

Solution to Homework 11

Task. In Homework 6, we considered the following pushdown automaton. This pushdown automaton has two states:

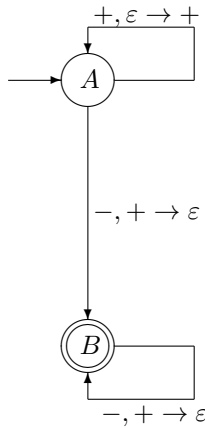
- the starting state A , and
- the final state B .

The transitions are as follows:

- From A to A , the transition is $+, \varepsilon \rightarrow +$;
- From A to B , the transition is: $-, + \rightarrow \varepsilon$;
- From B to B , the transition is $-, + \rightarrow \varepsilon$.

Use the general algorithm to transform this pushdown automaton into a context-free grammar. Show, step-by-step, how the resulting grammar will generate the word $++--$.

Solution. Our pushdown automaton has the following form:



Let us recall how the word

+ + --

is accepted by this automaton. We will list consequent states and the contents of the corresponding stacks, described from the top to bottom, and what symbols we see in the corresponding transitions:

- state A , stack is empty;

we read +;

- state A , stack has +;

we read +;

- state A , stack has ++;

we read -;

- state B , stack has +;

we read -;

- state B , stack is empty.

These transitions can be described as follows:

A	A	A	B	B
	+	+	+	

We start with the state A , we end up in the final state B . Thus, the first rule we apply is the rule $S \rightarrow A_{AB}$;



The first operation that we have is pushing the first + into the stack. This + is then popped at the very end. Let us list the two rules corresponding to pushing and popping:



In general, we have the two transitions



What do we need to plug in instead of p, q , etc. in the general 2-rule picture to come up with this particular picture:

- instead of p and q , we place A ;
- instead of r and s we place B ;
- instead of x and t , we place $+$; and
- instead of y , we place $-$.

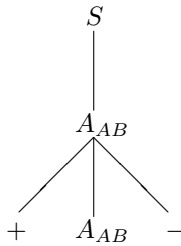
If we make these substitutions in the general rule:

$$A_{ps} \rightarrow xA_{qr}y$$

we get the rule

$$A_{AB} \rightarrow +A_{AB}-$$

Thus, the derivation so far takes the form:



We have covered the first transitions from A to A and the last transition from B to B , and we also covered the symbol $+$ that stays in the stack between the second occurrence of the state A and the first occurrence of the state B . Let us underline what we have covered already:

- state A , stack is empty;

we read $+$;

- state A , stack has $+$;

we read $+$;

- state A , stack has $++$;

we read $-$;

- state B , stack has $+$;

we read $-$;

- state B , stack is empty.

<u>A</u>	A	A	B	<u>B</u>
	<u>+</u>	<u>+</u>	<u>+</u>	

The first not-yet-covered state is the second occurrence of the state A . The first thing we do when in this state is we push the symbol $+$ into the stack. This symbol $+$ is popped at the transition from state A to state B . Here, we have the following rules:



In general, we have the two transitions



What do we need to plug in instead of p , q , etc. in the general 2-rule picture to come up with this particular picture:

- instead of p , q , and r , we place A ;
- instead of s , we place B ;
- instead of x and t , we place $+$; and
- instead of y , we place $-$.

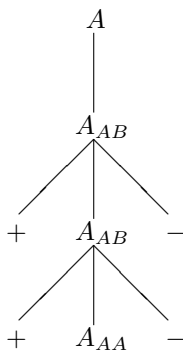
If we make these substitutions in the general rule:

$$A_{ps} \rightarrow xA_{qr}y$$

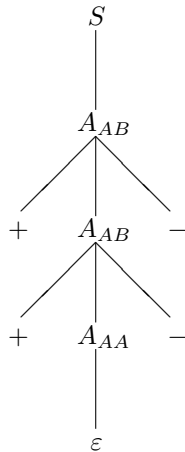
we get the rule

$$A_{AB} \rightarrow +A_{AA} - .$$

Thus, the derivation takes the form:



Now, we have covered all the transitions, so the only thing left to do is to use the trivial rule $A_{AA} \rightarrow \varepsilon$:



This is the desired derivation of the word $++--$ in the resulting grammar.

Which rules were used in this derivation? As we have mentioned, the full grammar has too many rules to be listed, but we can at least list the rules used in this derivation:

$$S \rightarrow A_{AB}, \quad A_{AB} \rightarrow +A_{AB}-, \quad A_{AB} \rightarrow +A_{AA}-, \quad A_{AA} \rightarrow \varepsilon.$$