

Solution to Homework Problem 16

Homework Problem 16. Design a Turing machine that, given a positive unary number n , add 2 to this number. Test it, step-by-step, on the example of $n = 2$.

Idea. This is similar to adding 1 to a unary number. The only difference is that when we reach the first blank space and replace it with 1, we are not done yet: we have one more 1 to add.

Solution. Here are the rules for the Turing machine:

start, $- \rightarrow R$, moving

moving, $1 \rightarrow R$

moving, $- \rightarrow 1$, R, adding2nd1

adding2nd1, $- \rightarrow 1$, L, back

back, $1 \rightarrow L$

back, $- \rightarrow \text{halt}$

Tracing.

_	1	1	-	-	-	...	start
-	<u>1</u>	1	-	-	-	...	moving
-	1	<u>1</u>	-	-	-	...	moving
-	1	1	_	-	-	...	moving
-	1	1	1	_	-	...	adding2nd1
-	1	1	<u>1</u>	1	-	...	back
-	1	<u>1</u>	1	1	-	...	back
-	<u>1</u>	1	1	1	-	...	back
_	1	1	1	1	-	...	back
_	1	1	1	1	-	...	halt