

Solution to Homework Problem 19

Homework Problem 19. As we discuss in class, a Turing machine can be described as a finite automata with two stacks:

- the right stack contains, on top, the symbol to which the head points; below is the next symbol to the right, then the next to next symbol to the right, etc.;
- the left stack contains, on top, the symbol directly to the left of the head (if there is a one), under it is the next symbol to the left, etc.

On the example a Turing machine that computes $n - 1$ for a binary number $n = 4$, show, step-by-step:

- how each state of the corresponding Turing machine can be represented in terms of two stacks, and
- how each transition from one state to another can be implemented by push and pop operations.

Solution.

1. At first, we have the following configuration:

-	0	0	1	-	-	...	start
---	---	---	---	---	---	-----	-------

Here, the left stack is empty, and the right stack has the form

-
0
0
1

2. Then, the configuration changes to:

-	<u>0</u>	0	1	-	-	...	moving
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Here, the two stacks have the following form:

-	0
	0
	1

To get to this configuration, we pop the symbol $-$ (meaning black space) from the right stack and push it into the left stack.

3. Then, the configuration changes to:

-	1	<u>0</u>	1	-	-	...
---	---	----------	---	---	---	-----

 moving

Here, the left stack has the following form:

1
-

and the right stack has the following form:

0
1

To get to this configuration, we replace 0 with 1, pop 1 from the right stack and push it into the left stack.

4. Then, the configuration changes to:

-	1	1	<u>1</u>	-	-	...
---	---	---	----------	---	---	-----

 moving

Here, the left stack has the following form:

1
1
-

and the right stack has the following form:

1

To get to this configuration, we replace 0 with 1, pop 1 from the right stack and push it into the left stack.

5. Then, the configuration changes to:

-	1	<u>1</u>	0	-	-	...
---	---	----------	---	---	---	-----

 back

Here, the left stack has the following form:

1
-

and the right stack has the following form:

1
0

To get to this configuration, we replace 1 with 0, pop 1 from the left stack and push it into the right stack.

6. Then, the configuration changes to:

-	<u>1</u>	1	0	-	-	...
---	----------	---	---	---	---	-----

 back

Here, the left stack has the following form:

-

and the right stack has the following form:

1
1
0

To get to this configuration, we pop 1 from the left stack and push it into the right stack.

7. Then, the configuration changes to:

-	1	1	0	-	-	...
---	---	---	---	---	---	-----

back

Here, the left stack is empty, and the right stack has the following form:

-
1
1
0

To get to this configuration, we pop - from the left stack and push it into the right stack.

8. Then, the configuration changes to:

-	1	1	0	-	-	...
---	---	---	---	---	---	-----

halt

Here, the contents of the tape did not change, and the location of the head did not change, so the stacks remain the same.