Formal Notations for Describing a Finite Automaton

How the book describes an automaton. The book described an automaton as a tuple $\langle Q, \Sigma, \delta, q_0, F\rangle$, where:

- Q is the set of the states;
- Σ (sigma capital) is the *alphabet*, i.e., the set of all possible symbols;
- δ (delta) is a function that describes, for each state q and for each symbol b, what will be the next state $\delta(q, b)$ of the automaton;
- q_0 is the start state, and
- F is the set of final states.

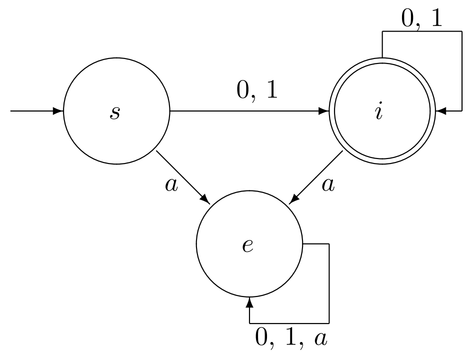
Why do we need such a description. This is easy to explain:

- we humans can easily draw pictures, we can easily understand pictures;
- however, for computers, pictures are a challenge.

So, if we want to implement an automaton as part of the compiler, we need to describe in a more formal way, easier to understand for a computer. Because of this:

- we need to learn how to describe a picture in these terms,
- and, vice versa, we need to learn how to draw a picture illustrating a formal definition.

Example: reminder. Let us consider the very first automaton with which we started:



Set of states. For this automaton, we have three states: s, i, and e, so the set of states is the set consisting of these three states:

$$Q = \{s, i, e\}.$$

Alphabet. For this automaton, we consider the situation when the input can have only three symbols: 0, 1, and a. Thus, here:

$$\Sigma = \{0, 1, a\}.$$

Starting state. The starting state here is the state s, so

$$q_0 = s.$$

The set of final states. For this automaton, we only have one final state, the state i. Thus, the set F of final states consists of only one element i:

$$F = \{i\}.$$

Important notice.

- It would be not correct to say that $F = i$: F is a set, it can have several elements – because there can be several different final states.
- It would also be not correct to say that $q_0 = \{s\}$. There is only one start state, there cannot be several start states – otherwise, we would not know where to start.

The function δ. In this case, if we are in the start state s and we see 1, we go to the state i. This means that $\delta(s, 1) = i$. Similarly:

$$\delta(s, 1) = i; \delta(s, 0) = i; \delta(s, a) = e;$$

$$\delta(i, 1) = i; \delta(i, 0) = i; \delta(i, a) = e;$$

$$\delta(e, 1) = e; \delta(e, 0) = e; \delta(e, a) = e.$$

This function can be naturally described as a table:

	1	0	a
s	i	i	e
i	i	i	e
e	e	e	e

Practice. Provide the formal description of other automata that we had so far.