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Chomsky

$S_0 \to \varepsilon$

$A \to BC$

$A \to a$

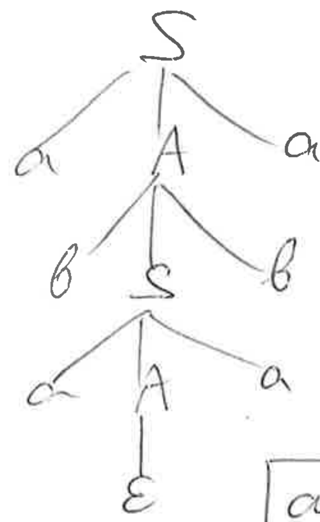
$S \to aAa$

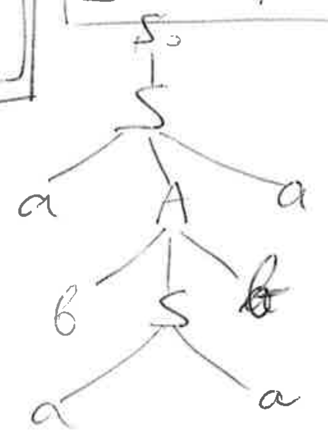
$A \to bSb$

$\cancel{A \to \varepsilon}$

$\cancel{S_0 \to S}$ ← prel step

$\boxed{S \to \cancel{A}a a}$ – add on stage 0

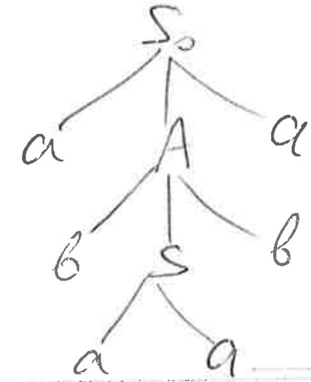




$S_0 \to aAa$

$S_0 \to aa$

$\boxed{abaaba}$



$V_a \to a$

$V_b \to b$

$S_0 \to V_a V_a$

$\cancel{S \to V_a A V_a}$

$\cancel{A \to V_b S V_b}$

$S \to V_a V_a$

$\cancel{S_0 \to V_a A V_a}$

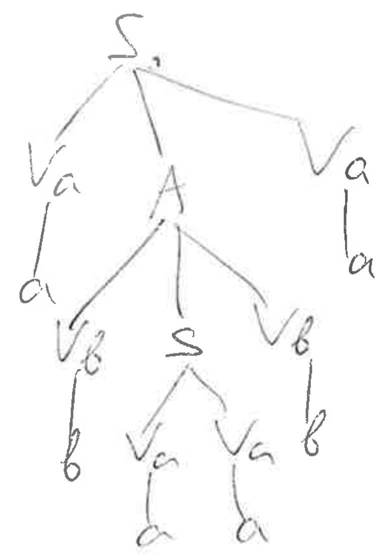
$S \to V_{aA} V_a$

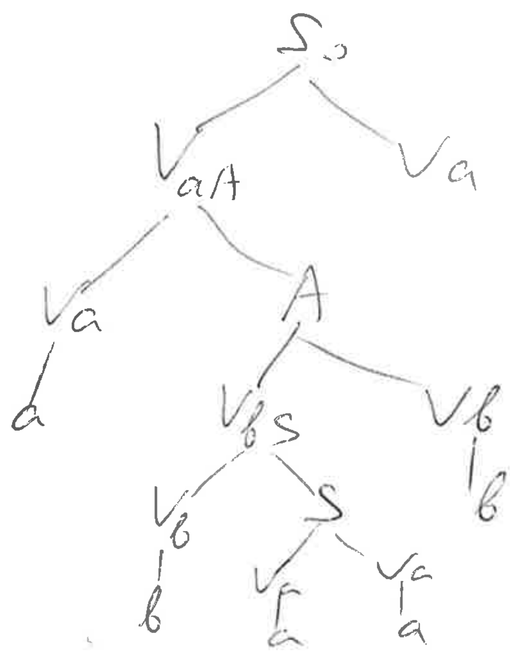
$S_0 \to V_{aA} V_a$

$V_{aA} \to V_a A$

$A \to V_{bS} V_b$

$V_{bS} \to V_b S$





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$S_0 \to 0A1$

$\cancel{S \to \varepsilon}$

$S \to 0A1$

$A \to 1S0$

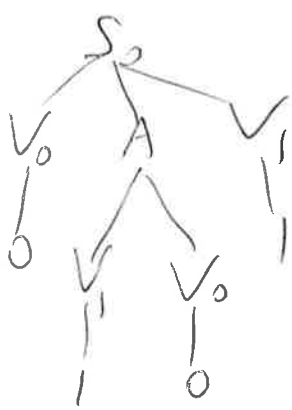
$\boxed{\cancel{S_0 \to S}}$

$A \to 10$

$S_0 \to \varepsilon$

$\boxed{0101}$

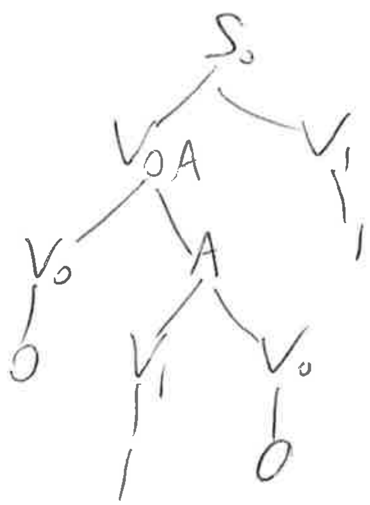
$V_0 \to 0$ $\quad$ $V_1 \to 1$ $\quad$ $\cancel{S \to V_0AV_1}$ $\quad$ $\cancel{A \to V_1SV_0}$



$\cancel{S_0 \to V_0AV_1}$ $\quad$ $A \to V_1V_0$ $\quad$ $S_0 \to \varepsilon$

$S \to V_{0A}V_1$ $\quad$ $V_{0A} \to V_0A$

$S_0 \to V_{0A}V_1$ $\quad$ $A \to V_{1S}V_0$ $\quad$ $V_{1S} \to V_1S$



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Pumping lemma for CFG

$\forall CFG_L\ \exists p\ \forall w \mid len(w) \geq p \rightarrow$
$\exists u, v, x, y, z\ (w = uvxyz\ \&\ len(vy) > 0\ \&\ len(vxy) \leq p\ \&\ \forall i\ (uv^i x y^i z \in L)$

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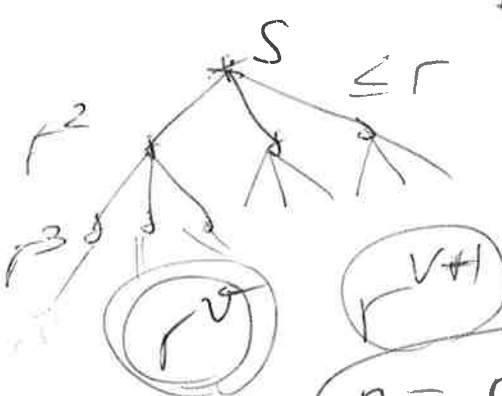
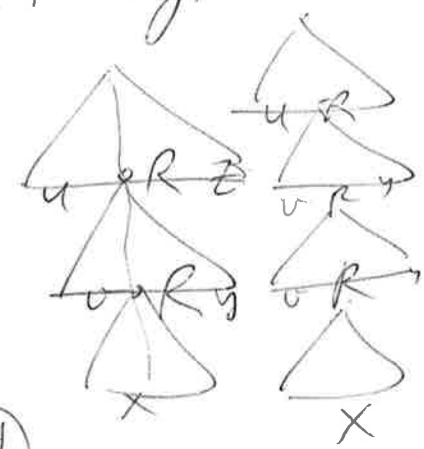
v variables

right-hand side $\leq r$ symbols

$v+1$ variables





$p = r^{v+1} + 1$

$S \rightarrow \varepsilon$

$S \rightarrow 0A1$

$A \rightarrow 1S0$

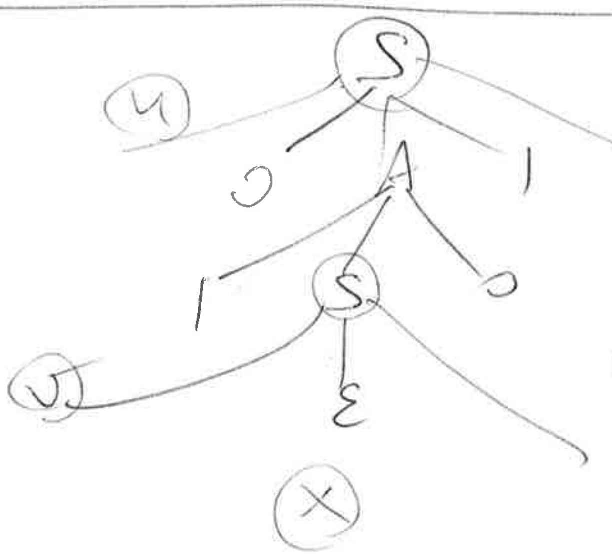
$u = \varepsilon$

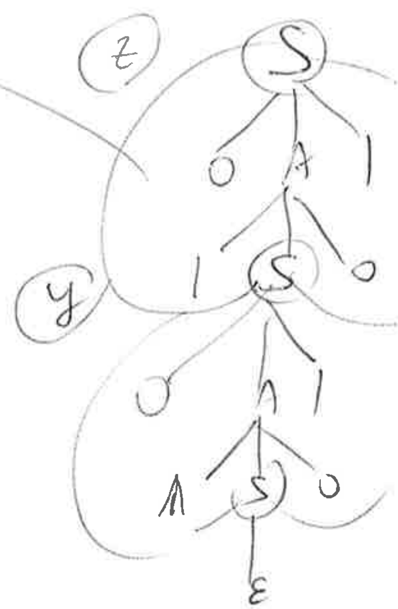
$v = 01$

$x = \varepsilon$

$y = 01$

$z = \varepsilon$





$u\,\underbrace{01}_{v}\underbrace{01}_{v}\,x\,\underbrace{01}_{y}\underbrace{01}_{y}\,z$

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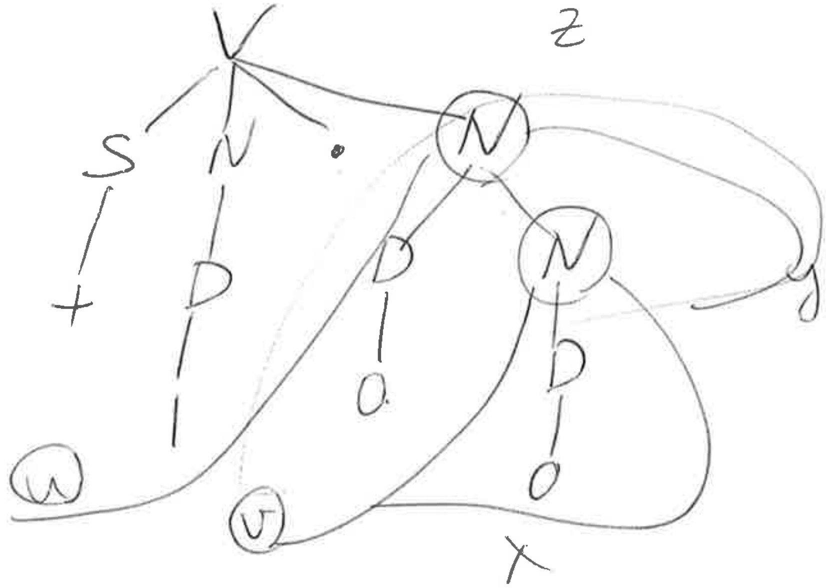
$V \to SN.N \qquad V \to N.N$

$S \to +,\ S \to -,\ N \to DN,\ N \to D,$

$D \to 0,\ D \to 1 \qquad +1.00$



$u = +1.$

$v = 0$

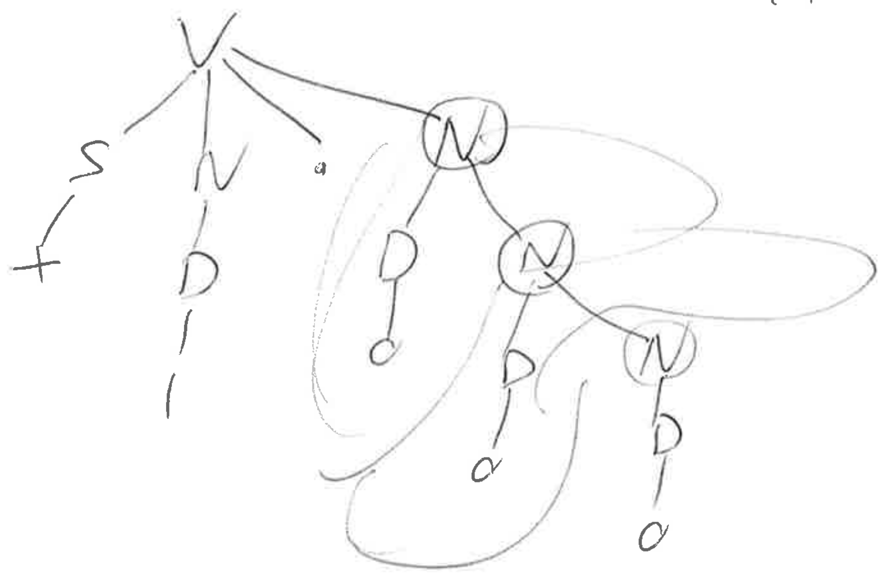
$x = 0$

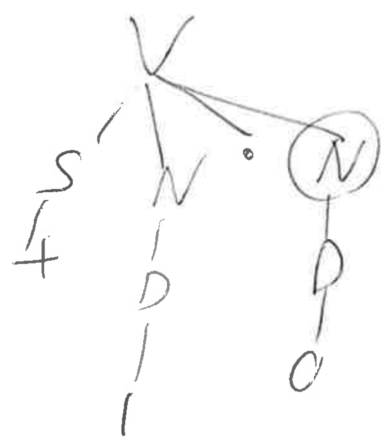
$y = \varepsilon$

$z = \varepsilon$

+1.0

+1.000





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Th. $L = \{a^n b^n c^n d^n : n = 0, 1, 2, \ldots\}$ not CF.
$\{\Lambda, abcd, aabbccdd, \ldots\}$

Proof by contradiction. Let's assume L is CFG. Then by pumping lemma $\exists p$
$\forall w \in L \ (len(w) \geq p \rightarrow \exists u, v, x, y, z \ (w = uvxyz \ \&$
$len(vy) > 0 \ \& \ len(\underbrace{vxy}) \leq p \ \& \ \forall i (uv^i x y^i z \in L)$.

Let's take the word $a^p b^p c^p d^p \in L$
$len(w) = 4p \geq p \Rightarrow$ we can apply pump. lemma.

$\underbrace{a \ldots a}_{p} \underbrace{b \ldots b}_{p} \underbrace{c \ldots c}_{p} \underbrace{d \ldots d}_{p}$

vxy is somewhere inside, its length is $\leq p$.

1) vxy is in a's, so when we go to uv^2xy^2z, we add a's - but we don't add b's, or c's or d's, so balance is disrupted, $uv^2xy^2z \notin L$

2) vxy in a's and b's - we add a's and b's - but not c's or d's so also balance is disrupted, $uv^2xy^2z \notin L$

3) Similarly, if vxy is in b's, or in b's & c's, or in c's, or in c's and d's - $uv^2xy^2z \notin L$

But by pumping lemma $uv^2xy^2z \in L$
So our assumption that L is CFG is wrong, so L is not CFG.

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Proof that $\sqrt{2}$ is not a rational number.

By contradiction, let's assume that $\sqrt{2}$ is rational $\sqrt{2} = \frac{m}{n}$. We can divide m, n by common divisor, so we can safely assume that m, n have no common divisors.

$\sqrt{2} = \frac{m}{n}$ $\quad 2 = \frac{m^2}{n^2}$ $\quad m^2 = \underbrace{2n^2}$

m^2 is divisible by 2, so m is divisible by 2, $m = 2k$ for some k

$4k^2 = 2n^2$, $\quad \underbrace{2k^2} = n^2$