# Solution to Homework Problem 16

**Homework Problem 16.** Design a Turing machine that, given a positive unary number $n$, add 2 to this number. Test it, step-by-step, on the example of $n = 3$.

**Idea.** This is similar to adding 1 to a unary number. The only difference is that when we reach the first blank space and replace it with 1, we are not done yet: we have more more 1 left to add.

**Solution.** Here are the rules for the Turing machine:
start, – → R, moving
moving, 1 → R
moving, – → 1, R, oneLeft
oneLeft, – → 1, L, back
back, 1 → L
back, – → halt

**Tracing.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\underline{-}$ | 1 | 1 | 1 | – | – | – | ... | start |
| – | $\underline{1}$ | 1 | 1 | – | – | – | ... | moving |
| – | 1 | $\underline{1}$ | 1 | – | – | – | ... | moving |
| – | 1 | 1 | $\underline{1}$ | – | – | – | ... | moving |
| – | 1 | 1 | 1 | $\underline{-}$ | – | – | ... | moving |
| – | 1 | 1 | 1 | 1 | $\underline{-}$ | – | ... | oneMore |
| – | 1 | 1 | 1 | $\underline{1}$ | 1 | – | ... | back |
| – | 1 | 1 | $\underline{1}$ | 1 | 1 | – | ... | back |
| – | 1 | $\underline{1}$ | 1 | 1 | 1 | – | ... | back |
| – | $\underline{1}$ | 1 | 1 | 1 | 1 | – | ... | back |
| $\underline{-}$ | 1 | 1 | 1 | 1 | 1 | – | ... | back |
| $\underline{-}$ | 1 | 1 | 1 | 1 | 1 | – | ... | halt |