Solution to Homework Problem 24

Homework Problem 24. Reproduce the proof of the halting problem in all the necessary detail.

Solution. We want to prove that no algorithm is possible that, given a program p and data d, checks whether p halts on d.

We will prove it by contradiction. Let us assume that there exists an algorithm – i.e., a Java program – halt-checker that:

- *given* two strings: a program p and data d,
- *returns* true if p halts on d, and false otherwise.

Let us now build the following auxiliary program:

```
public state int aux(String x){
  if (halt-checker(x,x))
    {while(True) x = x;}
  else {return 0;}}
```

Will this program aux halt if, as input, we give it the same string aux?

- If aux *halts* on aux, then, by definition of the halt-checker, the value

  halt-checker(aux,aux)

  is true. If you trace the above program aux, you will see that in this case, this program will go into an infinite loop – and thus, it will *not* halt.
- On the other hand, if aux *does not halt* on aux, then, by definition of the halt-checker, halt-checker(aux,aux) is false. If you trace the above program aux, you will see that in this case, this program *will halt* – namely, it will return 0.

In both case, we get a contradiction:

- if we assume that it halts, we conclude that it does not halt, and
- if we assume that it does not halt, then we conclude that it halts.

This contradiction shows that halt-checkers are indeed not possible.