Solutions to Homework 2

Task 2.1. Use the general algorithm that we learned in class to design a non-deterministic finite automaton that recognizes the language $(+ \cup -)(0 \cup 1)^*$.

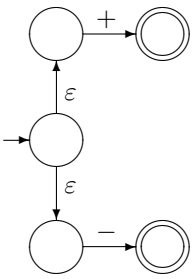
Reminder:

- 0 is a language consisting of only one 1-symbol word 0;
- AB means concatenation.

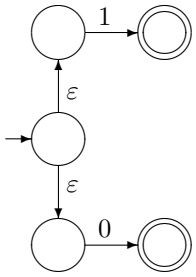
Solution. We start with the standard non-deterministic automata for recognizing:

- the language $+$ – that consists of a single word $+$, and
- the language $-$ – that consists of a single word $-$:

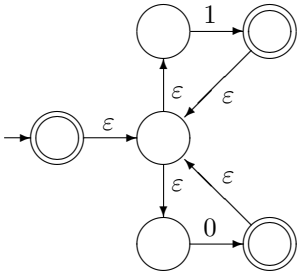
Then, we use the general algorithm for the union to design a non-deterministic automaton for recognizing the language $+ \cup -$:



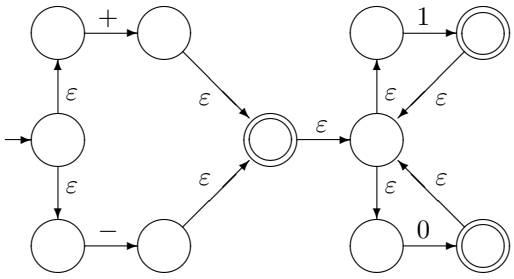
Similarly, we get a non-deterministic automaton for recognizing the language $0 \cup 1$:



Now, we apply a standard algorithm for the Kleene star, and we get the following non-deterministic automaton for $(0 \cup 1)^*$:

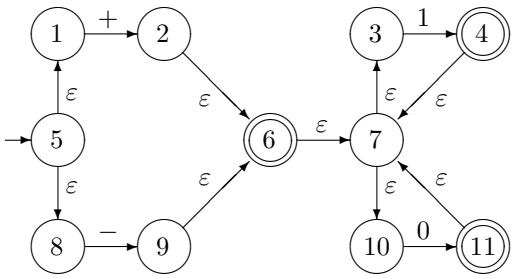


Now, we use the algorithm for concatenation for combine them: final states of the automaton for $+ \cup -$ are no longer final, and from each of them, we add a jump to the starting state of the automaton for $(0 \cup 1)^*$:



Task 2.2. Transform the resulting non-deterministic finite automaton into a deterministic one.

Solution. Let us first enumerate the states of the resulting non-deterministic automaton.



In the beginning, before we see any symbol, we are in state 5, and we can also jump to states 1 and 8. Thus, before we see any symbols, we can be in one of the states 1, 5, and 8. This set $\{1, 5, 8\}$ is thus the starting state of the desired deterministic finite automaton. Checking where we can go from this state and from the resulting states when we see one of the symbols $+$, $-$, 0, or 1, we arrive at the following deterministic automaton.