

Solution to Homework Problem 20

Homework Problem 20. As we discuss in class, a Turing machine can be described as a finite automata with two stacks:

- the right stack contains, on top, the symbol to which the head points; below is the next symbol to the right, then the next to next symbol to the right, etc.;
- the left stack contains, on top, the symbol directly to the left of the head (if there is a one), under it is the next symbol to the left, etc.

On the example a Turing machine that computes $n + 1$ for $n = 2$, show, step-by-step:

- how each state of the corresponding Turing machine can be represented in terms of two stacks, and
- how each transition from one state to another can be implemented by push and pop operations.

Solution.

1. At first, we have the following configuration:

-	1	1	-	-	...
---	---	---	---	---	-----

start

Here, the left stack is empty, and the right stack has the form

-
1
1

2. Then, the configuration changes to:

-	<u>1</u>	1	-	-	...
---	----------	---	---	---	-----

moving

Here, the two stacks have the following form:

-	1
	1

To get to this configuration, we pop the symbol $-$ (meaning black space) from the right stack and push it into the left stack.

3. Then, the configuration changes to:

-	1	<u>1</u>	-	-	...
---	---	----------	---	---	-----

 moving

Here, the stacks have the following form:

1
-

1

To get to this configuration, we pop 1 from the right stack and push it into the left stack.

4. Next, we have the following:

-	1	1	=	-	...
---	---	---	---	---	-----

 moving

Here, the stacks have the following form:

1
1
-

-

To get to this configuration, we pop 1 from the right stack and push it into the left stack.

5. Next, we have the following:

-	1	<u>1</u>	1	-	...
---	---	----------	---	---	-----

 back

Here, the stacks have the following form:

1
-

1
1

To get to this state, we:

- first, replaced the symbol - on top of the right stack with 1;
- then popped 1 from the left stack and pushed it into the right stack.

6. After this, we get the following configuration:

-	<u>1</u>	1	1	-	...
---	----------	---	---	---	-----

 back

Here, the stacks have the following form:

-

1
1
1

To get to this state, we popped 1 from the left stack and pushed it into the right stack.

7. Then, we get the following configuration:

=	1	1	1	-	...
---	---	---	---	---	-----

 back

Here, the left stack is empty, and the right stack has the following form:

–
1
1
1

To get to this state, we popped the symbol – from the left stack and pushed it into the right stack.

8. Finally, we get the configuration

–	1	1	1	–	...	halt
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In this configuration, the stacks remain the same as in the previous one.