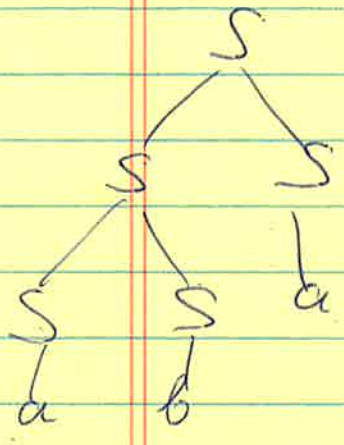
uvxyz

(-|-)

- $S \rightarrow SS$
- $S \rightarrow a$
- $S \rightarrow b$

- $u = a$
- $v = \Lambda$
- $x = b$
- $y = a$
- $z = \Lambda$

$a \Lambda ba a \Lambda$   
 $abaa$



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(-2-)

Ambiguous  $\equiv$  several derivations of the same word

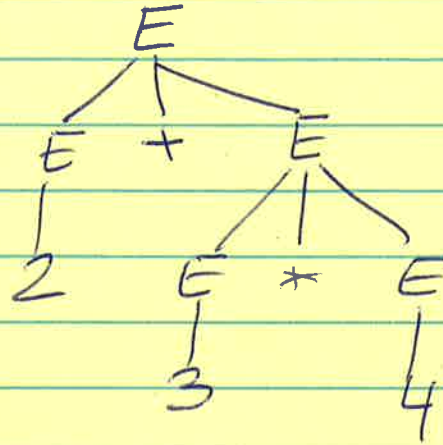
good - Unambiguous  $\equiv$  only one derivation



- ~~$E \rightarrow a$~~
- ~~$E \rightarrow b$~~
- ~~$E \rightarrow E + E$~~
- ~~$E \rightarrow E * E$~~
- ~~$E \rightarrow (E)$~~

2 + 3 \* 4

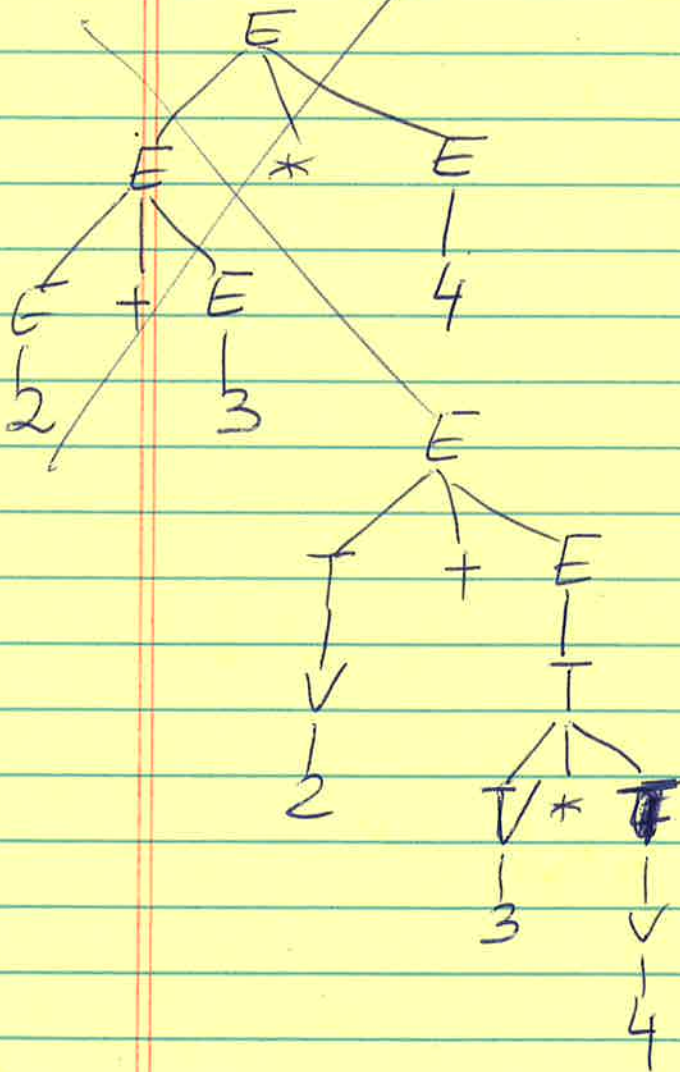
What we want



- $E \rightarrow T$
- $E \rightarrow T + E$
- $E \rightarrow T - E$
- $T \rightarrow V$
- $T \rightarrow V * T$
- $T \rightarrow V / T$
- $V \rightarrow a, \dots$
- $V \rightarrow 2, \dots$

$T \rightarrow (E)$

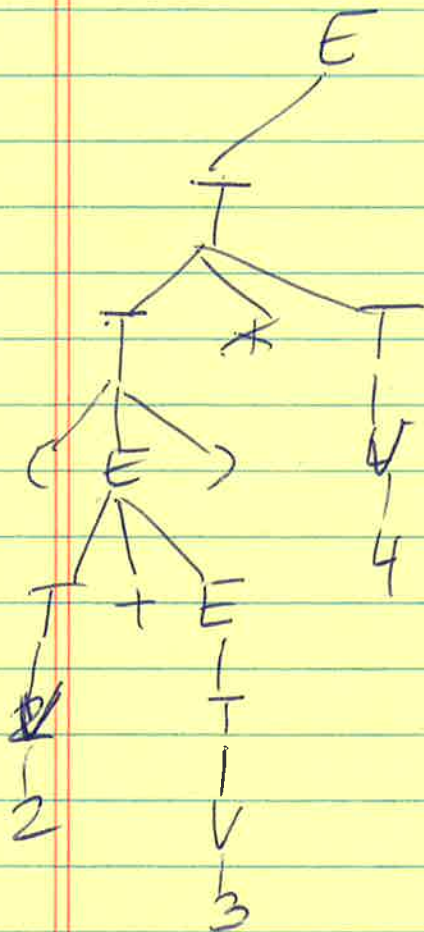
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$$(2+3) * 4$$



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# Chomsky normal form

Noam Chomsky

You are only allowed following rules:

$S \rightarrow \epsilon$  if  $S$  is a starting state

$V \rightarrow a$  variable goes to terminal symbol

$V \rightarrow AB$  a variable goes into two variable

Prelim step:

We introduce a new starting variable  $S_0$  and a rule  $S_0 \rightarrow S$

~~$S \rightarrow \epsilon$~~   
 ~~$S \rightarrow aSb$~~   
 $S \rightarrow aAb$   
 $A \rightarrow bSa$   
 ~~$A \rightarrow \epsilon$~~   
 ~~$A \rightarrow S$~~

Step 0: Deal w/ rules that have no symbols in RHS

If we have  $A \rightarrow \epsilon$ , we take all rules  $\rightarrow uAv$  and add rule  $\rightarrow uv$

step 0:

$A \rightarrow ba$
$S_0 \rightarrow \epsilon$
$S \rightarrow ab$

Step 1: we eliminate all rules  $A \rightarrow B$   
for every rule  $B \rightarrow u$ , we add  $A \rightarrow u$

step 1:

$A \rightarrow aAb$	$S_0 \rightarrow aAb$
$A \rightarrow ab$	$S_0 \rightarrow ab$

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$S \rightarrow \epsilon$   
 $S \rightarrow aAb$   
 $A \rightarrow bSa$   
 $A \rightarrow \epsilon$   
 $A \rightarrow S$

original  
CFG

$S \rightarrow \epsilon^x$   
 $S \rightarrow aAb$   
 $A \rightarrow bSa$   
 $A \rightarrow \epsilon^x$   
 $A \rightarrow S$   
 $S_0 \rightarrow S$

prelim. step

$S \rightarrow aAb$   
 $A \rightarrow bSa$   
 $A \rightarrow S$   $\times$   
 $S_0 \rightarrow S$   $\times$   
 $A \rightarrow ba$   
 $S_0 \rightarrow \epsilon$   
 $S \rightarrow ab$

Step 0

$S \rightarrow aAb$   
 $A \rightarrow bSa$   
 $A \rightarrow ba$   
 $S_0 \rightarrow \epsilon$   
 $S \rightarrow ab$   
 $A \rightarrow aAb$   
 $A \rightarrow ab$   
 $S_0 \rightarrow aAb$   
 $S_0 \rightarrow ab$

Step 1

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Step 2: For each terminal symbol  $a$ , we introduce a new variable  $V_a$  and a new rule  $V_a \rightarrow a$

$S \rightarrow V_a A V_b$	$V_a \rightarrow a$
$A \rightarrow V_b S V_a$	$V_b \rightarrow b$

$A \rightarrow V_b V_a$

$S_0 \rightarrow \epsilon$

$S \rightarrow V_a V_b$

$A \rightarrow V_a A V_b$

$A \rightarrow V_a V_b$

$S_0 \rightarrow V_a A V_b$

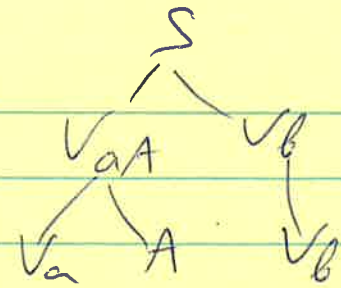
$S_0 \rightarrow V_a V_b$

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Step 3: eliminate all rules with 3 or more symbols in RHS

Idea: abc

$$\Gamma = a + b$$
$$\Gamma + C$$



$$S \rightarrow V_a A V_b$$

$$V_a A \rightarrow V_a A$$

$$A \rightarrow V_b S V_a$$

$$V_b S \rightarrow V_b S$$

$$A \rightarrow V_b V_a$$

$$S_0 \rightarrow \epsilon$$

$$S \rightarrow V_a V_b$$

$$A \rightarrow V_a A V_b$$

$$A \rightarrow V_a V_b$$

$$S_0 \rightarrow V_a A V_b$$

$$S_0 \rightarrow V_a V_b$$

$$V_a \rightarrow a$$
$$V_b \rightarrow b$$

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Step 0 :

$S \rightarrow A$   
 $S \rightarrow B$   
 ~~$A \rightarrow \epsilon$~~   
 ~~$B \rightarrow \epsilon$~~   
 $A \rightarrow AaB$   
 $B \rightarrow BbA$   
 $S_0 \rightarrow S$

~~$S \rightarrow \epsilon$~~   
 $A \rightarrow aB$   
 $B \rightarrow Bb$   
 $A \rightarrow Aa$   
 $B \rightarrow bA$   
 $A \rightarrow a$   
 $B \rightarrow b$   
 $S_0 \rightarrow \epsilon$