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$S \to AaA$

$A \to bS\cancel{B}b$

$\cancel{S \to \varepsilon}$

$\cancel{\boxed{S_0 \to S}}$

at step 0 — $S_0 \to \varepsilon$

step 1 — $S_0 \to AaA$

step 0 — $A \to bb$

$V_a \to a$ $\qquad$ $V_b \to b$

$S_0 \to \varepsilon$ $\qquad$ $\cancel{S \to AV_aA}$

$\cancel{A \to V_bSV_b}$ $\qquad$ $\cancel{S_0 \to AV_aA}$

$A \to V_bV_b$

after step 2

$\cancel{X}$ $A \to V_{bS}V_b$ $\qquad$ $V_{bS} \to V_bS$

$S \to V_{Aa}A$ $\qquad$ $V_{aA} \to V_aA$

$S_0 \to V_{Aa}A$

PDA $\longrightarrow$ CFG

A_{pq} $\equiv$ set of all words that, starting in state p with empty stack, lead to q with empty stack

$\boxed{A_{pp} \rightarrow \varepsilon}$ for all states p

$\boxed{A_{pr} \rightarrow A_{pq} A_{qr}}$ for all p, q, and r

$$\begin{tikzcd} p \arrow[r, "{a,\, \varepsilon \to t}"] & q \arrow[r, dashed] & r \arrow[r, "{b,\, t \to \varepsilon}"] & s \end{tikzcd}$$

$\boxed{A_{ps} \rightarrow a A_{qr} b}$ $\boxed{a, b, t \text{ could be } \varepsilon}$

Preliminary step: if we have a rule that both pops & pushes, we separate it into 2: (1) pop, (2) we push

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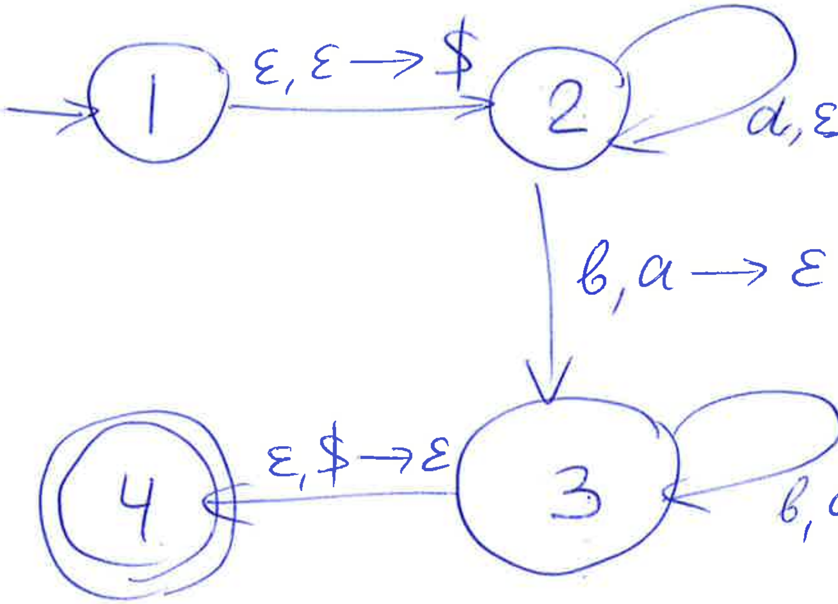
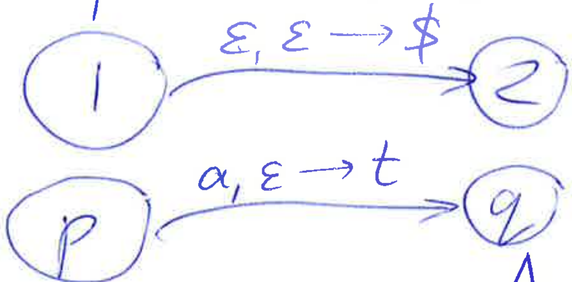
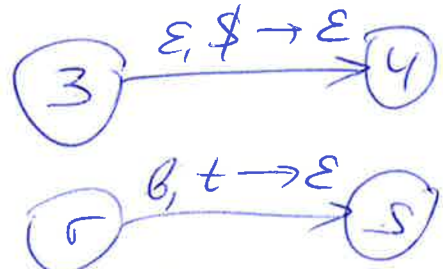


$a^n b^n$

$A_{11} \to \varepsilon \ \ldots$

$A_{pr} \to A_{pq} A_{qr} \ \ldots$

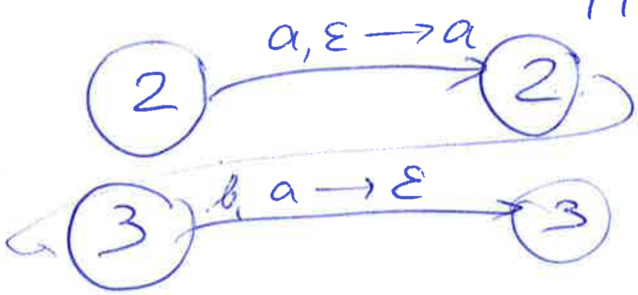


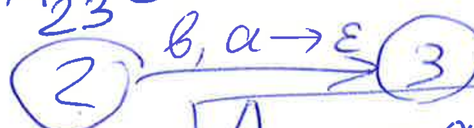


$A_{ps} \to a A_{qr} b$

$\boxed{A_{14} \to A_{23}}$

$A_{14} \to \varepsilon A_{23} \varepsilon$





$\boxed{A_{23} \to a A_{22} b}$

$\boxed{A_{23} \to a A_{23} b}$

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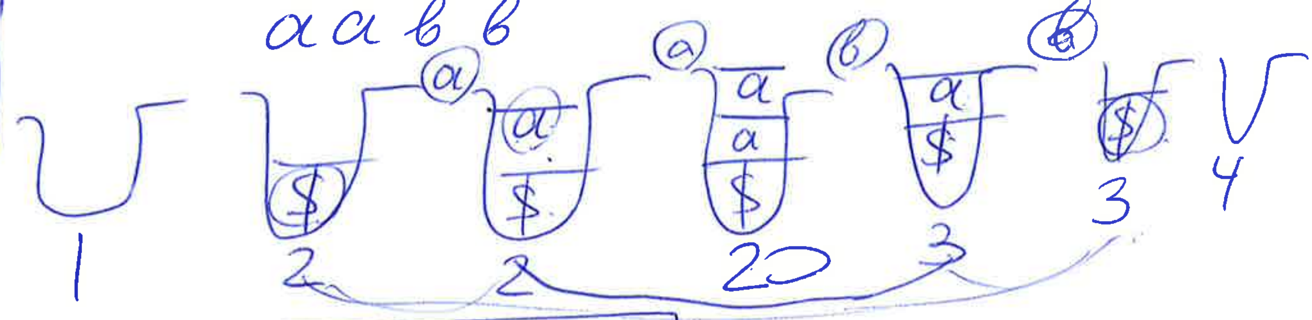


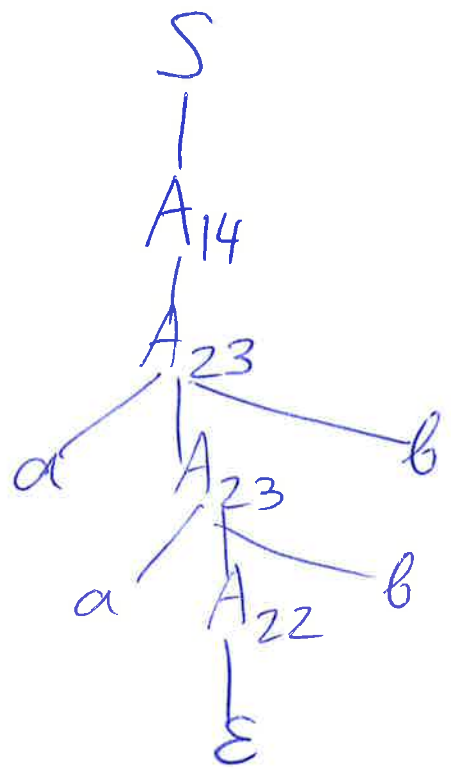
$A_{14} \rightarrow A_{23}$

$A_{23} \rightarrow a A_{22} b$

$A_{23} \rightarrow a A_{23} b$

$\boxed{S \rightarrow A_{sf}}$ ~~A_{14}~~ for every final state f

s is starting state



$\boxed{ab}$

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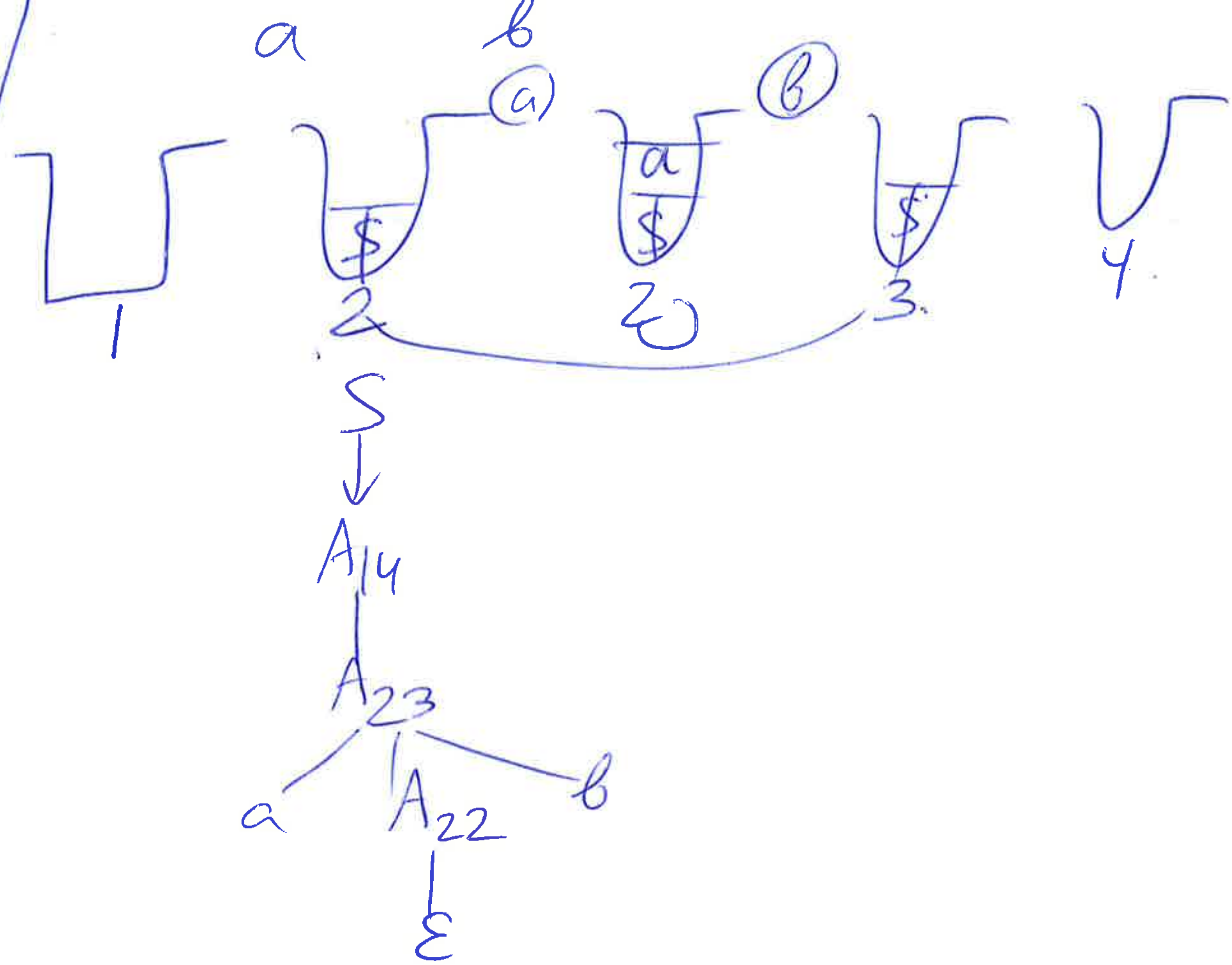

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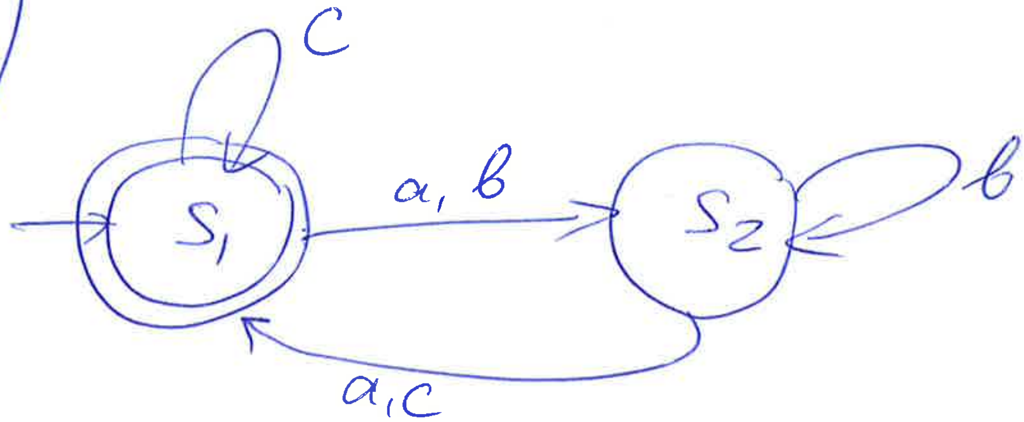
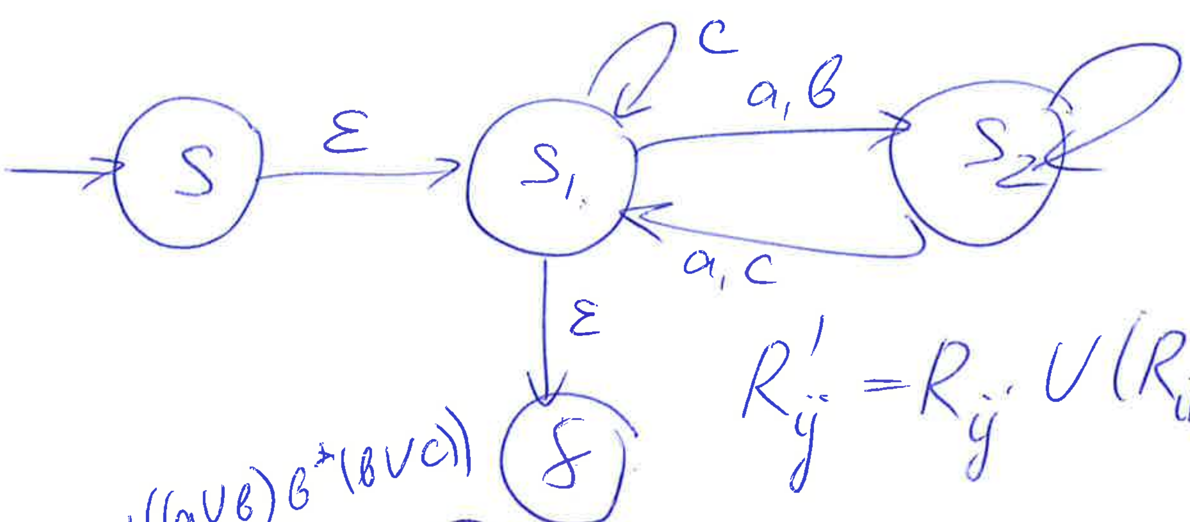
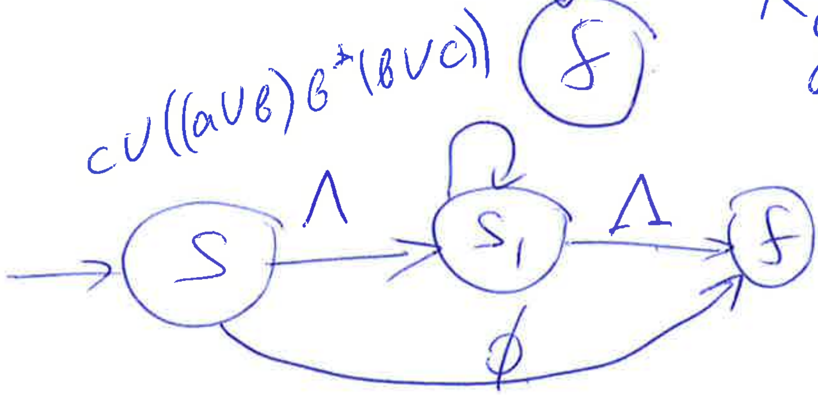




$$R'_{ij} = R_{ij} \cup (R_{ik} R^*_{kk} R_{kj})$$

$$k = 2$$



$$R'_{SS_1} = R_{SS_1} \cup (R_{SS_2} R^*_{S_2S_2} R_{S_2S_1})$$

$$\Lambda \cup \quad \phi \ldots$$

$$R'_{Sf} = R_{Sf} \cup (R_{SS_2} R^*_{S_2S_2} R_{S_2f})$$

$$\phi \qquad \phi \ldots$$

$$R'_{S_1f} = R_{S_1f} \cup (R_{S_1S_2} R^*_{S_2S_2} R_{S_2f})$$

$$\Lambda \cup (\underline{(a \cup b)\ b^* \emptyset})$$

$$R'_{S_1S_1} = R_{S_1S_1} \cup (R_{S_1S_2} R^*_{S_2S_2} R_{S_2S_1})$$

$$\boxed{c \cup ((a \cup b)\ b^* (a \cup c))}$$

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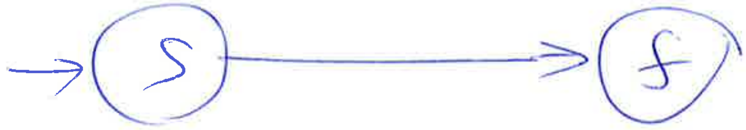
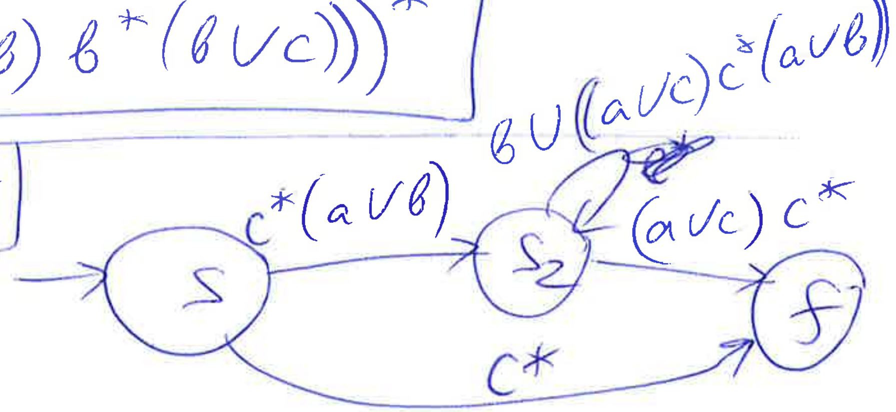


$$R'_{sf} = R_{sf} \cup (R_{ss_1} R^*_{s_1s_1} R_{s_1f})$$

$$\cancel{\emptyset} \cup (\cancel{\Lambda}(c \cup ((a\cup b)\, b^*(b\cup c)))^* \cancel{\Lambda})$$

$$\boxed{(c\cup((a\cup b)\, b^*(b\cup c)))^*}$$

If we eliminate s_1:



$$R'_{ss_2} = R_{ss_2} \cup (R_{ss_1} R^*_{s_1s_1} R_{s_1s_2})$$

$$\cancel{\emptyset} \cup (\cancel{\Lambda} c^*(a\cup b))$$

$$R'_{sf} = R_{sf} \cup (R_{ss_1} R^*_{s_1s_1} R_{s_1f})$$

$$\cancel{\emptyset} \quad \cancel{\Lambda} \quad c^* \quad \cancel{\Lambda}$$

$$R'_{s_2f} = R_{s_2f} \cup (R_{s_2s_1} R^*_{s_1s_1} R_{s_1f})$$

$$\emptyset \quad \underbrace{(a\cup c)\quad c^* \quad \Lambda}$$

$$R'_{s_2s_2} = R_{s_2s_2} \cup (R_{s_2s_1} R^*_{s_1s_1} R_{s_1s_2})$$

$$b \quad (a\cup c) \quad c^* \quad (a\cup b)$$

$$R'_{ss} = c^* \cup (c^*(a\cup b)(b\cup(a\cup c)c^*(a\cup b))^*(a\cup c)c^*$$