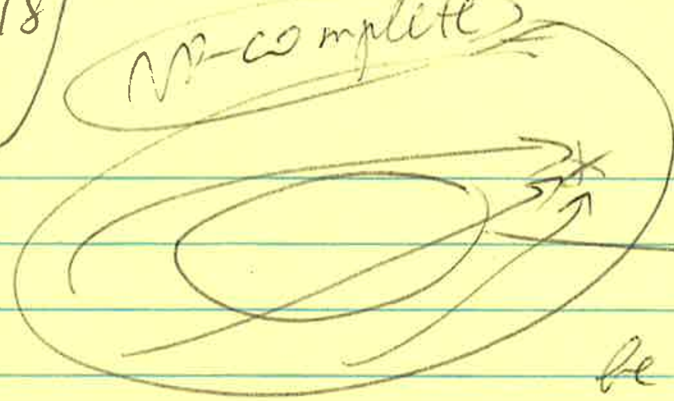
NP ≡ class of problems for which, once we have a candidate for a solution, we can check, in feasible time, whether it is indeed a solution.

Math: input: x statement
we want: y proof of x or of $\neg x$ (not x)
$C(x, y)$ - checking, $len(y) \leq P_\ell(len(x))$

Physics: input: x observations, expe. results
we want: y a formula
$C(x, y)$ easy to check, $len(y) < len(x)$

Engr: input: x specs
We want: a design y that satisfies these specs

Pair $(C(x, y), P_\ell)$, C is a feasible algorithm
P_ℓ is a polynomial
Given: x Find y such that $C(x, y)$ is true & $len(y) \leq P_\ell(len(x))$



NP – all problems

P – class of all problems that can be solved in feasible time.

Is P = NP? <u>Open problem</u> – No one knows.

There are some problems in NP about which we know that they are as hard as possible.

Most CS folks believe that P ≠ NP.

Any problem from NP can be reduced to this problem.

SAT

NP-complete ≡ no way to have an algorithm that always solves all instances of this problem in feasible time

Pumping lemma for CFG:

$$\forall CFG\ \exists p\ \forall w \in L\ (len(w) \geq p \rightarrow \exists u, v, x, y, z\ (w = uvxyz \ \&\ len(vy) > 0 \ \&\ len(vxy) \leq p \ \&\ \forall i\ (uv^i x y^i z \in L)))$$

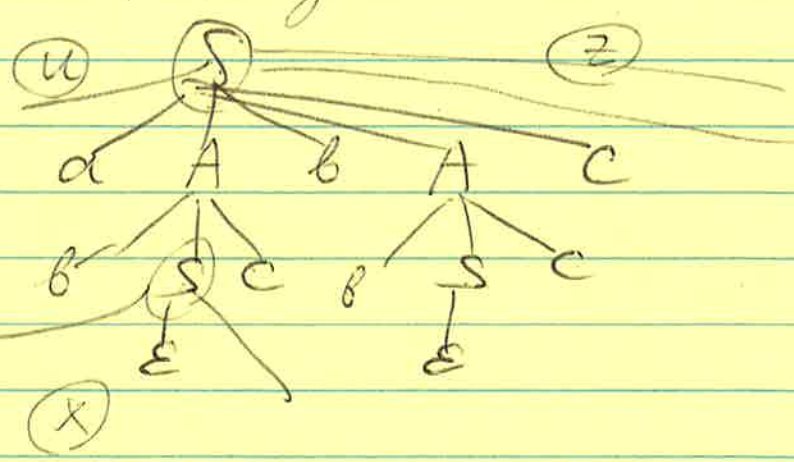
To use it, we find the lowest repetition of same variable on the same branch

- u – before 1st rep. to the left
- v – between 1 & 2 to the left
- x – below 2nd rep.
- y – between 1 and 2nd to the right
- z – before 1st to the right

$S \rightarrow aAbAc$

$A \rightarrow bSc$

$S \rightarrow \varepsilon$



$u = \varepsilon$

$v = ab$

$x = \varepsilon$

$y = cbbcc$

$z = \varepsilon$

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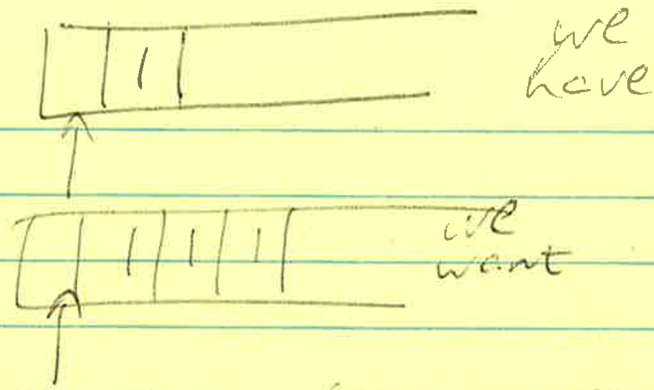
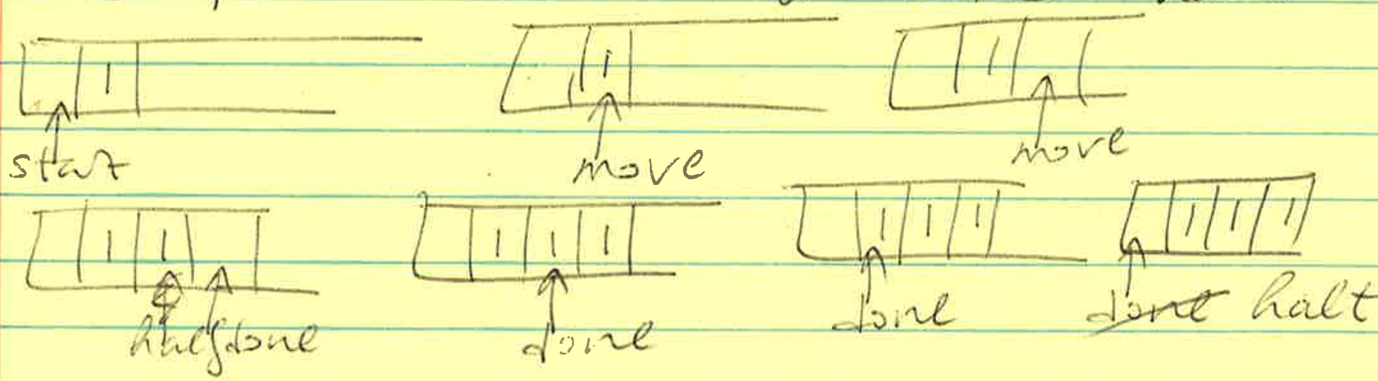


1. (start, $\sqcup$) $\longrightarrow$ (move, R)
2. (move, 1) $\longrightarrow$ R
3. (move, $\sqcup$) $\longrightarrow$ 1, R, halfdone
4. halfdone, $\sqcup$ $\longrightarrow$ 1, done, L
5. done, 1 $\longrightarrow$ L

6. done, $\sqcup$ $\longrightarrow$ halt



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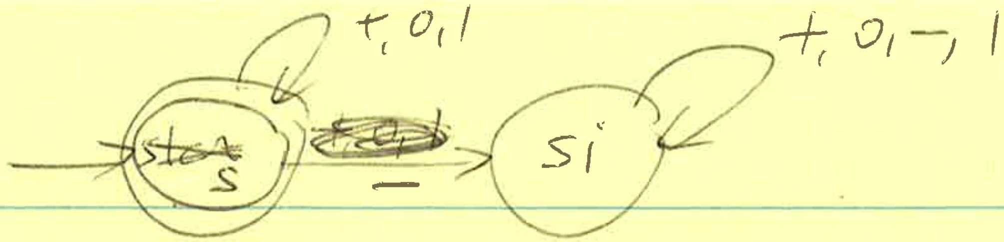
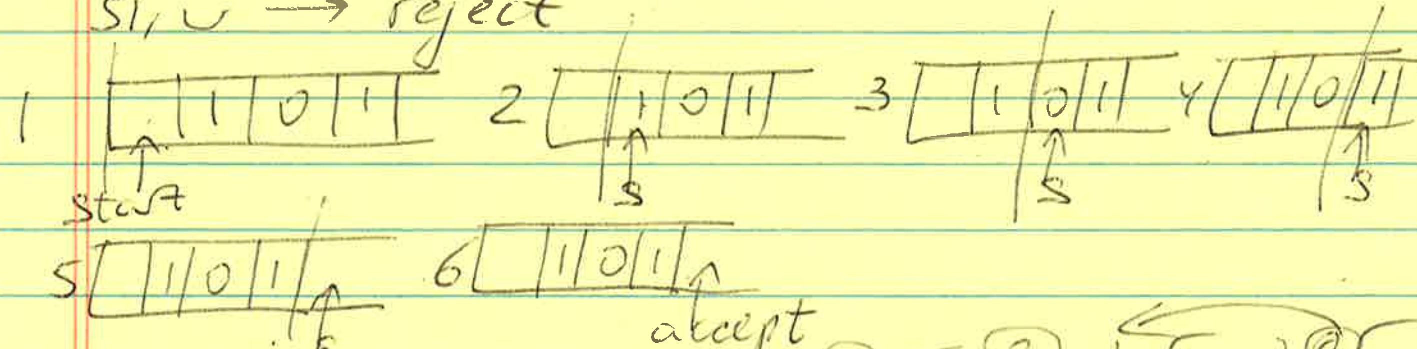


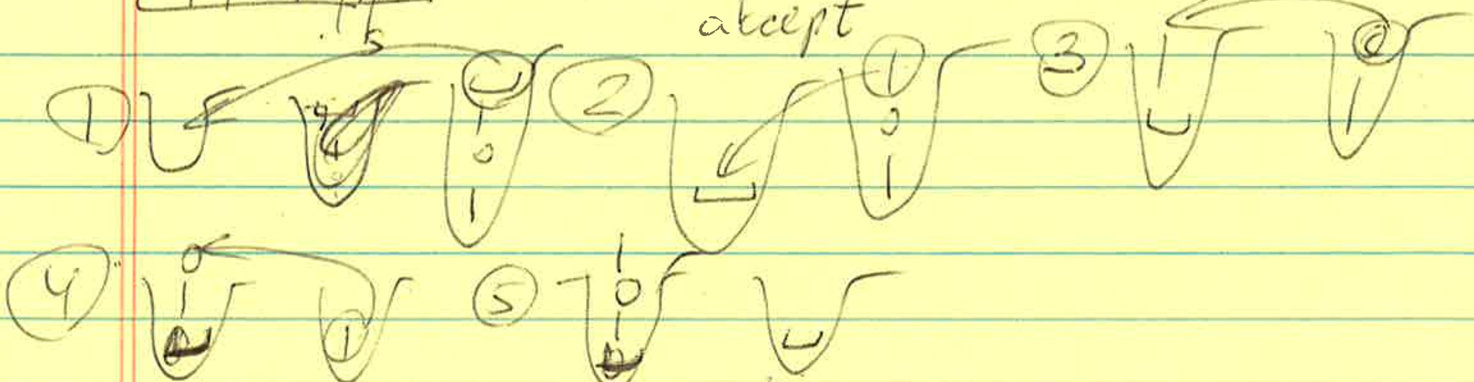
$\boxed{\text{start}, \sqcup \longrightarrow \cancel{si}\ s, R}$

s, + → s, R
s, 0 → s, R
s, 1 → s, R

s, − → si, R
si, + → si, R
si, 0 → si, R
si, − → si, R
si, 1 → si, R

s, ␣ → accept
si, ␣ → reject





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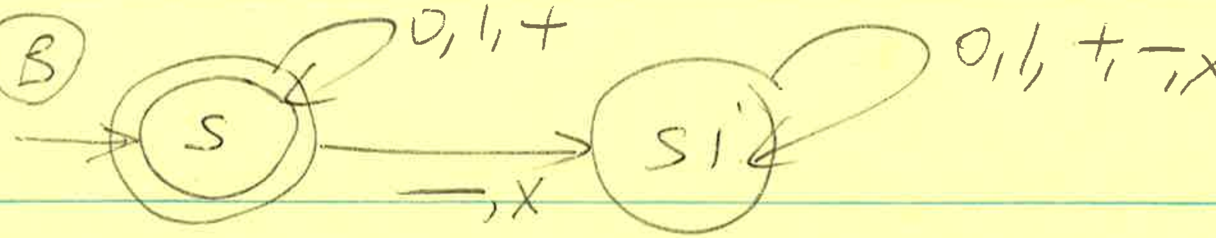
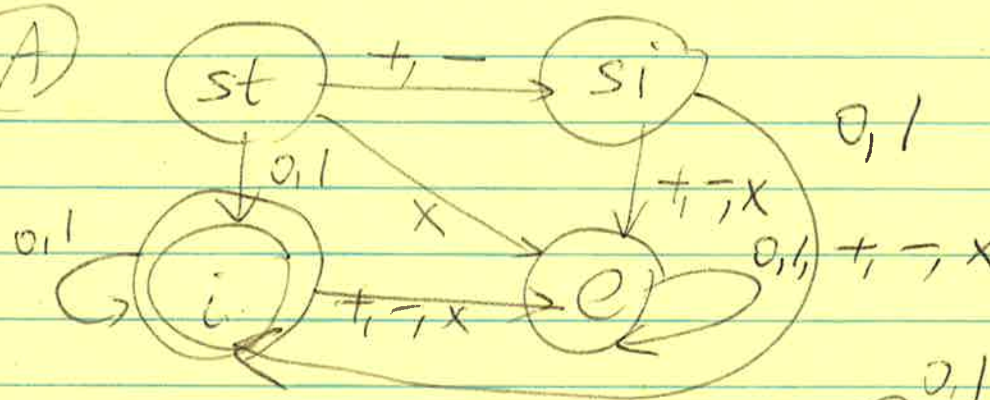
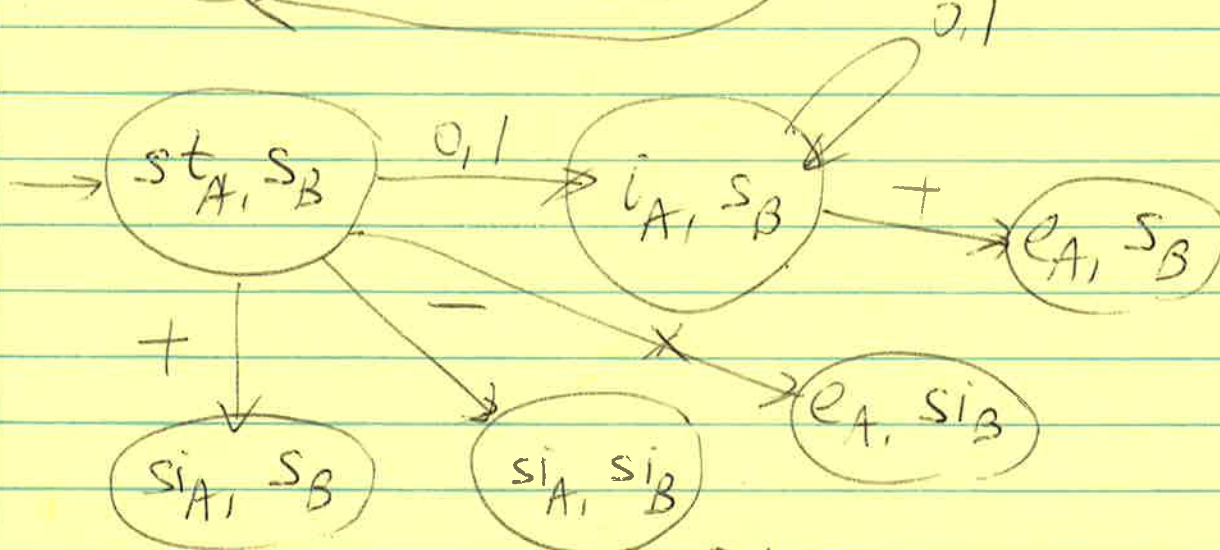
B

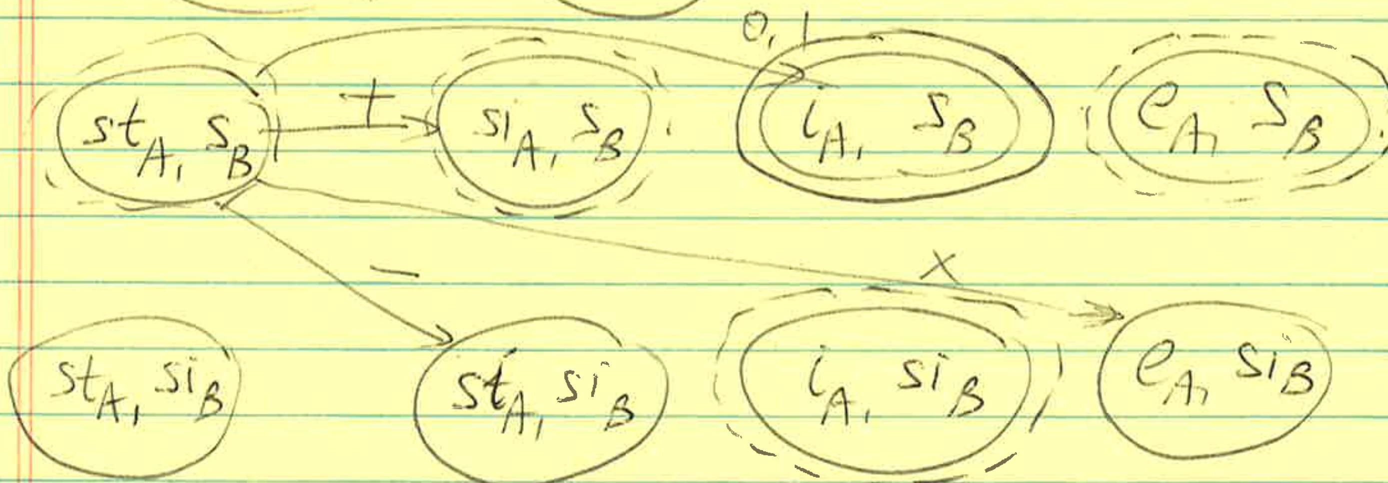


A



A∨B





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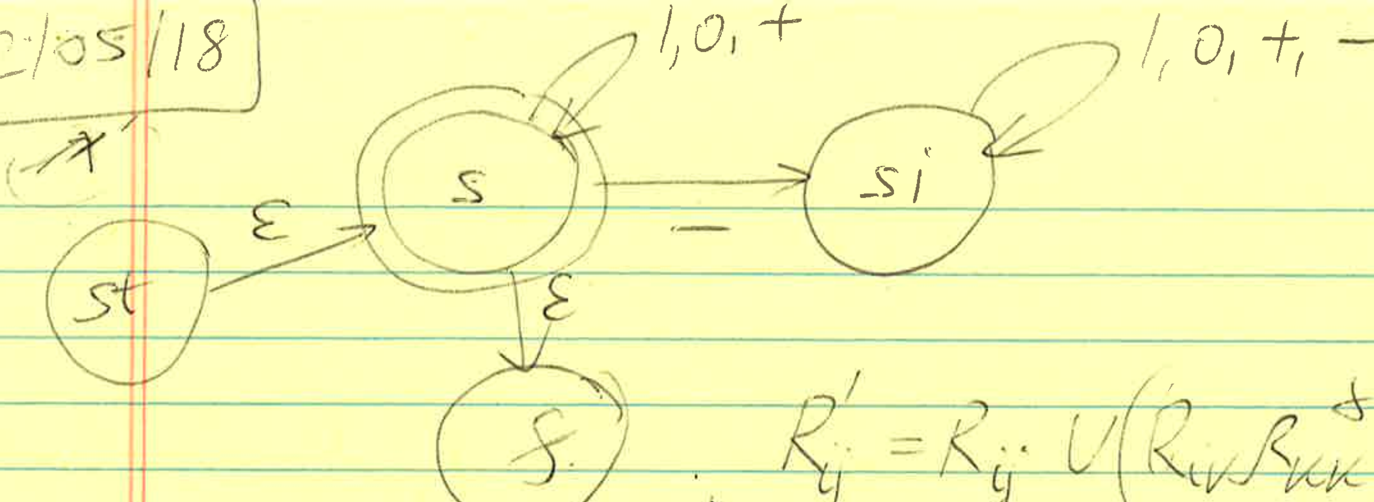
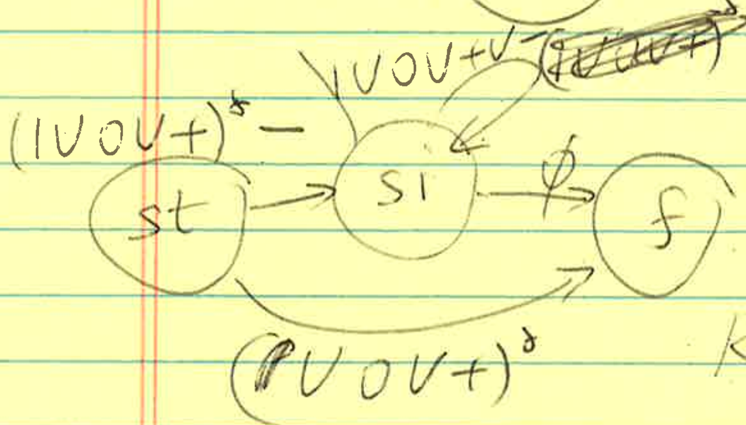


$$R'_{ij} = R_{ij} \cup (R_{ik} R_{kk}^* R_{kj})$$



$$R'_{st,si} = R_{st,si} \cup (R_{st,s} R_{s,s}^* R_{s,si})$$

$$\phi \qquad \Lambda(1 \cup 0 \cup +)^* -$$

$$R'_{st,f} = R_{st,f} \cup (R_{st,s} R_{s,s}^* R_{s,f})$$

$$\phi \cup \Lambda(1 \cup 0 \cup +)^* \Lambda$$

$$R'_{si,si} = R_{si,si} \cup (R_{si,s} R_{s,s}^* R_{s,si})$$

$$(1 \cup 0 \cup + \cup -) \cup (\phi \ \ldots$$

$$R'_{si,f} = R_{si,f} \cup (R_{si,s} R_{s,s}^* R_{s,f})$$

$$\phi \qquad \phi$$

$$R'_{st,f} = R_{st,f} \cup (R_{st,si} R_{si,si}^* R_{si,f})$$

$$(1 \cup 0 \cup +)^* \cup \underbrace{(\ldots \quad \ldots \quad \phi)}$$

$$\boxed{(1 \cup 0 \cup +)^*}$$